9 Models for propagation delays

9.4 Ionospheric model for radio techniques (Draft 22 July 2008)

Dispersive effects of the ionosphere on the propagation of radio signals are classically accounted for by linear combination of multi-frequency observations. In past years it has been shown that this approach induces errors on the computed time of propagation that can reach 100 ps for GPS. For wide-band VLBI observations, the induced errors might reach a couple of ps. In this section the estimation of the effect of higher-order neglected ionospheric terms and possible conventional models are summarized.

9.4.1 Ionospheric delay dependence on radio signals including higher order terms

The delay $\delta\rho_I$ experienced by the transionospheric electromagnetic signals, travelling from the transmitter T at $\vec{r}_T$ to the receiver R at $\vec{r}_R$, separated by a distance ρ, can be expressed by the integral of the refractive index n along the ray path:

$$\delta\rho_I = \int_{\vec{r}_T}^{\vec{r}_R} c\frac{dl}{v} - \rho = \int_{\vec{r}_T}^{\vec{r}_R} (n-1)dl \tag{1}$$

where v is the actual transionospheric signal propagation velocity at the given place, $c = 299792458$ m/s is the light speed in free space and dl is the differential length element.

Effects on carrier phase data

Such refractive index for the carrier phase, n_p, can be expressed by the Appleton expression, for both ordinary (upper sign) and extraordinary (lower sign) waves, as (see for instance Davies 1990, page 72):

$$n_p^2 = 1 - \frac{X}{1 - iZ - \frac{Y_T^2}{2(1-X-iZ)} \pm \left[\frac{Y_T^4}{4(1-X-iZ)^2} + Y_L^2\right]^{\frac{1}{2}}} \tag{2}$$

where:

$$X = \frac{\omega_p^2}{\omega^2} \tag{3}$$

$$Y_L = -\frac{\omega_g}{\omega}\cos\theta \tag{4}$$

$$Y_T = -\frac{\omega_g}{\omega}\sin\theta \tag{5}$$

$$Z = \frac{\gamma}{\omega} \tag{6}$$

being $i = \sqrt{-1}$, θ the angle between the magnetic field $\vec{B}$ and the electromagnetic (EM) propagation direction $\vec{k}$. In general $\omega = 2\pi f$ is the *circular* frequency corresponding to frequency f. This applies to the carrier circular frequency ω, and the plasma and gyro circular frequencies associated to the free electrons of the ionosphere:

$$\omega_p^2 = \frac{N_e q^2}{m_e \epsilon_0} \tag{7}$$

$$\omega_g = \frac{Bq}{m_e} \tag{8}$$

where N_e is the number density of free electrons and B is the magnetic field modulus (both depending on time and position along the EM ray), $q \simeq 1.6022 \cdot 10^{-19}$C is the absolute value of the electron charge, $m_e \simeq 9.1094 \cdot 10^{-31}$kg is the electron mass and $\epsilon_0 \simeq 8.8542 \cdot 10^{-12}$F/m is the electric permittivity in free space (vacuum). Finally γ is the frictional force factor, acting such force on each free electron, modelled as $\gamma m_e \dot{z}$, being z the coordinate over the propagation direction $\vec{k}$.

From the Appleton equation, if we neglect the frictional force ($Z = 0$), assuming that we are in a cold, collisionless, magnetized plasma such as the Ionosphere, the squared phase index of refraction can be approximated as:

$$n_p^2 = 1 - \frac{X}{1 - \frac{Y_T^2}{2(1-X)} \pm \left[\frac{Y_T^4}{4(1-X)^2} + Y_L^2\right]^{\frac{1}{2}}} \tag{9}$$

