Notes on antenna polarization and VLBI observables

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The purpose of this note is to show explicitly the dependence of the cross-correlation products on source polarization and feed cross-polarization, for feeds that are nominally either linearly or circularly polarized, and to propose linear combinations of the linear-polarization products with some advantages and disadvantages.

Preliminaries

The relationship between the electric field incident on an antenna and the input voltage to the A/D converter may be viewed as the product of two complex gain terms: (1) the conversion factor between the incident field and the voltage induced by it in the feed, and (2) the voltage gain from the feed to the A/D. The product of these two terms will be referred to herein as the complex gain G, which is a function of frequency and polarization channel. In general, the feed voltage includes contributions from both the desired polarization and the orthogonal polarization.

The cross-correlation products will be expressed in terms of the Stokes parameters I, Q, U, and V of the incident field. For their definitions, see, e.g., Thompson, Moran, and Swenson (2001) or Kraus (1966). I represents the total intensity, Q and U define the linearly polarized components, and V measures the circularly polarized component.

Two effects can cause the plane of polarization of a linearly polarized field from a celestial source to rotate relative to an antenna feed: (1) rotation of the feed itself (for an alt-az mount, the angle of rotation is the parallactic angle), and (2) ionospheric Faraday rotation, which can amount to of order 10° at S-band. In these notes, ϕ refers to the sum of these two angles, Δ to the difference angle $\phi_1 - \phi_2$ between two stations, and Σ to the sum $\phi_1 + \phi_2$.

Circularly polarized feeds

VLBI polarimetry with circularly polarized feeds has been treated extensively in the literature. See, e.g., Roberts, Wardle, and Brown (1994) and Kemball (1999), and references therein.

Current standard practice in geodetic VLBI is to observe with feeds nominally sensitive to RCP fields inicident on the antenna. The output voltage from such a feed may be expressed as

$$V_R \;=\; G_R\,(E_R\,e^{-i\phi} + D_R\,E_L\,e^{i\phi}) \tag{1}$$

where D_R represents the complex fractional response to the undesired (LCP) polarization.

The RCP-RCP cross-correlation product $R_1 R_2^*$, which is the time average $\langle V_{R1} V_{R2}^* \rangle$, is then

$$\begin{aligned} R_1 R_2^* / G_{R1} G_{R2}^* \;=\; & (I+V)\,e^{-i\Delta} \;+\; D_{R1}\,D_{R2}^*\,(I-V)\,e^{i\Delta} \\ & + D_{R1}\,(Q-iU)\,e^{i\Sigma} \;+\; D_{R2}^*\,(Q+iU)\,e^{-i\Sigma} \end{aligned} \tag{2}$$

The synchrotron sources typically observed in geodetic VLBI are circuarly polarized by no more than a few tenths of a percent, so the V term may be neglected. With this simplication, the response, normalized to the response to a purely RCP source, is:

$$\frac{R_1 R_2^*}{R_1 R_2^* \,(\mathrm{RCP\ \ source})} \;=\; 1 \;+\; D_{R1}\, D_{R2}^*\, e^{2i\Delta} \;+\; D_{R1} \frac{Q-iU}{I}\, e^{2i\phi_1} \;+\; D_{R2}^* \frac{Q+iU}{I}\, e^{-2i\phi_2} \quad (3)$$

Linear polarization is typically a few percent, and at most ∼10%. The magnitudes of the D terms at S/X on geodetic antennas are typically between 0.05 and 0.15 (–26 to –16 dB), with the VLBA antennas among the better performers (Corey and Titus 2006). From equation (3), the fringe phase error caused by a source with 10% linear polarization observed by an antenna with a –20 dB D term can be as large as ∼0.01 radian; the corresponding delay error across 500 MHz is 3 ps. Two antennas with –20 dB D terms can cause an error of similar magnitude on an unpolarized source.

Linearly polarized feeds

While many connected-element arrays employ linear feeds (e.g., Westerbork, ACTA, ATA), I am unaware of any VLBI polarimetry having been done with linear feeds, and there is a dearth of literature on the subject.

Let $v(t,\nu)$ be the component of the E-field from the source that is linearly polarized in the North-South (vertical) direction, before entering the ionosphere, and let $h(t,\nu)$ be the orthogonally polarized (horizontal) component.

Let x and y be the feed output voltages for the polarization channels most sensitive to v and h, respectively, when the feed rotation angle ϕ is zero. For a perfect feed with no cross-polarization,

$$x = G_x(v\cos\phi + h\sin\phi) \quad (4)$$
$$y = G_y(-v\sin\phi + h\cos\phi) \quad (5)$$

