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"Performance of Broadband Delay (BBD) Sequences"

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Memo: Performance of Broadband Delay (BBD) Sequences

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Introduction

The broadband delay (bbd) technique involves using several (e.g. 3-5) IF bands spread over a wide RF frequency range to resolve the interferometer phase, and hence achieve about an order of magnitude improvement in delay precision compared to current group delays. The goal with bbd is to achieve phase resolution even at modest (~10 per band) SNR's thus helping to minimize the interferometer sensitivity requirements and enable the use of smaller faster antennas. Improvement of delay measurement precision is one of the VLBI2010 strategies suggested in the IVS Working Group 3 final report.

In the original studies of bbd sequences, RF bandwidths of about 2-15 GHz were considered. It now appears that the high frequency cut-off may be technically difficult to achieve with some feeds, in particular, the generally preferred Kildal "eleven" style feed. In addition, the lowest frequencies in the original band may also become unusable. The lower frequencies already have significant RFI contamination, and this is expected to get worse with time.

In this study, we consider:

- The impact of the number of IF bands used,
- The impact of increasing the low frequency cut-off of the RF band,
- The impact of decreasing the high frequency cut-off of the RF band,
- The suitability of discontinuous frequency coverage, based on a combination feed proposed by Per Simon Kildal,
- The impact of using 0.5 GHz bandwidth IF bands, as are used in the NASA proof-of-concept test.
- The performance of two fallback techniques, one based on resolved phase differences only and the other based on multi-band group delays.

Method for selecting a sequence

Sequences are selected based on an exhaustive search through frequency space. The frequency of each band is stepped across the RF frequency range in steps of 0.1 GHz. All non-overlapping combinations of bands are then tested to find the one that requires the lowest SNR per band to resolve phase.

In each case, the SNR required to resolve phase is determined using the following steps:

1. For all bands in an observation, relations are developed between SNR and both group delay error, $\delta\tau_i^G$, and phase delay error, $\delta\tau_i^\phi$. This is based on the usual

equations, i.e. $\delta\tau_i^G = \dfrac{1}{2\cdot\pi\cdot SNR\cdot BW\cdot bw_{factor}}$ and $\delta\tau_i^\phi = \dfrac{f_i}{2\cdot\pi\cdot SNR}$, where i is the band index, f_i is the mid-band frequency, BW is the bandwidth and bw_{factor} is the bandwidth factor. The bandwidth factor is always set to 0.35, since it is assumed that half of the maximum data rate is used in all cases leaving room in the spectrum for bandwidth optimization.

2. Assuming SNR=1, a least squares covariance inversion is done of the group delay observations to produce the unit error of both the ionosphere term (which is proportional to the line of site electron content), K, and the non-dispersive interferometer delay, τ. The inversion is based on the equation $\tau_i^G = \tau + \dfrac{K}{f_i^2}$. Arbitrary values of $\delta\tau$ and δK can be determined by dividing by SNR.
3. All adjacent pairs of bands are analyzed to find the pair where the phase difference ambiguity can be resolved at lowest possible SNR. This is based on the equation $\phi_i = \dfrac{\tau_i^\phi}{f_i} = \tau - \dfrac{K}{f_i}$ and the SNR relations developed in step 2.
4. The SNR required to resolve the phase difference in step 3 with 5-sigma confidence is noted and a new pseudo group delay observable is created from the resolved phase difference.
5. Steps 2 to 4 are repeated, each time adding the new pseudo group delay into step 2 and removing the resolved pair of bands from those considered in step 3 until all band pairs have been resolved. At that point the phase is completely connected from the lowest frequency band to the highest. The highest SNR from from all times through step 3 is the SNR required to resolve all phase differences, i.e. the last column in Table 1.
6. Assuming SNR=1, a least squares covariance inversion is done of all group and phase delays to get the unit error of the phase offset, $\delta\phi$. Arbitrary values of $\delta\phi$ can be determined by dividing by SNR. Using this relation, the SNR required to resolve the phase offset with 5-sigma confidence can be determined. The larger of the SNR's from step 5 and 6 is the minimum BBD SNR, i.e. the third last column in Table 1.