From this point, the following second-order Taylor approximation can be done for small values of δ:

$$(1+\delta)^\eta \simeq 1 + \eta\delta + \frac{\eta(\eta-1)}{2}\delta^2 \tag{10}$$

retaining only terms up to f^{-4} (i.e. ω^{-4}), similarly to Bassiri and Hajj (1993) approach, and making the proper approximation of n_p^2 for transionospheric signals with frequencies $\omega >> \omega_p$, such as those of GNSS (see Datta-Barua 2006 for a detailed discussion of several approximation ways adopted by different authors):

$$n_p = 1 - \frac{1}{2}X \pm \frac{1}{2}XY_L - \frac{1}{8}X^2 - \frac{1}{4}X \cdot Y^2(1+\cos^2\theta) \tag{11}$$

where

$$Y^2 = Y_L^2 + Y_T^2 = \left(\frac{\omega_p}{\omega}\right)^2 \tag{12}$$

and again upper sign represents ordinary wave, and lower sign represents extraordinary wave which can be typically associated to right hand polarized EM signals such as those of GPS antennas, and most L and S Band antennas that receive satellite signals.

The following explicit expression for n_p can be obtained for extraordinary EM signal in terms of the main physical constants and parameters, after substituting X, Y_L, Y from previous relationships (equations 3, 4 and 5):

$$n_p = 1 - \frac{q^2}{8\pi^2 m_e \epsilon_0} \cdot \frac{N_e}{f^2} - \frac{q^3}{16\pi^3 m_e^2 \epsilon_0} \cdot \frac{N_e B \cos\theta}{f^3} - \frac{q^4}{128\pi^4 m_e^2 \epsilon_0^2} \cdot \frac{N_e^2}{f^4} - \frac{q^4}{64\pi^4 m_e^3 \epsilon_0} \cdot \frac{N_e B^2(1+\cos^2\theta)}{f^4} \tag{13}$$

From equation 13, and applying Eq. 1, the following ionospheric dependent terms in the carrier phase, up to third (f^{-4}) order, are obtained:

$$\delta\rho_{I,p} = -\frac{s_1}{f^2} - \frac{s_2}{f^3} - \frac{s_3}{f^4} \tag{14}$$

being the first, second and third order coefficients, s_1, s_2 and s_3, after substituting the above introduced physical constants, m_e, q, ϵ_0, with 5 significant digits, in International System of Physical Units (SI):

$$s_1 = 40.309 \int_{\vec{r}_T}^{\vec{r}_R} N_e dl \tag{15}$$

$$s_2 = 1.1284 \cdot 10^{12} \int_{\vec{r}_T}^{\vec{r}_R} N_e B \cos\theta dl \tag{16}$$

$$s_3 = 812.42 \int_{\vec{r}_T}^{\vec{r}_R} N_e^2 dl + 1.5793 \cdot 10^{22} \int_{\vec{r}_T}^{\vec{r}_R} N_e B^2 \left(1 + \cos^2\theta\right) dl \tag{17}$$

These expressions are fully equivalent for instance to equations 2 to 5 in Fritsche et al. 2005.

It can be seen in last expressions 14 to 17 that the ionospheric delay on the carrier phase pseudorange is negative, associated to an increase of the phase velocity in the EM signal transionospheric propagation.

In order to asses the importance of the different ionospheric terms for $\delta\rho_{I,p}$ in equation 14, we can start on the first term, assuming a high value of Slant Total Electron Content (STEC see subsection 9.4.3 below for more details) of $S = \int_{\vec{r}_T}^{\vec{r}_R} N_e dl \sim 300 \cdot 10^{16} \mathrm{m}^{-2}$:

$$\delta\rho_{I,p,1} = -\frac{40.309 S}{f^2} \sim -\frac{1.2 \cdot 10^{20}}{f^2} \tag{18}$$

This brings, for example, to values of the order of magnitude in the first ionospheric order term $\delta\rho_{I,p,1}$ up to several km of delay for $f \simeq 150$ MHz (negative for the carrier phase), corresponding to the lower frequency of the NIMS satellite system (U.S. Navy Ionospheric Measuring System, formerly TRANSIT), and up to several tens of meters for $f = 1575.42 MHz$ (L_1 GPS carrier frequency).

