Space Geodesy, VLBI, and the Fourth Pillar of Geodesy – Spacetime Curvature

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Abstract Typically geodesy is described as having "three pillars": the variations in Earth's shape, gravity field, and rotation. These pillars form the conceptual and observational basis for the celestial and terrestrial reference frames required for Earth and space observations. However, it is no longer adequate to base the conceptual and observational basis on only three pillars. Spacetime curvature as described by the General Theory of Relativity (GTR) is an integral component of all space geodesy techniques and influences all measurements, techniques, and data reduction. Spacetime curvature is therefore the fourth pillar. It is the measurement of the shape of spacetime and its variations. Due to accuracies of Very Long Baseline Interferometry (VLBI) and optical celestial reference frame measurements reaching the tens of micro-arcsecond level in the near future, it is essential to recognize the impact of spacetime seeing on the accuracy objectives of the Global Geodetic Observing System. Spacetime seeing (resulting from spacetime curvature) is analogous to astronomical seeing (resulting from atmospheric conditions), as all of spacetime is affected by microlensing/weak lensing to some extent as a result of mass (normal baryonic and darkmatter) distribution, placing a limit on the realization of the celestial reference frame.

Keywords ICRF, VLBI, General Theory of Relativity, microlensing, weak lensing, spacetime curvature seeing, space geodesy

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1 Introduction

Currently the focus of the International Association of Geodesy (IAG) is Earth-centric to a large extent; the IAG's main perspective is towards Earth. This is reflected in the structure and tasks of its four commissions. These are briefly summarized as:

- Commission 1: Reference Frames
- Commission 2: Gravity Field
- Commission 3: Earth Rotation and Geodynamics
- Commission 4: Positioning and Applications

These commissions fulfill valuable roles in the continued maintenance and improvement of the terrestrial and celestial reference frames, gravity field modeling and measurement, determination of orbits and their modeling, Earth orientation, tectonic measurement and modeling, ocean level changes, planetary and lunar dynamics, precise positioning, and applications of space geodesy for ionospheric and atmospheric studies. This list is not comprehensive as the applications and implications of geodesy are diverse and encompass subfields of many disciplines. In addition to the four commissions, there are nine joint study groups.

1. JSG 0.1: Application of time series analysis in geodesy
2. JSG 0.2: Gravity field modeling in support of height system realization
3. JSG 0.3: Comparison of current methodologies in regional gravity field modeling
4. JSG 0.4: Coordinate systems in numerical weather models
5. JSG 0.5: Multi-sensor combination for the separation of integral geodetic signals

6. JSG 0.6: Applicability of current GRACE solution strategies to the next generation of inter-satellite range observations
7. JSG 0.7: Computational methods for high-resolution gravity field modeling and nonlinear diffusion filtering
8. JSG 0.8: Earth system interaction from space geodesy
9. JSG 0.9: Future developments of ITRF models and their geophysical interpretation

It is quite clear that these study groups cover a wide field; they are however Earth-centric. That is of course no surprise, as the commissions and joint study groups support the requirements of the three classical pillars of geodesy. These basically are:

1. changes in Earth's shape (geokinematics),
2. its gravity field, and
3. rotation and orientation of Earth.

Taken together, these three classical pillars provide an underlying conceptual framework as well as observational and technological systems for the realization of the reference frames required for Earth observation. In addition, these 'three pillars' are synergistically interlinked and relate to one another through the Earth system processes.

2 Roles of the Reference Frames

The most important component in all of geodesy and in particular space geodesy is the International Celestial Reference Frame (ICRF) as determined by Very Long Baseline Interferometry (VLBI). No doubt this statement will be frowned upon by some, but this is simply due to the fact that the ICRF provides the only absolute frame of reference, which needs to be of the highest possible accuracy. In addition, all Earth-bound reference frame requirements, which depend on the International Terrestrial Reference Frame (ITRF), access the ICRF through the Earth orientation parameters (EOPs). This necessitates ultra-high accuracy EOPs. These accuracy requirements spill over into specifications for the Global Geodetic Observing System (GGOS), where the long-term accuracy and maintenance of the reference frames at very high level are major requirements. The ICRF, ITRF, and EOPs are the mandate of the International Earth Rotation and Reference Systems Service (IERS). EOPs and the ITRF are determined as multi-technique solutions and rely on high quality contributions from the different space geodetic techniques. High accuracy objectives such as maintenance of the ITRF at an accuracy level of 1 mm and stability level of 0.1 mm per year are difficult objectives (a factor of ten or more improvement on the current ITRF2008), which can only be attained if the ICRF and its variations in time can be maintained and the ICRF can be densified at a sufficiently accurate level. In addition to extreme demands by users on Earth (e.g., requirements of ocean level variation due to climate change), more demands for high accuracy space geodetic products for application in the solar system, as well as for astrometric VLBI, tests of GTR, interplanetary (and beyond) navigation will soon become common. When one therefore considers the combination of space and terrestrial geodetic techniques, measuring the complexities of the Earth system, through the three classical pillars of geodesy, the reference frames are the most important common denominator, with the ICRF being the crème de la crème. If the ICRF cannot be measured and maintained to a very high level (eventually micro-arcsecond level), which includes the optical and radio reference frames, it will not be possible to meet the GGOS objectives for the ITRF. From the viewpoint of its instrumental capacity, GGOS has six major levels of instrumentation and reference objects. These are not all fully developed.

