Dependency of Satellite Geodesy on UT1-UTC from VLBI

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Abstract Satellite-based techniques need to know UT1 when Earth rotation parameters and orbital elements shall be estimated in a global solution because of the correlation between the ascending nodes of the satellite orbits with the UT1 parameter. In this context the satellite techniques depend on the UT1 results from VLBI. On the other hand, the estimated satellite orbits first established in inertial space are transformed back into the Earth-fixed frame using the estimated Earth rotation parameters. In this sense the quasi-inertial frame for the satellite orbit determination can also be seen as an intermediate frame for a particular solution.

The largest acceleration acting on the satellite, except for the gravitational attraction of the Earth, is the gravitational attraction by the Moon and the Sun. The positions of these celestial bodies are given in the inertial frame. As the satellites are usually observed from sites on the Earth's surface, the transformation from the terrestrial to the celestial reference system is required, including the correct UT1 value, in order to calculate the gravitational accelerations from Moon and Sun to the satellite orbit. Because of the large distances of the masses from the satellite, an error in UT1 of 10 ms can be accepted to limit the effect on the satellite orbit to a value smaller than 1 cm, also for longer satellite arcs containing several revolutions.

Keywords UT1, Satellite techniques, Satellite orbit modeling

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1 Introduction

The coordinates of the ground stations used for satellite technique solutions are normally expressed in an Earth-fixed, geocentric coordinate system, as defined by the ITRS (International Terrestrial Reference System), realized in the series of International Terrestrial Reference Frames (ITRF). This coordinate system is rotating with the Earth in space, which is why it is not an inertial system, as required for modeling the satellite motion. As the satellite orbits are primarily affected by interactions with the Earth's body, an Earth-centered coordinate system not rotating with the Earth but moving with the Earth around the Sun is best suited for satellite orbit modeling. Such a quasi-inertial system is the Geocentric Celestial Reference System (GCRS) formally realized by a GCRF (Geocentric Celestial Reference Frame).

All satellite techniques measure certain relations between the satellite positions and locations on the Earth's surface (e.g., the distance between a ground station and a satellite or the distance from the satellite to the ocean or ice surface) with different technologies. Therefore, all satellite-geodetic techniques have to take into account the relation between the two systems. When the elements on the Earth surface are expressed in the ITRF and the satellite position is available in the GCRF from orbit modeling, the transformation between both systems is necessary to process the satellite-based measurements related to the Earth surface. This transformation consists of the Earth orientation parameters as described in Chapter 5 of the IERS Conventions ([1], where IERS stands for International Earth Rotation and Reference Systems Service).

As all Earth-related information is expressed in the ITRF, it makes sense to transform the satellite positions at the epoch of the measurements (or with a regular sampling allowing for a numerical interpolation) from the GCRF into the ITRF as well. It is an advantage of this approach that only the ITRF needs to be maintained with regular realizations as ITRFxx and the GCRF can be kept as an intermediate system realized for satellite orbit modeling in the solutions using observations from a satellite-based technique.

In order to perform the transformation from the ITRF into GCRF (and vice versa) according to the IERS Conventions, the satellite techniques need in particular the UT1 values from VLBI (Very Long Baseline Interferometry). Subsequently we discuss the impact of missing or degraded UT1 values on satellite-geodetic analyses.

2 Transformation between ITRF and GCRF

The ITRS as well as the GCRS are both Cartesian, rectangular coordinate systems. The principal plane of the GCRS refers to the instantaneous rotation axis of the Earth, whereas the ITRS is defined by a long-term mean location of the rotation axis. The major difference is the definition of the coordinate axes within this plane: for the GCRS it is the vernal equinox and for the ITRS the zero-meridian pointing towards one particular meridian on the Earth surface, e.g., to the Greenwich meridian. The principle is illustrated by Figure 1.