The relative importance of the first ($\delta\rho_{I,p,1} = -s_1/f^2$), second ($\delta\rho_{I,p,2} = -s_2/f^3$) and third order terms ($\delta\rho_{I,p,3} = -s_3/f^4$) is also dependent on the frequencies, being the higher order terms less important for higher frequencies (such as in the case of several VLBI frequencies compared with GPS frequencies for instance). Indeed, from equations 14, 15, 16 and 17:

$$\frac{\delta\rho_{I,p,2}}{\delta\rho_{I,p,1}} = \frac{2.7994 \cdot 10^{10}}{f} \cdot \frac{\int_{\vec{r}_T}^{\vec{r}_R} N_e B \cos\theta dl}{\int_{\vec{r}_T}^{\vec{r}_R} N_e dl} \tag{19}$$

By taking raw values reflecting order of magnitude of $|B_0 \cos\theta_0| \simeq 10^4$nT at a given effective height to evaluate both integrals, the order of magnitude of the relative value between second and first order ionospheric terms can be approximated by:

$$\frac{\delta\rho_{I,p,2}}{\delta\rho_{I,p,1}} \simeq \frac{2.7994 \cdot 10^{10}}{f} |B_0 \cos\theta_0| \sim \frac{2.8 \cdot 10^5}{f} \tag{20}$$

This brings to relative values for $\delta\rho_{I,p,2}$ and $\delta\rho_{I,p,1}$ of the less than 1% for $f \simeq 150$ MHz (NIMS), and less than 0.1% for $f = 1575.42 MHz$ (GPS L_1 carrier).

Similarly, the order of magnitude of the relative value between third and second order ionospheric terms can be estimated as:

$$\frac{\delta\rho_{I,p,3}}{\delta\rho_{I,p,2}} = \frac{7.1998 \cdot 10^{-10}}{f} \cdot \frac{\int_{\vec{r}_T}^{\vec{r}_R} N_e^2 dl}{\int_{\vec{r}_T}^{\vec{r}_R} N_e B \cos\theta dl} + \frac{1.3996 \cdot 10^{10}}{f} \cdot \frac{\int_{\vec{r}_T}^{\vec{r}_R} N_e B^2 \left(1 + \cos^2\theta\right) dl}{\int_{\vec{r}_T}^{\vec{r}_R} N_e B \cos\theta dl} \tag{21}$$

Considering the above taken raw values reflecting order of magnitude of $|B_0 \cos\theta_0| \simeq 10^4$nT at a given effective height to evaluate the integrals, and an intermediate angle of $\theta_0 = 45$ deg, and taking $N_0 \simeq 10^{12} \mathrm{m}^{-3}$ a raw order of magnitude value of effective electron density, fulfilling $N_0 \cdot \int_{\vec{r}_T}^{\vec{r}_R} N_e dl = \int_{\vec{r}_T}^{\vec{r}_R} N_e^2 dl$, we get the following relative order of magnitude value between third and second order ionospheric terms:

$$\frac{\delta\rho_{I,p,3}}{\delta\rho_{I,p,2}} \simeq \frac{1}{f}\left(1.3996\cdot 10^{10}\cdot|3B_0\cos\theta_0| + 7.1998\cdot 10^{-10}\frac{N_0}{|B_0\cos\theta_0|}\right) \sim \frac{4.3\cdot 10^5 + 7.2\cdot 10^7}{f} \tag{22}$$

Then the order of magnitude of the ratio between third and second order ionospheric terms can be as high as about 50% for NIMS frequency $f \simeq 150MHz$ but less than 10% for $f = 1575.42MHz$, the L_1 GPS carrier frequency.

Another conclusion from this approximation is that the second integral in the third order term s_3 (see 17) can be typically neglected compared with the second integral depending only on the electron density (it can be seen in equation 22 that it is about two orders of magnitude smaller):

$$s_3 \simeq 812\int_{\vec{r}_T}^{\vec{r}_R} N_e^2 dl \tag{23}$$

Finally in order to show that third order ionospheric approximation should be adequate for most of the radio astronomic-geodetic techniques, we can consider the fourth order term in the carrier phase $\delta\rho_{I,p,4}$. It can be deduced in a similar way that first to third order terms, but now keeping the terms in f^{-5} from equations 9 and 10 in the corresponding fourth order term $\delta n_{p,4}$ of the carrier phase ionospheric refraction index term

