

To: Ad Hoc Working Group on HF-EOP
From: Richard Ray (GSFC)
Re: Indexing and argument conventions for tides
Date: December 4, 2017

Summary

I would like to urge this working group—and hopefully eventually the wider tidal EOP community—to adopt tidal indexing and tidal argument conventions that are more consistent with those commonly used by the Earth tide and ocean tide communities. More specifically I urge adoption of a convention that follows more closely Doodson’s elegant 1921 scheme, slightly extended.

An immediate advantage of doing this would be that advances in ocean-tide modeling can be more readily converted into new models for the geodetic EOP community, without the on-going confusions and errors that occur now. But aside from the issue of consistency among different communities, there are other reasons to use standard ocean-tide conventions, since they more readily help us understand underlying physical differences among models. Current EOP models in the form now tabulated by the IERS tempts many users to think of these models as black boxes of random numbers. One could thus even argue there are strong *aesthetic* reasons for following Doodson.

What I will here call the Woolard convention dates back to the 1950s and to his development of the tidal potential for use by the nutation community¹. Woolard (1953) expressed his tidal arguments as linear combinations of the fundamental variables that E. W. Brown (and others earlier) employed in his lunar theory. The tidal arguments adopted by Woolard (1959) are closely related. In my humble opinion, Doodson’s earlier convention is much preferable. As is well known, Doodson, who also used Brown’s lunar theory, re-expressed his final series in terms of six fundamental variables, all relatable to those used by Woolard. One great advantage of Doodson’s system stems from the very different temporal rates of the six variables, so that when tidal lines are tabulated by frequency, the integer expansion coefficients fall into a simple, orderly pattern. Or as Doodson himself wrote: “it is a curious fact that if we

¹I’ll be indebted to anyone wishing to correct my (limited) historical understanding of how we ended up with the present conventions.

classify in terms of τ [lunar mean time], with a sub-classification with regard to s [the moon’s mean longitude], and a further sub-classification with regard to h [the sun’s mean longitude], the constituents are completely separated into groups with no overlapping of speeds [frequencies]. It is still more curious that to the order required the same process can be continued for all the variables. Owing to this, a rather elegant and very useful form of presentation of the results is possible” (Doodson, 1921).

Let us write the integer Doodson argument indices (without the often-added ‘5’s) as the 6-integer set $(k_1, k_2, \dots k_6)$. To this standard Doodson argument I find that adding a seventh index, k_7 , being a simple integer multiple of 90° , is useful and makes the whole argument clearer to everyone. This multiple of 90° then permits the cosine function to be used consistently with all arguments and ensures all amplitudes are positive. (The original expansions of Doodson and Cartwright-Tayler, and likewise Yoder et al., required both positive and negative amplitudes, and sine functions for diurnal waves.) Sometimes this additional k_7 is left unmentioned, which tends to confuse non-experts. Many modern texts (e.g., Pugh & Woodworth, 2014, Table 4.1²) do include this extra multiple of 90° , at least implicitly, if not explicitly as the additional k_7 .

To see how this plays out, consider Table 1 where the UT1 coefficients of a few of the major diurnal constituents are listed (by frequency) in the two different conventions. As Doodson’s quotation above stresses, his integer indices fall in a clear orderly pattern, automatically sorting the constituents by frequency. The Woolard indices, aside from the first one that denotes a diurnal wave, appear in almost random order.

Or consider the two spectral lines that make up the (gravitational) S_1 constituent (I didn’t include these in Table 1). In Doodson’s scheme we have

τ	s	h	p	N'	p_s
1	1	-1	0	0	-1
1	1	-1	0	0	1

Anyone looking at these indices would know immediately that the two spectral lines are of almost identical frequencies, differing only by 2 cycles in 20,000 years. The Woolard scheme for the S_1 lines is

γ	l	l'	F	D	Ω
1	0	1	-2	2	2
1	0	-1	0	0	0

I defy anyone to say they can look at these indices and know immediately that the two lines have nearly the same frequency!

²But watch for a typographical error in Pugh-Woodworth’s argument of K_1 .