UT1 is the central element to define the relation between the axes of the ITRS and GCRS. An error in UT1 results in a misorientation of the GCRF for the orbit computation in the quasi-inertial space with respect to the ITRF. The ascending node of the orbital plane is computed with respect to the X-axis of the GCRF and is therefore contaminated by such a misorientation of the coordinate axes. When using the erroneous UT1 value together with the corresponding contaminated angle for the ascending node, the orbital plane will keep the same orientation with respect to the Earth surface. The analysis of the satellite measurements related to the Earth-fixed frame is not affected.

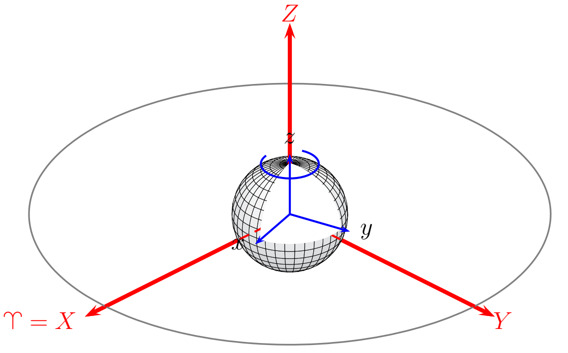


Fig. 1 Principle of ITRS (blue system) and GCRS (red system).

From a pure mathematical point of view, the two coordinate systems can be transformed with three rotations around each of the axes (Eulerian angles). The representation with polar motion, Earth rotation, precession, and nutation is a conventional one by considering the individual effects contributing to the physics of Earth rotation. When not following the convention but just estimating three angles instead, the representation of the satellite orbit with respect to the Earth-fixed frame is still possible—but the related rotation angles cannot be physically interpreted anymore if the error in UT1 becomes too large (e.g., hours).

Also, with a non-conventional realization of the GCRF, satellite measurements can be processed and used for the conventional ITRF realizations. Accelerations acting on the satellite and based on Earth-fixed models (e.g., the Earth gravity field) can also be transformed into a misaligned GCRF with the same effect on the satellite trajectory. In this sense a degraded UT1 series does not harm the satellite techniques.

The only remaining problems with a non-conventional realization of the GCRS are corrections to the Earth rotation parameters for effects with a higher resolution in time than that used for the estimation of the polar motion and Earth rotation angles. While a technique like Satellite Laser Ranging (SLR) is estimating not more than one offset per component and day, other techniques like GNSS (Global Navigation Satellite Systems) allow even for a higher resolution of up to 15 minutes [2].

As long as the UT1 errors are small (e.g., below a second) high frequency polar motion corrections can still be used without limitations. At this magnitude, UT1 errors do not harm the self-consistent use of Earth-fixed and quasi-inertial frames for analyzing satellite data.

3 Celestial Bodies other than the Earth

A degraded UT1 value may have an effect on the position of celestial bodies other than the Earth. Sun, Moon, and the most massive planets of the solar system are taken into account when analyzing measurements from satellite-geodetic techniques. The orders of magnitude of the most important accelerations acting on an artificial Earth satellite are listed in Table 1. The effects are discussed in detail in this section.

3.1 Solid Earth Tides for the Ground Stations

To compute the displacements of the ground stations due to solid Earth tides according to [1], the positions of the Sun and the Moon are needed. They are available in the ICRF, the inertial frame where the Earth (together with the satellites) is moving around the Sun. In order to obtain the positions of the celestial bodies in the Earth-fixed frame to compute the corrections for the station displacement, the transformation using the conventional definition of the Earth's orientation in space is needed. Otherwise the positions of the Sun and Moon are slightly misplaced.

The most important term in this context is UT1. An error in UT1 corresponds to an epoch error when the effect is computed. As a rough guess, we assume that the peak-to-peak deformation is about 40 cm with two minima and maxima per day — six hours for the transition from the minimum to maximum deformation. This results in a velocity of the ground station when following this deformation of $\frac{0.4}{6}\,\frac{\mathrm{m}}{\mathrm{h}} = \frac{400}{360}\,\frac{\mathrm{mm}}{\mathrm{min}} \approx 1\,\frac{\mathrm{mm}}{\mathrm{min}}$. This order of magnitude implies that even with an error of one minute in UT1 the deformations for ground stations due to Solid Earth tides can be computed with millimeter accuracy.