For an imperfect feed, each output is a linear combination of the perfect x and y responses:

$$x' = x + D_x y \quad (6)$$
$$y' = y + D_y x \quad (7)$$

The four cross-correlation products can be evaluated in their full generality from the above equations, but for our purposes the following expressions, calculated to first order in the D terms and in the polarization Stokes parameters Q, U, and V, will suffice:

$$2\,\langle x_1' x_2'^* \rangle \,/\, G_{x1}G_{x2}^* = +I\cos\Delta + Q\cos\Sigma + U\sin\Sigma - iV\sin\Delta + I(-D_{x1} + D_{x2}^*)\sin\Delta \quad (8)$$
$$2\,\langle y_1' y_2'^* \rangle \,/\, G_{y1}G_{y2}^* = +I\cos\Delta - Q\cos\Sigma - U\sin\Sigma - iV\sin\Delta + I(+D_{y1} - D_{y2}^*)\sin\Delta \quad (9)$$
$$2\,\langle x_1' y_2'^* \rangle \,/\, G_{x1}G_{y2}^* = +I\sin\Delta - Q\sin\Sigma + U\cos\Sigma + iV\cos\Delta + I(+D_{x1} + D_{y2}^*)\cos\Delta \quad (10)$$
$$2\,\langle y_1' x_2'^* \rangle \,/\, G_{y1}G_{x2}^* = -I\sin\Delta - Q\sin\Sigma + U\cos\Sigma - iV\cos\Delta + I(+D_{y1} + D_{x2}^*)\cos\Delta \quad (11)$$

Note that each product involves all four Stokes parameters as well as a linear combination (different in each case) of two D terms. Furthermore, as is obvious without going through all the math, the fringe power from the dominant I term passes from the parallel-hands products to the cross-hands as the difference angle Δ goes from zero to 90°. Therefore no one single product can be relied upon always to give strong fringes (unless we schedule observations only over restricted ranges of Δ, which seems like a *very* bad idea).

One possibility for creating an observable whose magnitude is much less dependent on Δ is to construct pseudo-circular observables from the four products. For instance, RCP-RCP and LCP-LCP products can be formed as

$$\begin{aligned} RR &\equiv \langle\, (x_1'/G_{x1} + iy_1'/G_{y1})\, (x_2'/G_{x2} + iy_2'/G_{y2})^*\, \rangle & (12) \\ &= [\, (I+V) + \frac{i}{2} I(-D_{x1} + D_{y1} + D_{x2}^* - D_{y2}^*)\,]\, e^{-i\Delta} & (13) \\ LL &\equiv \langle\, (x_1'/G_{x1} - iy_1'/G_{y1})\, (x_2'/G_{x2} - iy_2'/G_{y2})^*\, \rangle & (14) \\ &= [\, (I-V) - \frac{i}{2} I(-D_{x1} + D_{y1} + D_{x2}^* - D_{y2}^*)\,]\, e^{+i\Delta} & (15) \end{aligned}$$

Unlike the case with circularly polarized feeds, where the D terms enter only in second order, here they appear in first order. For a single –20 dB D term, the resulting fringe phase error can be as large as 0.05 radian. But the D terms are typically smaller for linear feeds than for circular, especially for broadband feeds. Furthermore, to the extent the D_x and D_y terms for a given feed are the same, cancellation between the D terms will reduce their effect. The most important fact about the D term contribution in equations 13 and 15 is that their Δ-dependence is the same as for the $I+V$ term, so the systematic error caused by the D terms should be constant from scan to scan.

There remains one overriding problem with the pseudo-circular products, however: their construction requires knowledge of the complex gains. It is not necessary to know the absolute gains, but only the relative gains between the polarization channels at each station, as can be seen by recasting the definition of RR as

$$RR = \langle \frac{1}{G_{x1} G_{x2}^*} (x_1' + i\, y_1'\, g_1\, e^{i\psi_1})(x_2' + i\, y_2'\, g_2\, e^{i\psi_2})^* \rangle \qquad (16)$$

where $g = |G_x/G_y|$ and $\psi = \arg(G_x/G_y)$. But even the task of determining just the relative gain is nontrivial. In VLBI polarimetry with circular feeds, the relative gains (and D terms) are routinely determined from the cross-correlation products themselves. But certain simplifying assumptions that can be made with circular feeds cannot be made with linear feeds, and the four linear products (equations 12-15) are more tightly coupled together than are the circular products. Whether gain and D term estimation from linear-feed data is even feasible is an open question, according to Bill Cotton (priv. comm.); he is, however, optimistic about the answer. An alternative may be to estimate the relative gain magnitude from the fringe amplitudes on weakly polarized sources, and to use the noise or phase cal signals to monitor gain variations at other times. A phase cal signal radiated into the feed could be used to measure the relative phase between the two channels. If the D terms need to be estimated, they can be measured using a linearly polarized test signal transmitted at the far-field distance, which is 14 km at 15 GHz for a 12m-diameter dish.

If correlating all four cross products is deemed too heavy a load on the correlator, another option is to construct mixed linear-circular observables, which use only two products, such as

$$\begin{aligned} XR &\equiv \langle\, (x_1'/G_{x1})\, (x_2'/G_{x2} + iy_2'/G_{y2})^*\, \rangle & (17) \\ &= (I + V - iD_{x1})\, e^{-i\Delta} + (Q - iU)\, e^{i\Sigma} + I\, (D_{x2}^* \sin\Delta - D_{y2}^* \cos\Delta) & (18) \end{aligned}$$

A disadvantage of this observable is its dependence on the linear-polarization Stokes parameters. There will also be misclosures around station triangles, as one station will have to be correlated differently on the two baselines, e.g., X_1R_2, X_1R_3, and X_2R_3.

I have not yet done an SNR analysis for these pseudo-circular observables to see whether the SNR differs from the circular-feed case.

Mixed mode: linear feed correlated against circular feed

Someday, maybe.