$$\delta n_{p,4} = -\frac{1}{2}XY_L\left(\frac{X}{2} + Y^2\left[1 + \frac{1}{8}\sin^2\theta\tan^2\theta\right]\right) \tag{24}$$

which is expressed with the same notation than previous expressions. Substituting in terms of the dependences on the corresponding physical and mathematical constants and applying equation 1, the 4th order ionospheric term in delay can be expressed as:

$$\delta\rho_{I,p,4} = -\frac{s_4}{f^5} \tag{25}$$

where

$$s_4 = \frac{q^5}{128\pi^5 {m_e}^3{\epsilon_0}^2}\int_{\vec{r}_T}^{\vec{r}_R} N_e^2 B\cos\theta dl + \frac{q^5}{64\pi^5{m_e}^4\epsilon_0}\int_{\vec{r}_T}^{\vec{r}_R} N_e B^3 f(\theta) dl \tag{26}$$

being

$$f(\theta) = \cos\theta\left(1 + \frac{1}{8}\sin^2\theta\tan^2\theta\right) \tag{27}$$

and substituting the constant values we get:

$$s_4 = 4.5481\cdot 10^{13}\int_{\vec{r}_T}^{\vec{r}_R} N_e^2 B\cos\theta dl + 8.8413\cdot 10^{32}\int_{\vec{r}_T}^{\vec{r}_R} N_e B^3 f(\theta) dl \tag{28}$$

Taking into account equations 25, 28, 14 and 23, the ratio between fourth and third ionospheric order terms can be written as:

$$\frac{\delta\rho_{I,p,4}}{\delta\rho_{I,p,3}} = \frac{1}{f}\left(5.5982\cdot 10^{10}\frac{\int_{\vec{r}_T}^{\vec{r}_R} N_e^2 B\cos\theta dl}{\int_{\vec{r}_T}^{\vec{r}_R} N_e^2 dl} + 1.0883\cdot 10^{30}\frac{\int_{\vec{r}_T}^{\vec{r}_R} N_e B^3 f(\theta) dl}{\int_{\vec{r}_R} N_e^2 dl}\right) \tag{29}$$

Taking into account the same approximations and particular values than before, the ratio can be expressed as:

$$\frac{\delta\rho_{I,p,4}}{\delta\rho_{I,p,3}} \simeq \frac{1}{f}\left(5.6\cdot 10^{10}|B_0\cos\theta_0| + 1.1\cdot 10^{30}\frac{|B_0\cos\theta_0|^3 f(\theta_0)}{N_0|\cos^3\theta_0|}\right) \sim \frac{1}{f}\left(5.6\cdot 10^5 + 2.3\cdot 10^3\right) \tag{30}$$

Table 1: Delays (in millimeters) are corresponding to the different higher order ionospheric terms, from first to fourth (in columns) for a representative subset of working frequencies in radio astronomy and geodesy: they correspond to a particular case of $|B_0 \cos\theta_0| \sim 10^4$ nT, $\theta_0 = \pi/4$, $N_0 = 10^{12}\mathrm{m}^{-3}$ and $S = 3 \cdot 10^{18}\mathrm{m}^{-2}$. The values that can be typically neglected (those lower than 1 mm) are clearly identified by its negative exponent.