Table 1 Perturbing accelerations acting on a GPS satellite [3].

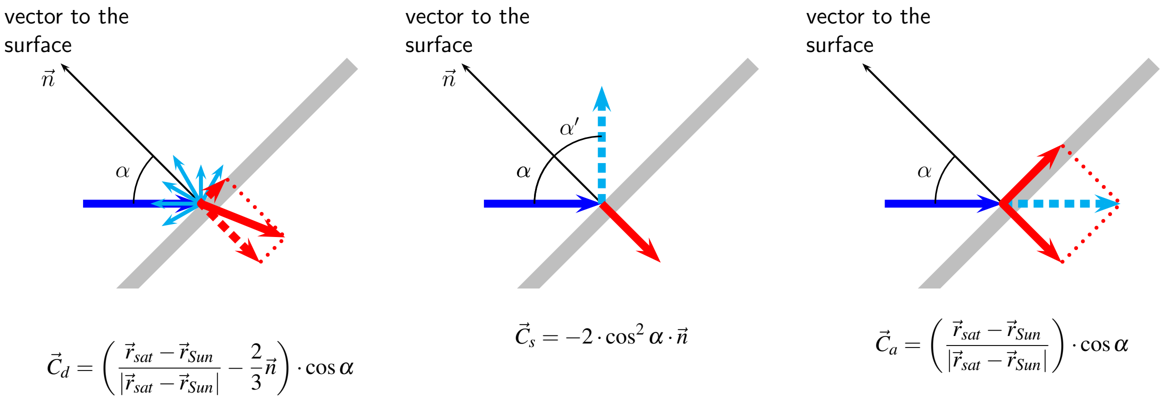
Perturbation	Acceleration $\mathrm{m/s^2}$	Orbit Error after one day (m)
Two–Body Term of Earth's Gravity Field	0.59	∞
Oblateness of the Earth	$5 \cdot 10^{-5}$	$10,000$
Lunar Gravitational Attraction	$5 \cdot 10^{-6}$	$3,000$
Solar Gravitational Attraction	$2 \cdot 10^{-6}$	800
Other Terms of Earth's Gravity Field	$3 \cdot 10^{-7}$	200
Radiation Pressure (direct)	$9 \cdot 10^{-8}$	200
Y–Bias	$5 \cdot 10^{-10}$	2
Solid Earth Tides	$1 \cdot 10^{-9}$	0.3

3.2 Solar Radiation Pressure Modeling

Solar radiation pressure is an essential component for orbit modeling of many satellite types emerging from the interaction of solar radiation with the satellite surface. The direction towards the Sun from the illuminated surface area $\vec{r}_{sat} - \vec{r}_{Sun}$ with an intersection angle α are the input features for the computation of these accelerations (see Figure 2). Which of the effects matter for a given surface element depends on the optical properties. The impact on the orbit depends finally on the area of the surface elements and the mass of the satellite [4]. The largest uncertainty emerges from the level of detail in the decomposition of the satellite structure and the use of the correct inflight properties.

The location of the Sun with respect to the satellite is needed to compute the orientation of the particles interacting with a satellite surface. It appears in the vector difference $\vec{r}_{sat} - \vec{r}_{Sun}$ and the intersection angle α as indicated in Figure 2. As α is the angle between the normal vector $\vec{n}$ of the plane and the unit vector pointing from the satellite towards the Sun $\vec{r}_{Sun}$, we need only the angle α to assess the impact of a wrong location of the Sun when computing the effect of solar radiation. In the worst case the misorientation of the Sun resulting from a wrong UT1 value fully coincides with the intersection angle. The angle α appears in all formulas within a cos- or $\cos^2$-function. A 1% error in the $\cos\alpha$-term corresponds to 0.5 degrees (or two minutes in time). The uncertainty of the knowledge of the optical properties of the satellites after a certain time in orbit is usually more important.