Impact of the number of bands used

The cases of 2, 3, 4, and 5 bands are considered. For the sequences studied, the high frequency cut-off is either 11 or 14 GHz, the low frequency cut-off is either 2.2 or 3.2 GHz, and the bandwidth is 1 GHz. It is assumed that the bands are only half full so that an rms bandwidth factor of 0.35 is used. In figure 1, the minimum SNR to resolve interferometer phase is plotted against the number of bands used. The solid line is for low and high frequency cut-offs of 3.2 and 11 GHz respectively, the dashed line 3.2 and 14 GHz, the dotted line 2.2 and 11 GHZ, and the dash dotted line 2.2 and 14 GHz. Four bands were selected for use in VLBI2010 and will be assumed from this point onward in the memo.

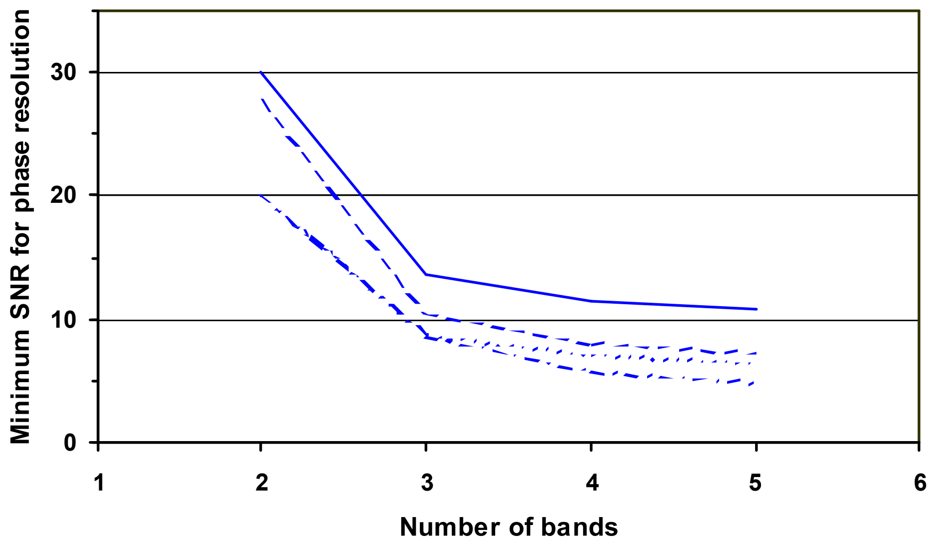

Figure 1. SNR to resolve phase.

Case Studies

The remaining analysis is based on a number of case studies. These are summarized in Table 1. In each case, 4 bands are assumed. The 15 options are grouped in threes. The first member of each group of three assumes a lower frequency cut-off of 2.2 GHz (i.e. the case of manageable S-band RFI); the second assumes a lower frequency cut-off of 3.2 GHz (i.e. unmanageable S-band RFI); and the third case includes, for backwards compatibility, standard geodetic S/X-band frequencies (i.e. one band at 2.2 GHz and one band at 8.0 GHz). There are 5 groups of 3 cases. The first 4 groups assume a bandwidth per band of 1 GHz and the last group assumes a bandwidth per band of 0.5 GHz (i.e. compatible with the iBOB1's being used in the NASA proof of concept tests). In all cases, the bit rate is assumed to be half maximum, i.e. for the 1 GHz bandwidth, the bit rate is 4 Gbps per band. This is based on 2 polarizations, Nyquist sampling, 2-bits per sample and half of the available channels selected. This is a deviation from previous thinking where full bit rate was assumed. Reducing the bit rate has the negative impact of roughly doubling the time to achieve target SNR's but provides significant room for RFI avoidance and allows the channels to be optimally placed to increase rms bandwidth factor from 0.29 to about 0.35, which is assumed in the cases studied. The first 3 group assume a single broadband feed of the Kildal "eleven" style with high frequency cut-off of 9 GHz for group 1, 11 GHz for group 2 and 14 GHz in group 3. For group 4, the frequency range is split into two sections, one from 2.2 to 9 GHz and the other from 14 to 18 GHz. This is based on a suggestion made by Per Simon Kildal at the 2008 IVS General Meeting in St. Petersburg to use an "eleven" feed for the 2.2 to 9 GHz range and an integrated waveguide feed for the range 14-18 GHz. The final (5th) group is intended to emulate the system used for the NASA proof-of-concept tests with high frequency cut-off of 13 GHz and bandwidth of 0.5 GHz.