f / MHz	Technique	$\delta\rho_{I,p,1}$ / mm	$\delta\rho_{I,p,2}$ / mm	$\delta\rho_{I,p,3}$ / mm	$\delta\rho_{I,p,4}$ / mm
150	NIMS	$-5.3 \cdot 10^6$	$-9.9 \cdot 10^3$	$-4.8 \cdot 10^3$	$-1.8 \cdot 10^1$
400	NIMS / Doris	$-7.5 \cdot 10^5$	$-5.2 \cdot 10^2$	$-9.4 \cdot 10^1$	$-1.3 \cdot 10^{-1}$
1228	GPS (L2)	$-8.0 \cdot 10^4$	$-1.8 \cdot 10^1$	$-1.1 \cdot 10^0$	$-5.0 \cdot 10^{-4}$
1575	GPS (L1)	$-4.8 \cdot 10^4$	$-8.5 \cdot 10^0$	$-3.9 \cdot 10^{-1}$	$-1.4 \cdot 10^{-4}$
2000	Doris	$-3.0 \cdot 10^4$	$-4.2 \cdot 10^0$	$-1.5 \cdot 10^{-1}$	$-4.2 \cdot 10^{-5}$
2300	Low VLBI f.	$-2.3 \cdot 10^4$	$-2.8 \cdot 10^0$	$-8.8 \cdot 10^{-2}$	$-2.2 \cdot 10^{-5}$
8400	High VLBI f.	$-1.7 \cdot 10^3$	$-5.7 \cdot 10^{-2}$	$-4.9 \cdot 10^{-4}$	$-3.3 \cdot 10^{-8}$
12000	Time trans. low Ku f.	$-8.3 \cdot 10^2$	$-1.9 \cdot 10^{-2}$	$-1.1 \cdot 10^{-4}$	$-5.2 \cdot 10^{-9}$
14000	Time trans. high Ku f.	$-6.1 \cdot 10^2$	$-1.2 \cdot 10^{-2}$	$-6.2 \cdot 10^{-5}$	$-2.5 \cdot 10^{-9}$

From this expression it can be seen than, in particular, the relative weight of fourth to third order ionospheric terms is less than 1% for $f \simeq 150$MHz (NIMS) and less than 0.1% for the L1 GPS carrier at $f = 1575.42MHz$. Another conclusion from this development is that the fourth order term can be approximated by the first term in equation 29:

$$s_4 \simeq 4.55 \cdot 10^{13} \int_{\vec{r}_T}^{\vec{r}_R} N_e^2 B \cos\theta dl \tag{31}$$

Finally in Table 9.4.1 you can see as the different terms are translated in delays, for different frequencies of interest in radio astronomic-geodetic research, with the same approximations and particular values than above ($|B_0 \cos\theta_0| \sim 10^4 nT$, $N_0 \sim 10^{12}\mathrm{m}^{-3}$ and $S \sim 3 \cdot 10^{18}\mathrm{m}^{-2}$). It can be seen, taking as significant threshold the delay value of 1mm, that:

- First order ionospheric term, as expected, is significant for all the considered frequencies.
- Second order ionospheric term should be also taken into account in all the frequencies, excepting for the high VLBI and time transfer Ku band ones.
- Third order ionospheric term should be taken into account in NIMS and Doris low frequencies, being in the significance limit for GPS and high Doris frequencies. It can be neglected for VLBI and Time transfer Ku band frequencies.
- Fourth order can be neglected, excepting for the very low NIMS frequency of 150 MHz.

Effects on code pseudorange data

The corresponding effect can be computed for the code pseudorange measurements, by using the well known relationship between phase and code refractive indices, n_p and n_c respectively, relating the phase velocity with the group (code) velocity (see for instance Davies, 1990, pag. 13):

$$n_c = n_p + f \frac{dn_p}{df} \tag{32}$$

A similar relationship is fulfilled between the code and carrier phase ionospheric delays, $\delta\rho_{I,c}$ and $\delta\rho_{I,p}$, after introducing equation 32 in equation 1:

$$\delta\rho_{I,c} = \delta\rho_{I,p} + f \frac{d}{df} \delta\rho_{I,p} \tag{33}$$

Applying equation 33 to equation 14, the ionospheric effect on code ionospheric delay, up to third order term, is:

$$\delta\rho_{I,c} = \frac{s_1}{f^2} + 2\frac{s_2}{f^3} + 3\frac{s_3}{f^4} \tag{34}$$

It can be seen from this relationship, taking into account equations 15, 16 and 17, that the ionospheric delay on the code pseudorange is positive, associated to a decrease of the EM signal group velocity in the transionospheric propagation.