Because GNSS satellites do orient their solar panels towards the Sun, the empirical parameters are typically also expressed in a Sun-oriented coordinate system at the satellite [5]. A displacement of the Sun as described above would also result in a misorientation of the principal axes of this coordinate system. As long as these are only fractions of a degree (as assumed above), the resulting discrepancy between the force acting on the satellite and the representation in the empirical orbit model is absorbed by the estimated orbit parameters.



$$\vec{C}_d = \left(\frac{\vec{r}_{sat} - \vec{r}_{Sun}}{|\vec{r}_{sat} - \vec{r}_{Sun}|} - \frac{2}{3}\vec{n} \right) \cdot \cos\alpha$$

$$\vec{C}_s = -2 \cdot \cos^2\alpha \cdot \vec{n}$$

$$\vec{C}_a = \left(\frac{\vec{r}_{sat} - \vec{r}_{Sun}}{|\vec{r}_{sat} - \vec{r}_{Sun}|} \right) \cdot \cos\alpha$$

(a) Diffuse reflection (b) Specular reflection (c) Absorption

Fig. 2 Different versions of direct solar radiation pressure.

Other effects where the position of the Sun is needed in the context of solar radiation pressure modeling are the eclipse enter and exit epochs, when it is assumed that the radiation pressure due to the Sun is switched on or off. Assuming the worst constellation between Sun, Earth (or Moon), and satellite, the epoch is wrong by the full UT1 error. As long as the UT1 error is below one minute, this inconsistency can be absorbed by the empirical parameters of the Solar radiation pressure model.

3.3 Gravitational Force of the Sun and Moon

The gravitational attraction of the Earth is the most important component to keep a satellite on an orbit around the Earth. The oblateness of the Earth is causing an acceleration of $5 \cdot 10^{-5}\,\frac{\mathrm{m}}{\mathrm{s}^2}$ for a satellite with a semi-major axis of about 26,400 km (see Table 1). Satellites in a lower orbit (e.g., altimeter satellites with a height of about 1,000 km above the Earth surface) show a larger effect, by a factor of $\frac{(26{,}400\ km)^2}{(7{,}400\ km)^2} \approx 12$.

The Earth with a mass M_E, Sun, Moon, and other planets with masses M_i generate the following gravitational acceleration on a satellite at position $\ddot{\vec{r}}_{sat}$:

$$\ddot{\vec{r}}_{sat} = -GM_E \frac{\vec{r}_{sat}}{|\vec{r}_{sat}|^3} - G \cdot \sum_{i=1}^{n} M_i \frac{\vec{r}_i - \vec{r}_{sat}}{|\vec{r}_i - \vec{r}_{sat}|^3} \qquad (1)$$

Using the masses and the mean distances to the Sun and Moon, results in accelerations of $2 \cdot 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}^2}$ and $5 \cdot 10^{-6}\,\frac{\mathrm{m}}{\mathrm{s}^2}$, respectively (see Table 1). In this term the height of the satellite above the Earth surface is of secondary importance because the variation of the distance to the Moon or Sun due to the revolution of the satellite around the Earth is small compared to the mean distance to these celestial bodies.

The main effect from a degraded UT1 value is an erroneous direction to Moon and Sun in equation (1) resulting in an acceleration on the satellite misplaced by this angle. If the celestial body is located perpendicular above the orbital plane, a pure effect in the out-of-plane direction results, whereas along-track and radial effects are expected when the perturbing body is located in the orbital plane. With empirical accelerations estimated in orbit determination, the influence of potentially degraded positions of the celestial bodies on the solution can be greatly reduced.

Most of the analysis centers of the International DORIS Service (IDS, where DORIS stands for Doppler Orbit Determination and Radiopositioning Integrated on Satellite) include empirical once-per-revolution terms in along-track and cross-track directions in their solutions [6]. In this sense the related solutions are expected to be quite robust with respect to a dislocation of the Sun and Moon when computing the gravitational forces acting on these satellites.