The fourteen columns of the table are:

- Index of the case being studied
- Bandwidth per band
- Low frequency cut-off of the sequence
- High frequency cut-off of the sequence
- Low frequency cut-off of the second frequency range, for the combination feed only
- High frequency cut-off of the second frequency range, for the combination feed only
- Frequency of the lower edge of band 1
- Frequency of the lower edge of band 2
- Frequency of the lower edge of band 3
- Frequency of the lower edge of band 4
- Phase delay precision at SNR=10 of the fully resolved phase delay (Note: The phase delay is not, in all cases, fully resolve at SNR=10. In these cases, the SNR target will need to be raised above 10.)
- SNR required to fully resolve the interferometer phase
- Delay precision at SNR=10 assuming all phase differences are resolved, but not the phase.
- SNR required to resolve all phase differences.

Case Index	BW (GHz)	Low freq limit (GHz)	Hi freq limit (GHz)	Low freq limit 2 (GHz)	Hi freq limit 2 (GHz)	Band 1 freq (GHz)	Band 2 freq (GHz)	Band 3 freq (GHz)	Band 4 freq (GHz)	BBD dTau @ SNR=10 (ps)	SNR per band to resolve BBD	Delphi dTau @ SnR=10 (ps)	SNR per band to resolve all delphis
1.1	1.0	2.2	9			2.2	3.8	4.8	8.0	2.1	9.5	9.4	7.2
1.2	1.0	3.2	9			3.2	4.6	5.6	8.0	2.4	17.6	15.2	5.9
1.3	1.0	2.2	9			2.2	3.8	4.8	8.0	2.1	9.5	9.4	7.2
2.1	1.0	2.2	11			2.2	4.3	5.8	10.0	1.5	7.2	6.3	7.2

2.2	1.0	3.2	11			3.2	5.2	6.2	10.0	1.7	11.7	8.9	8.5
2.3	1.0	2.2	11			2.2	4.7	8.0	10.0	1.4	8.3	7.0	6.1
3.1	1.0	2.2	14			2.2	4.4	7.1	12.3	1.2	6.0	4.5	6.0
3.2	1.0	3.2	14			3.2	6.0	7.8	13.0	1.2	8.1	5.3	7.9
3.3	1.0	2.2	14			2.2	5.7	8.0	12.3	1.1	6.3	4.7	6.3
4.1	1.0	2.2	9	14	18	2.2	6.5	14.0	16.4	0.7	7.0	3.1	7.0
4.2	1.0	3.2	9	14	18	3.2	6.9	14.6	17.0	0.8	7.2	3.7	7.2
4.3	1.0	2.2	9	14	18	2.2	8.0	14.0	16.7	0.7	7.7	3.3	7.7
5.1	0.5	2.2	13			2.2	4.1	5.6	9.3	1.7	7.7	7.2	7.7
5.2	0.5	3.2	13			3.2	5.3	7.1	11.9	1.4	9.0	6.3	9.0
5.3	0.5	2.2	13			2.2	4.5	8.4	9.8	1.4	8.5	7.5	8.5

Table 1. Summary of bbd case studies

The phase-connected results (columns 13 and 14 of Table 1) are a fallback from the bbd results (columns 11 and 12) since the phase differences can be resolved more easily than the actual interferometer phase, and are less influenced by source structure. An even more secure fallback option though is an expanded group delay solution where three of the four bands are bunched near the top of the frequency range. The 5 groups that are in Table 1 are also summarized in Table 2 with the case numbers continuing in each group where they left off in Table 1.

Case Index	BW (GHz)	Low freq limit (GHz)	Hi freq limit (GHz)	Low freq limit 2 (GHz)	Hi freq limit 2 (GHz)	Band 1 freq (GHz)	Band 2 freq (GHz)	Band 3 freq (GHz)	Band 4 freq (GHz)	Group dTau @ SNR=10 (ps)	SNR per band to resolve Group delay
1.4	1.0	2.2	9			2.2	6.0	7.0	8.0	13.2	7.0
1.5	1.0	3.2	9			3.2	6.0	7.0	8.0	18.0	7.0
2.4	1.0	2.2	11			2.2	7.0	8.5	10.0	8.8	7.0
2.5	1.0	3.2	11			3.2	7.0	8.5	10.0	12.4	7.0
3.4	1.0	2.2	14			2.2	9.0	11.5	13.0	7.6	7.0
3.5	1.0	3.2	14			3.2	9.0	11.5	13.0	9.2	7.0
4.4	1.0	2.2	9	14	18	2.2	14.0	15.5	17.0	7.2	7.0
4.5	1.0	3.2	9	14	18	3.2	14.0	15.5	17.0	7.6	7.0
5.4	0.5	2.2	13			2.2	10.5	11.5	12.5	11.6	7.0
5.5	0.5	3.2	13			3.2	10.5	11.5	12.5	14.4	7.0