Table 1: Coefficients of selected diurnal UT1 constituents*

Woollard convention (IERS, 2010)									
Tide	γ	l	l'	F	D	Ω		Cos	Sin
σ_1	1	0	0	-2	-2	-2		-0.39	1.19
Q_1	1	-1	0	-2	0	-2		-2.50	5.12
ρ_1	1	1	0	-2	-2	-2		-0.47	0.91
O_1	1	0	0	-2	0	-2		-12.07	16.02
M_1	1	-1	0	0	0	0		0.75	-0.86
P_1	1	0	0	-2	2	-2		-3.10	5.51
K_1	1	0	0	0	0	0		8.55	-17.62
θ_1	1	-1	0	0	2	0		0.04	-0.29
J_1	1	1	0	0	0	0		0.19	-1.61
OO_1	1	0	0	2	0	2		-0.04	-1.44

Doodson convention (Chao, 1996)									
Tide	τ	s	h	p	N'	p_s	90°	Cos	Sin
σ_1	1	-3	2	0	0	0	-1	1.21	0.37
Q_1	1	-2	0	1	0	0	-1	5.03	2.45
ρ_1	1	-2	2	-1	0	0	-1	0.89	0.47
O_1	1	-1	0	0	0	0	-1	16.05	12.10
M_1	1	0	1	0	0	0	1	0.90	0.78
P_1	1	1	-2	0	0	0	-1	5.16	2.86
K_1	1	1	0	0	0	0	1	17.71	8.64
θ_1	1	2	-2	1	2	0	1	0.27	0.03
J_1	1	2	0	-1	0	0	1	1.52	0.08
OO_1	1	3	0	0	0	0	1	1.38	-0.24

* The same tide model is displayed in the two indexing conventions. The numerical coefficients differ slightly because the bottom table takes the coefficients directly from the paper of Chao et al. (1996), whereas the IERS coefficients went through various transformations, in and out of orthotides.

The numerical coefficients themselves also are more easily interpretable in the Doodson scheme. In Table 1, all sine and cosine coefficients have the same sign across the band, except at the edge of the band where the sine component of J_1 turns negative. All constituents have roughly similar phase lags on the potential, of order 30° or so, until again at the high-frequency part of the band above θ_1 where the phases become very small and then turn negative at J_1 . Dividing these coefficients by the magnitude of each constituent’s driving potential would yield admittance curves that are (mostly) a smooth function of frequency, a fact that Richard Eanes tried to exploit when he urged adoption of the response (orthotide) formalism; again this smoothness across the band is fairly evident from the tabulated coefficients (so long as we are aware of the rough relative sizes of equilibrium tides), thanks to the logic behind the Doodson layout.

Finally, it is worth noting that some standard nomenclature for tides makes sense only within the confines of Doodson’s scheme. Consider the technical definitions of the terms species, group, and constituent: All terms with the same value of k_1 constitute a tidal “species,” with different species separated in frequency by about one cycle per lunar day. All terms with the same (k_1, k_2) constitute a tidal “group,” with different groups separated by about one cycle per month. And all terms with the same (k_1, k_2, k_3) constitute a tidal “constituent,” with different constituents separated by one cycle per year. (Thus, despite the loose terminology often seen in the literature, a tidal constituent consists in general of a cluster of spectral lines, not a single isolated line.) As Table 1 shows, the identification of tidal species, groups, and constituents becomes automatic.

To those who argue that the Woolard convention is now firmly entrenched within the EOP community and things should therefore not be upended, I would argue that this may be true within the nutation community, but not the geodetic EOP community. In fact, the paper on which the current IERS-2010 model of EOP is based (Chao et al., 1996) uses the Doodson convention. Moreover, those worried about overturning traditions should consider Woolard’s original work on the tidal expansion, which was less comprehensive and (as argued above) less elegant than Doodson’s work done some thirty years earlier. In fact, Woolard (1959) later acknowledged Doodson’s earlier work, when he noted that his expansion could be checked by comparing it with Doodson’s.

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