- **Earthbound:**
  - **Level 1:** Terrestrial geodetic infrastructure (e.g., VLBI, SLR, GNSS, DORIS, and InSAR).
- **Spacebound:**
  - **Level 2:** LEO satellite missions;
  - **Level 3:** GNSS and the Lageos-type SLR satellites;
  - **Level 4:** Planetary missions and geodetic infrastructure on the Moon and planets;
  - **Level 5:** Specialized geodetic space platforms (e.g., orbiting laser transponders, VLBI satellites, Moon-bound tethered reference satellites, optical reference frame satellites); and
  - **Level 6:** Extragalactic objects.

3 Geodesy is Not *Ge*odesy Anymore

The term *geodesy* has become a bit of a misnomer ("terme inapproprié") as geodesy is not Earth-centric anymore. The term *ge* (earth) must be read in a wider context today. As example, what does the 'geodesy of the Moon' mean? Geodesy is derived from the Greek $\gamma\varepsilon\omega\delta\alpha\iota\sigma\iota\alpha$ (geodaisia), of which the literal meaning is 'division of the Earth'. However, in modern times, components of geodesy, in particular 'space geodesy', are moving towards measurement and representation of the solar system, and its place in spacetime curvature, which brings us to the Greek *chronochora*, an abstract model of space and time, i.e., *chronochora-daisia*, or *spacetimedesy*. As example, measuring the deflection of a star's light or a quasar's radio emission as it bends in the spacetime curvature of a massive object such as the Sun or Jupiter is spacetimedesy, not space geodesy. The VLBI technique in particular measures in spacetime curvature, through vast distances, through the 'local universe' and beyond. Observing through the galactic bulge must be considered as a current spacetime curvature problem and has to be accounted for when ICRF sources are being observed. Space geodesy is a good tool to test GTR. In particular, VLBI is suitable for deflection tests (Müller et al., 2008; Heinkelmann and Schuh, 2009). If GGOS is developed to its full potential, space geodesy will remain a strong contender in reliable tests of relativity, possibly reaching adequate test accuracy to evaluate alternative (scalar-tensor) theories, which predict small deviations from GTR values at a level of $|\gamma| \approx 10^{-6} - 10^{-7}$ (Combrinck, 2012).

4 Through the Looking Glass of Spacetime Curvature

The effect of weak gravitational lensing on the stability of the ICRF was already considered to some extent (*cf.* Hosokawa et al., 1997; Zharov et al., 2000), concluding that weak gravitational lensing places a limit on the realization of the ICRF. The implications on GGOS objectives have not been quantified. A certain amount of effort will have to be dedicated to improving our understanding of how spacetime affects our observations and long term reference frame stability. Spacetime curvature, i.e., the 'gravity field' of space is already incorporated in space geodesy measurements and data analysis as a matter of routine to some level as it affects VLBI, SLR, and GNSS measurements. Locally, in the solar system, VLBI has to account for deflection of the radio signal emitted from the VLBI source due to spacetime curvature caused by the Sun and large planets such as Jupiter. In the case of GPS, one has to consider that due to spacetime curvature a clock runs faster in a weaker 'gravity field', but slower due to relativistic motion. In the case of GPS, spacetime curvature wins, so the onboard clocks are set slower before being placed in orbit. The SLR model includes acceleration described by the Schwarzschild metric, frame dragging (Lense-Thirring) and geodetic (de Sitter) precession, and correction for spacetime curvature (Shapiro delay). These models and relativistic corrections are discussed in detail in Combrinck (2013) and with considerable detail concerning VLBI in Schuh and Böhm (2013).

5 So Spacetime Is Not 'flat'

Known gravitational lenses in the VLBI databases are excluded from ICRF solutions as discussed in IERS Technical Note 35 (IERS, 2009). As spacetime curvature (microlensing/weak lensing) has become an observable (Figure 1), and will become so more and

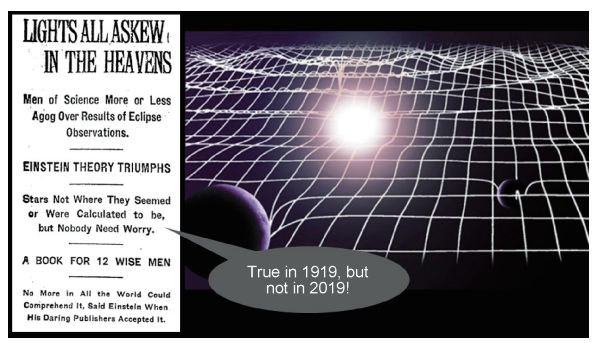

Fig. 1 Spacetime curvature has become an observable as we move into μas measuring accuracies.

more as the geodesy community strives to meet GGOS requirements (GGOS requirements for the ICRF is 25 μas accuracy, with a stability of 3 μas per year) the fourth pillar of geodesy must be taken into account. As a summary of the four pillars of geodesy we then have measurements of:

- **First pillar:** changes in Earth's shape (geokinematics),
- **Second pillar:** its gravity field,
- **Third pillar:** rotation and orientation of Earth, and
- **Fourth pillar:** spacetime curvature:
  - Positions and orientation in spacetime,
  - reference frames in spacetime, and
  - how these affect the three classical pillars.