Table 2. Summary of fallback group delay case studies

Observations, Summary and Conclusions

- There is a very large bbd performance improvement going from 2 bands to 3 bands (see Figure 1) with diminishing returns for 3 bands and above. The VLBI2010 choice of 4 bands should be a robust choice.
- Raising the low frequency cut-off from 2.2 to 3.2 GHz to avoid RFI degrades performance both in terms of delay precision and the SNR required to resolve phase. (See Table 1)
    - Effort should be expended to reduce the impact of RFI so that it becomes more likely that the low frequency cut-off of 2.2 GHz can be used, e.g. narrower frequency binning in the DBE should be used to allow more efficient RFI avoidance, and lower data rates should be used to leave room in the band for RFI avoidance.
- Performance in general improves as the high frequency cut-off increases.
    - The impact of the low frequency cut-off (e.g. 2.2 to 3.2 GHz) becomes smaller as the high frequency cut-off increases from 9 to 18 GHz.
        - The degradation caused by going from 2.2 to 3.3 GHz lower limit is serious enough for the 9 and 11 GHz upper limit cases that the SNR=10 target needs to be revised significantly upwards for secure phase resolution.
- Best performance overall was achieved with the split frequency range (cases 4.1, 4.2, and 4.3 in Table 1 and cases 4.4 and 4.5 in Table 2) based on the combination "eleven" and waveguide feed proposed by Per Simon Kildal.
    - The high frequency component of this feed has the benefit that source structures errors should be smaller there, and although approaching the water vapour line the high frequency limit is low enough to still be reasonably robust against bad weather.
    - With the high frequency component of this feed, it is possible to use the low frequency cut-off of 3.2 GHz with minimal degradation (e.g. case 4.2 in Table 1 and case 4.5 in Table 2), thus increasing resistance to RFI.
    - This feed option should be pursued further with Per Simon Kildal.
- The backwards-compatible modes of operation (cases 1.3, 2.3, 3.3, 4.3, and 5.3 in Table 1) worked almost identical to the optimized modes (cases 1.1, 2.1, 3.1, 4.1, and 5.1 in Table 1).
- Performance is degraded but acceptable for the 0.5 GHz bandwidth cases (e.g. cases 5.1, 5.2, and 5.3 in Table 1 and cases 5.4 and 5.5 in Table 2). The narrower bandwidths will only be used temporarily for the NASA proof-of-concept tests.
- A first fallback mode of operation was considered in which only the phase differences are used in the delay calculation.
    - For this mode of operation, the delay precision is degraded by about a factor of about 4 over bbd. In all but one case, they are less than 10 ps and

in cases 4.1, 4.2, and 4.3 they are less than 4 ps. Monte Carlo simulations show that degradation of delay precision from 4 ps and 8 ps results in only about 10% degradation in geodetic results.

  - This mode of operation is generally more robust than the BBD mode since the actual interferometer phase does not need to be resolved. However, in cases with high high-frequency cut-off, frequency differences between bands can still be large increasing the chance of significant source structure differences between bands.

- A second fallback mode of operations was considered based on enhanced group delays where three of the four bands were bunched near the top of the frequency range and treated as a single "super band". This fallback mode has many advantages:
  - It is fairly likely to work.
  - Delay precision better than about 10 ps at SNR=10 can be achieved in many cases.
  - Minimum SNR of 7 per band is adequate.
  - For case 4.5 in Table 2, an improvement over current 1 Gbps S/X observations of a factor of 24 can be expected. In other words, if everything else were kept the same, the same delay precision could be achieved for a 24 times weaker source, a 5 times small pair of dishes, or a 575 times shorter integration interval. Even without going to bbd, this fallback group delay solution is a significant advance over current 1 Gbps S/X observations.