9.4.2 Correcting the ionospheric effects on code and phase

The more efficient way of correcting the ionospheric effects is by combining simultaneous measurements in k different frequencies, which allows to cancel out the ionospheric effects up to order $k-1$, taking into account relationships 14 and 34 for carrier phase or code. A well know example is the case of actual GPS system with two frequencies, which allows to cancel out the first order ionospheric effect by the so called ionospheric-free combination of observables (see below). And in the future, with Galileo and modernized GPS systems, the full correction can be extended to second order ionospheric terms too.

9.4.3 Correcting the ionospheric term for single frequency users

If the user is only able to gather measurements at a single frequency f, then his main problem is to correct as much as possible (or at least *mitigate*) the first order ionospheric terms in phase and code measurements, $\delta\rho_{I,p,1}$ and $\delta\rho_{I,c,1}$, which explains more than 99.9% of the total ionospheric delays, as we have shown above. The first ionospheric order terms are only dependent on the Slant Total Electron Content $S = \int_{\vec{r}_T}^{\vec{r}_R} N_e dl$ and the signal frequency (equations 14, 34, 15):

$$\delta\rho_{I,p,1} = -40.309\frac{S}{f^2}\delta\rho_{I,c,1} = +40.309\frac{S}{f^2} \tag{35}$$

There are different available external sources for the STEC S. Many of them provides the vertically integrated ionospheric free electron density, so called Vertical Total Electron Content (VTEC, V), globally or at least at regional scale.

From the VTEC values (V) corresponding to the observation time, the STEC S can be estimated thanks to a factor approximating the pass from the vertical to the slant Total Electron Content: the so called *ionospheric mapping function*, M. Indeed:

$$S = M \cdot V \tag{36}$$

Typically a thin shell spherical layer model, at a fixed *effective ionospheric height* h, is applied:

$$M = \frac{1}{\sqrt{1 - \frac{r^2 \cos^2 E}{(r+h)^2}}} \tag{37}$$

being r and E the geocentric distance and ray spherical elevation taken from the user receiver. In the case of IGS the adopted effective height is $h = 450km$. This approximation can introduce significant errors as well, of 5% or more, specially when the 3D nature of the electron density distribution N_e can affect more to the integrated (total electron content) values: at low elevation or low latitude observations (see for instance Hernández-Pajares et al. 2005).

Some of the more common sources of electron content are:

- Global Vertical Total Electron Content (VTEC) maps, such as those computed by the International GNSS Service (IGS, http://www.igs.org) from a global network of dual-frequency receivers. The user can compute its STEC, S, from interpolating the VTEC maps and applying the corresponding mapping function (equations 36 and 37 with $h = 450km$ in IGS). The IGS VTEC maps have typically errors of 10 to 20% (see for instance Hernández-Pajares 2004 and Orus et al. 2002).
- Predicted VTEC models such as those used by GNSS: Klobuchar model broadcasted in GPS navigation message, or NeQuick (http://www.itu.int/ITU-R/study-groups/software/rsg3-p531-electron-density.zip) for future Galileo system. They can show average errors up to 50% (up to 30% at low latitude, see for instance Orus et al. 2002, Aragon et al. 2004).

- Regional VTEC models, which provide better accuracy by means of a better temporal and spatial resolution, thanks to the availability of dense networks of permanent receivers (such as the cases of Japan, Europe or USA).
- Empirical standard models of the Ionosphere, based on all available data sources, such as the International Reference Ionosphere (IRI, Bilitza 1990, http://modelweb.gsfc.nasa.gov/ionos/iri.html) or PIM (Daniell et al. 1995, http://www.cpi.com/products/pim/pim.html). If they are adjusted to the actual conditions by means of one or several parameters (such as the Sun Spot Number, SSN, Bilitza et al. 1998), these empirical models can provide at least similar performance than predicted VTEC models for GNSS. Otherwise the performance can be logically poor, depending on the region and time.