Spacetime curvature (i.e., the gravity field of space) thus becomes the fourth pillar of modern geodesy, or if you want to, 'spacetimedesy'. This fourth pillar affects all high accuracy space geodesy measurements.

6 Current Assumptions

For most general cases relevant to gravitational lensing, the assumption is made that the Friedmann-Lemaître-Robertson-Walker metric describes the geometry of the universe and that the matter which leads to the lensing are no more than local perturbations (Naryan and Bartelmann, 1995).

When considering microlensing (temporary increase of source amplitude as a result of magnification) some assumptions are usually made:

- The effect of spacetime curvature is dominated by a single localized collection of matter at some point between source and observer (thin lens approximation).
- Therefore lensing is assumed to occur at a single distance.
- In most astronomical cases this is justified as the 'lens thickness' and it is relatively small compared to the typical distances of order few Gpc between observer and lens or lens and background quasar/galaxy, respectively (and galaxy sizes $\sim$50 kpc, galaxy cluster $\sim$2 Mpc).

These assumptions are only valid if the Newtonian potential is small $|\Phi| \ll c^2$ and if the relative velocities of lens, source, and observer are small compared to the velocity of light so that $v \ll c$.

A complex two-dimensional magnification distribution in the plane of the source results from the microlenses which is made up of many caustics which correspond to locations of formally infinite magnification. Apparent brightness and positional changes can occur as a function of time due to relative motion of the observer, lens, and source (Wambsganss (1998). In terms of the ICRF, the effects due to amplitude variation and apparent positional changes should be evaluated. Microlensing effects will have to be considered individually for each reference quasar, with the possible development of a subset of 'super stable' ICRF sources, i.e., those with minimal microlensing position and amplitude distortion. There are several problems with the basic assumptions normally made for microlensing:

- Dark matter: baryon acoustic oscillations have left voids regularly of $\sim$150 Mpc diameter, surrounded by the galaxies, but what is in between? Dark matter creates the gravitational framework for baryonic material, it thus creates a spacetime framework for the ICRF. Deflection of VLBI source signals is dependent on the total matter density and does not differentiate between dark or ordinary matter. What do we know about the halo's of dark matter around galaxies concerning the ICRF?
- Dark energy.
- The 'thin lens' may be stacked lenses (multiple lenses).
- Ultra-weak lensing effect for high-resolution, high-accuracy astrometry may be a cumulative effect creating 'spacetime curvature seeing' effects.
- Eventually, we will find that all VLBI source structure stability is affected by spacetime curvature dynamics at some level, impacting on our 'stable' ICRF.

As example, the galaxy cluster Abell 2218 is the widest separation lens system to be detected in the radio spectrum so far; maximum source separation is 41 arcsec. We have to consider that 'subtler' curvature effects than is seen in Abell 2218, e.g., weak lensing (which may have light bending effects that cannot be determined under the assumptions previously mentioned, but rather in a statistical way) could have an adverse impact on the ICRF, leaving us with the question: how much do we know about the effects of microlensing/weak lensing on the stability of the ICRF? Weak lensing is likely to be the main problem considering the ICRF, as basically all of spacetime is effected by the geometric distribution and variations in spacetime curvature, which is also dynamic in time, demanding continuous monitoring and maintenance of a spacetime curvature defined ICRF.

7 Recommendations

- The three pillars of geodesy should be extended to have a fourth pillar (spacetime curvature) as achieving the GGOS objectives of 1 mm accuracy in the ITRF (i.e., all the space geodetic techniques) and 0.1 mm ITRF stability is finally constrained by our understanding of the geometry of spacetime (and its variations in time).
- A working group, in close collaboration with the IERS/IVS Working Group should be formed to consider aspects of spacetime curvature (microlensing, weak lensing, local solar system deviations from asymptotically flat space) and its effects on the ICRF and how it will affect the three classical pillars of geodesy. To some extent this exists, but not formally, so this issue is perhaps not being addressed to the extent it should.

8 Conclusions

Support by GGOS for VLBI, SLR, and LLR will improve validations of GTR, but only to its fullest extent if all aspects of GGOS are addressed: networks, spacetime curvature, equipment, models, observing strategy, and processing strategies. Spacetime curvature dynamics could create "spacetime curvature seeing" effects. The possible effects of spacetime seeing should be considered in the light of extreme accuracy requirements for the ICRF and ITRF. In the near future, spacetime curvature as an observable cannot be excluded from any ICRF measurements and maintenance. It is suggested that a sub-set of super stable ICRF sources be identified which show minimum positional distortion and amplitude variation due to microlensing or weak lensing to ensure high level integrity of the ICRF.

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