As opposed to this setup, empirical accelerations are usually estimated in the Sun-oriented coordinate system located in the satellite when processing GNSS

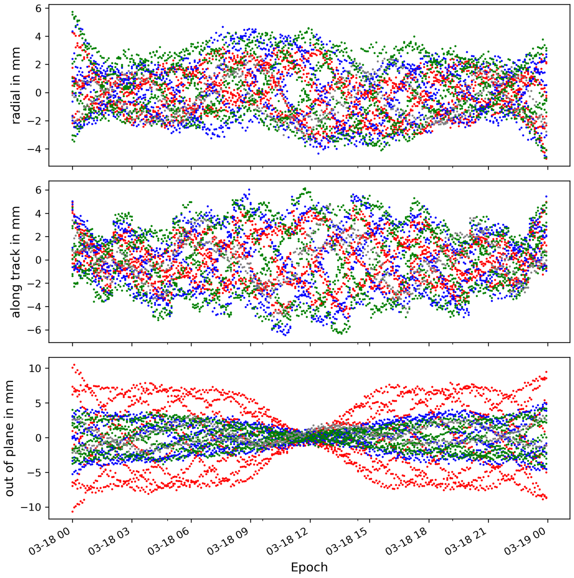


Fig. 3 Influence on the orbits of Galileo satellites in a one-day arc due to the direct gravitational forces by shifting UT1 by 100 ms in the positions of Moon and Sun.

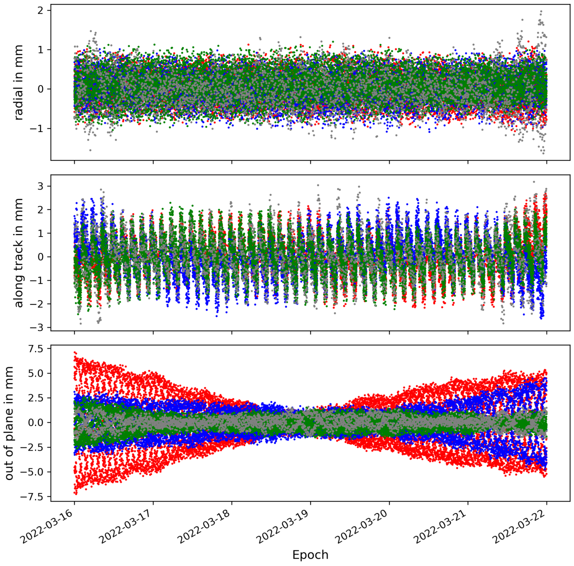


Fig. 4 Influence on the orbits of Galileo satellites in a six-day arc due to the direct gravitational forces by shifting UT1 by 10 ms in the positions of Moon and Sun.

data. Once-per-revolution terms are set up in one of the components of this coordinate system (see [7, 5]). Therefore, the effects caused by degraded UT1 values on the orbits and the solutions in general are larger for these satellites.

The errors introduced in the Galileo satellite positions by errors in UT1 are illustrated by Figures 3 and 4. An error of 100 ms in UT1 is underlying Figure 3, one of 10 ms Figure 4. Figure 3 shows the effect on a one-day orbit (about two revolutions of each satellite), Figure 4 on a six-day orbit (about 12 revolutions). In both cases the differences between the original (true) and the computed orbits with the modified positions of Moon and Sun do not exceed 1 cm.

The colors red, blue, and green in the figures correspond to the three orbital planes of the Galileo constellation (the gray dots represent the two satellites in specific elliptical orbits). The three planes show different effects as a result of the misorientation of the celestial bodies. Most prominent is the effect in the out-of-plane component. The satellites in the red plane show the largest effect, whereas the effects on the satellites in the green and blue planes are smaller. The elevation of the Sun above the orbital plane is about -56° for the red, $+25^\circ$ for the blue, and -20° for the green plane.

Two differences in the assumptions underlying Figures 3 and 4 matter: First, the size of the assumed UT1 error is increased by a factor of ten from Figure 4 to Figure 3 and the arc length is increased by a factor of six from Figure 3 to Figure 4. Finally, the comparison of the two figures confirms that the longer the arc (e.g., more revolutions shall be represented by one set of orbit parameter), the more the orbit solution becomes sensitive to the gravitational acceleration due to Moon and Sun. The computation of this impact is the most relevant influence of a potential error in UT1.