9.4.4 Correcting the ionospheric term for dual frequency users

In case the user is able to gather two simultaneous measurements in two frequencies, f_a and f_b ($f_a > f_b$), the situation is much better, because the first order term can be cancelled, and this means more than 99.9% of the total ionospheric delay. Indeed, by applying as weight factors f_a^2 and $-f_b^2$, the corresponding first-order-ionospheric-free combination $\rho_p^{(1)}$,

$$\rho_p^{(1)}(a,b) = \frac{f_a^2 \rho_p^{(a)} - f_b^2 \rho_p^{(b)}}{f_a^2 - f_b^2} \tag{38}$$

leads to the following new ionospheric dependences, for carrier phase and code ($\delta\rho_{I,p}^{(1)}$ and $\delta\rho_{I,c}^{(1)}$ respectively), after considering equations 14, 34:

$$\delta\rho_{I,p}^{(1)} = \frac{f_a^2 \delta\rho_{I,p}^{(a)} - f_b^2 \delta\rho_{I,p}^{(b)}}{f_a^2 - f_b^2} = \frac{s_2}{f_a f_b (f_a + f_b)} + \frac{s_3}{f_a^2 f_b^2} \tag{39}$$

$$\delta\rho_{I,c}^{(1)} = \frac{f_a^2 \delta\rho_{I,c}^{(a)} - f_b^2 \delta\rho_{I,c}^{(b)}}{f_a^2 - f_b^2} = -\frac{2s_2}{f_a f_b (f_a + b_b)} - \frac{3s_3}{f_a^2 f_b^2} \tag{40}$$

where s_2 and s_3 will depend on electron density N_e and magnetic field $\vec{B}$, following expressions 16 and 23. The following approximations can be done to facilitate the computations:

$$s_2 = 1.1284 \cdot 10^{12} \int_{\vec{r}_T}^{\vec{r}_R} N_e B \cos\theta dl \simeq 1.1284 \cdot 10^{12} B_p \cos\theta_p \cdot S \tag{41}$$

where B_p and θ_p are the magnetic field modulus and projecting angle regarding to the propagation direction, at a convenient effective pierce point p, and S is the integrated electron density, or STEC S (this approximation is used by, among in other works cited above, Kedar et al., 2003 and by Petrie et al., 2008). The third order coefficient can be approximated in terms of the maximum electron density along the ray path N_m:

$$s_3 \simeq 812 \int_{\vec{r}_T}^{\vec{r}_R} N_e^2 dl \simeq 812 \eta N_m S \tag{42}$$

It can be taken $\eta \simeq 0.66$ and N_m can be expressed as function of the slab thickness H (which can be modelled as function on the latitude and local time) and the VTEC V (see more details in Fritsche et al. 2005 and mentioned references).

These expressions typically leads in GPS to values of up to few centimeters for second order ionospheric correction: for instance $\delta\rho_{I,p}^{(1)} \simeq 2$ cm for a given observation with high STEC values (such as $S \simeq 300$ TECU $= 10^{18}$ m^{-3}) and magnetic field projection of $B\cos\theta \simeq 3 \cdot 10^4 nT$.

Moreover the curvature (or bending) of the ray can be considered as an additional correction Δs_3 (up to few millimeters at low elevation for GPS frequencies), appearing as a f^{-4} dependence too, which can be easily added to s_3 coefficient of equation 42. In particular Jakowski et al. 1994 derived a simplified expression for GPS which, with the above introduced notation, the coefficient of the f^{-4} term approximating the bending effect is:

$$\Delta s_3 \simeq 2.495 \cdot 10^8 [(1 - 0.8592 \cos^2 E]^{-1/2} - 1] \cdot S^2 \tag{43}$$

not being the units in SI system in this case: the STEC S in TECU=10^{16}m^{-3}, the factor Δs_3 in mm$\cdot$(MHz)^4.

