To: Ad Hoc Working Group on HF-EOP

From: John Gipson

Re: Time scales used in calculation of HF-EOP

Date: October 3, 2017

**Summary**

This note grew out of conversations with Richard Ray on the appropriate times to use in the calculation of HF-EOP variation. Should you use UT, TT, or some combination? It turns out that you should use TT for calculation of some things, and UT for calculations of others.

A different, but related issue, is that in studying some of the software written to compute HF-EOP (including that at use at Goddard) at least some of the software uses a single kind of time to compute everything. It was not initially clear to me if this would lead to significant errors or not. The answer is basically no.

* If you use UT for all calculations, the maximum error in Polar motion is about 0.2 μas and the maximum error in UT1 is about 0.02 μts.
* Using TT to calculate GMST results in an error which is 20-30 times as large. For Polar Motion it is about 7 μas, and for UT1 it is about 0.5 μts.

If you are going to use just one time, it should be UT, and not TT.

**Tidal Form of IERS model and Correct Time to use.**

There are two forms of the IERS model. One form gives a model in terms of ortho-tides, and the other in terms of coefficients at specific tides. In this note I only focus on the tidal form of the model. The tidal terms are specified by 6 time-varying angles $\vec{α}$. The general form of the tidally induced variation in, e.g., UT1 is:

 $δUT1=\sum\_{}^{}A\_{j}sin(\vec{a\_{j}}∙\vec{α\_{j}})+\sum\_{}^{}B\_{j}cos(\vec{a\_{j}}∙\vec{α\_{j}})$

The $A\_{j}$ and $B\_{j}$ are the amplitudes for a particular tide in the model (NOT the amplitude of the tidal potential), and the $\vec{a\_{j}}$ are a set of 6 integers specifying the tide: The K1 tide is (0,0,0,0,0,0,1); The M2 tide is (0,0,-2,0,-2,2). The period of the fundamental arguments and GMST is summarized in the table below.

|  |  |  |
| --- | --- | --- |
| Argument | Description | Period |
| l  | Mean anomaly of the moon | 27.65D |
| l’ | Mean anomaly of the sun | 365.25D |
| F | Mean longitude of the moon-Omega | 27.21D |
| D | Mean Elongation of the moon | 29.53D |
| Ω | Mean longitude of ascending node of the moon | 6798.29D |
| GMST | Green mean sidereal time  | 1.00D |

Note from this it is clear that K1 is a diurnal tide, since the multiplier of GMST is 1, and M2 is diurnal, with a multiplier of 2.

In calculating the $\vec{α}$ you should use two different kinds of time.

1. For the 5 fundamental arguments of nutation you should use TT.
2. For GMST+π you should use UT.

I will call this time dependence TT\_UT.

In the routines I have seen the calculation of the 6 angles specified above are all done in a single routine which is passed a single time value, the Julian day (including fractional part), although it is not clear if the Julian days are TT or UT. In any case, this leads to two kinds of errors.

1. In calculating the fundamental arguments you use UT instead of TT. I call this time dependence UT\_UT.
2. In calculating GMST you use TT instead of UT. I will call this time dependence TT\_TT.

The difference between UT and TT depends on the number of accumulated leap-seconds plus a constant offset, and hence depends on the epoch. In 2008 it was 62.18sec, with TT being ahead of UT. Hence (in 2008) using UT instead of TT, or TT instead of UT, is the same as the time being off by 62 seconds.

One can get a rough sense of the error if one uses TT when computing GMST, as follows: The error in time is about 1 minute. For a semi-diurnal tide, the frequency is ~30°/h, so a 1-minute time error gives a phase error of 0.5°. For a constituent of amplitude 20 μts, the error could be as much as 20 sin(0.5°) = 0.2 μts for a single term. Since the change in UT1 is the sum over many tides, the total error might be much larger. Going through a similar argument for using UT instead of TT when computing the fundamental arguments leads to the conclusion that the effect should be very small. I verify both of these below.

**Method**

Using one of the Goddard HF-EOP models, I computed the HF variation in Polar Motion and UT1 on an hourly basis for the full year 2008. I looked at three different cases.

1. TT\_UT: TT is used to calculate the fundamental arguments. UT is used to calculate GMST. This is what you should do.
2. UT\_UT: UT is used to calculate the fundamental arguments. UT is used to calculate GMST. This is incorrect.
3. TT\_TT: TT is used to calculate the fundamental arguments. TT is used to calculate GMST. This is also incorrect.

I then found the time difference in the Polar Motion and UT1 time series between cases 1&2 and cases 1&3. The results are summarized in the table below, and in the following figures.

|  |  |  |  |
| --- | --- | --- | --- |
|  | X (μas) | Y(μas) | UT1(μts) |
| TT\_UT-UT\_UT: RMS | 0.09 | 0.06 | 0.01 |
| TT\_UT-UT\_UT:Max | 0.24 | 0.18 | 0.02 |
| TT\_UT-TT\_TT: RMS | 2.56 | 1.71 | 0.18 |
| TT\_UT-TT\_TT:Max | 6.93 | 4.45 | 0.50 |

Note that the effect of using UT to calculate the fundamental arguments is very small, and results in a maximum error in Polar motion ~ 0.2 μas and a maximum error in UT1 of 0.02 μts. Using TT to calculate GMST results in an error which is 20-30 times as large. For Polar Motion it is ~ 7 μas, and for UT1 it is about 0.5 μts. The conclusion from this is that if you are going to use a single kind of time to calculate both, it should be UT. (But, given the current level of accuracy, it is probably OK to use TT.)

It turns out that at Goddard we used the third kind of time arguments when we used our HF-EOP model. Effectively this means that our estimates for the $A\_{j}$ and $B\_{j}$ are slightly off, on the order of 0.2 μts for a 20 μts tide. However if you use our estimated coefficients, together with our software to calculate the value of the fundamental arguments, the answers should be roughly correct.

The figures on the following pages plot the difference using the correct kinds of time, and the two types of error. For all three EOP components I include plots of all of 2008 and of just the first week.

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