In conclusion, for GNSS solutions an error for UT1 below 10 ms is advisable in order to avoid a degradation of the results.

4 Earth Orientation Parameters from GNSS

As described in Section 2, satellite-based techniques are not able to contribute to UT1 because of the correlation with ascending node of the satellite orbit that need to be estimated for the orbit determination. Other components of the Earth orientation parameters can be obtained from GNSS, like polar motion (standard solutions from the International GNSS Service, IGS, contain daily offsets and rates) and in particular Length of Day (LOD). These GNSS-based estimates of LOD can be used to define a GNSS-derived series of UT1-UTC relative to an initial epoch t_0:

$$(UT1-UTC)_{GNSS}(t_i) = (UT1-UTC)_{VLBI}(t_0) - \sum_{k=0}^{i-1}(LOD)_{GNSS}(t_k)$$

Such a series was created based on CODE contribution to the IGS repro3 [8] and is displayed in Figure 5. The series is based on a three-day long-arc solution as described in [9]. One set of orbit parameters covers 72 hours. The Earth rotation parameters in the three-day long-arc solution from CODE is generated using a piece-wise linear representation with nodal points at midnight (i.e., four per component). One of the UT1 values is fixed on the CO4 value, whereas the other three are estimated. From their variations in time, the LOD is derived. For the orbit and Earth rotation parameters the middle of the three days is extracted as the solution. For comparison, Figure 5 also contains the blue curve where the cumulated LOD series is extracted from the corresponding ony-day solutions.

Figure 5 indicates that it should indeed be possible to extract a GNSS-derived UT1-UTC time series of acceptable quality even over a long time span. It should be noted that the solution contains between 250 to 300 tracking stations and includes GPS (since 1994), GLONASS (since 2002), and Galileo (since 2012). Towards the end of the series, more than 80 satellites do contribute.

Already 20 years ago, [10] showed that it is even possible to extract information about precession and nutation from GNSS using the same principles as those outlined for UT1-UTC.

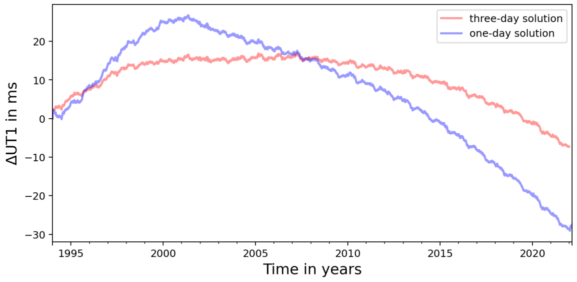


Fig. 5 Difference between UT1-UTC from C04(14)-series and the cumulated LOD series from CODE's contribution to IGS repro3 effort.

5 Summary

Most of the computations and modeling in the analysis of satellite orbits and related measurements is done either in the Earth-fixed (ITRS) or the quasi-inertial (GCRS) coordinate systems. For the transformation between the two systems Earth rotation matrices need to be applied. UT1 deficiencies can be absorbed to a large extent by the initial conditions and other parameters of the satellite orbits. In this way, both reference frames are mutually consistent where only a huge deficiency of several hours in UT1 leads to polar motion estimates that do not agree with the convention and that can therefore not be interpreted anymore. Moderate deficiencies in UT1 of less than one minute do not harm the consistency.

The biggest effect from celestial bodies from outside of this system is the gravitational force from the Moon and Sun on the satellite. Deficiencies in UT1 result in a wrong position of the Moon and Sun when the gravitational force is computed for orbit modeling. An error of up to 10 ms can be compensated by usually estimated orbit parameters and do not introduce a significant degradation of the obtained satellite orbit.

With a GNSS solution the cumulated LOD parameters may follow UT1 over years, in particular with a long-arc solution and continuity conditions on the Earth rotation parameters. GNSS solutions with few stations and satellites and in particular based on short arcs have noisier rates for the polar motion—and more relevant in this context—LOD estimates [11]. Other techniques, e.g., SLR with a limited number of tracking stations and non-uniformly distributed observations, have less favorable capabilities for reconstructing LOD.

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