Then, to evaluate $\delta\rho^{(1)}_{I,p}$ and $\delta\rho^{(1)}_{I,c}$ we need as well an STEC source for S, as in the case of single frequency users (see previous subsection). In this case, the double frequency measurements can be used, to provide a direct estimate of S, from the first order term which contains more than 99.9% of it. For instance in GPS S can be estimated from the ionospheric (geometry-free) combination of carrier phases $L_I = L_1 - L_2$ and codes $P_I = P_2 - P_1$, being L_i and P_i the carrier phase and code measurements for carrier frequency f_i, in length units. Indeed, writing L_I and P_I in terms of corresponding carrier phase ambiguity B_I and interfrequency delay code biases (DCBs) for receiver and transmitter D and D'

$$L_I = S + B_I \tag{44}$$

$$P_I = S + D + D' \tag{45}$$

the STEC S can be estimated as

$$S = L_I - < L_I - P_I > - D - D' \tag{46}$$

being $< \ \cdot \ >$ the average along a carrier phase continuous arch of transmitter-receiver data (with no phase cycle-slips). This way of computing the STEC has certain advantages, specially when no external sources of STEC are available (such as in real-time conditions) or at low latitudes and elevations (see Hernández-Pajares et al. 2007 for corresponding discussion).

Moreover, taking into account equations 39 and 40, a source of magnetic field is needed, which should be more realistic than the dipolar one, such as the International Magnetic Reference Field (IMRF, http://www.ngdc.noaa.gov/IAGA/vmod/igrf.html) or the Comprehensive Model (see http://core2.gsfc.nasa.gov/CM/), to reduce errors up to more than 60% in certain regions (see as well discussion in Hernández-Pajares et al. 2007). Equations 39 to 42, with an adequate source of STEC and magnetic field (see above) provide a conventional method to correct the ionospheric higher order terms for dual frequency users.

An alternative approach to correcting the GPS measurements is to apply the second order ionospheric correction by means of redefining the first-order ionospheric free combination of observables (Brunner and Gu 1991), for instance in terms of the line-of-sight magnetic field projection term[1]. This approach has the disadvantage of producing a time dependent carrier phase bias. More details on pros and cons of different approaches for higher order ionospheric corrections (including regional models such as Hoque and Jakowski 2006) can be found in Hernández-Pajares et al. 2008.

9.4.5 Correcting the ionospheric term for multi(three or more)-frequency users

GNSS systems offering simultaneous observations in 3 or more frequencies should be available soon. Thence it will be possible to cancel out, from these k simultaneous observations of the same transmitter-receiver pair, up to the first $k-1$ ionospheric order terms.

As an example, and from equation 38 applied to two pairs of three consecutive frequencies (f_a, f_b and f_c), is possible to define a combination of of carrier phase observables first and second order ionospheric free, $\rho^{(2)}_p$:

$$\rho^{(2)}_p = \frac{f_a f_b (f_a + f_b) \rho^{(1)}_p (a,b) - f_b f_c (f_b + f_c) \rho^{(1)}_p (b,c)}{f_a f_b (f_a + f_b) - f_b f_c (f_b + f_c)} \tag{47}$$

[1] From equation 44 and definition of first-order ionospheric free combination of carrier phases $L_c \equiv (f_1^2 L_1 - f_2^2 L_2)/(f_1^2 - f_2^2) = \rho^\star + B_c$ (being $\rho^\star$ the non-frequency dependent terms –including geometric distance, clock errors and tropospheric delay– and B_c the carrier phase bias), an apparently first and second order iono free combination of carrier phases can be easily derived $L'_c = \rho^\star + B'_c$, being $L'_c = L_c - s_2 L_I/(f_1 f_2 (f_1 + f_2))$ and $B'_c = B_c - s_2 B_I/(f_1 f_2 (f_1 + f_2))$ the new combination of observables and time-varying carrier phase bias, respectively.

From here and from equation 39 the following remaining higher order ionospheric dependence can be deduced:

$$\delta\rho_{I,p}^{(2)} = \frac{s_3}{f_a f_c (f_b^2 + f_b[f_a + f_c])} \tag{48}$$

A similar definition to equation 47 can be done for the code observations, bringing (by using equation 40) to the following remaining higher order ionospheric dependence:

$$\delta\rho_{I,c}^{(2)} = \frac{-2s_3}{f_a f_c (f_b^2 + f_b[f_a + f_c])} \tag{49}$$

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