IVS combination: Correlations between the different input series

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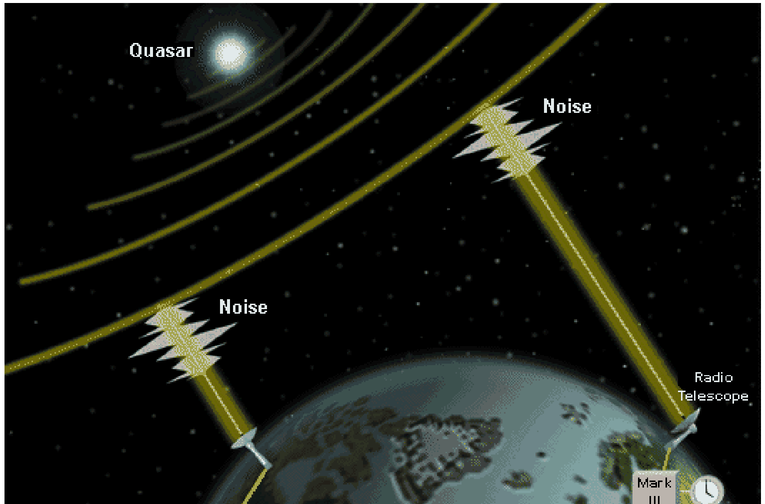
Motivation



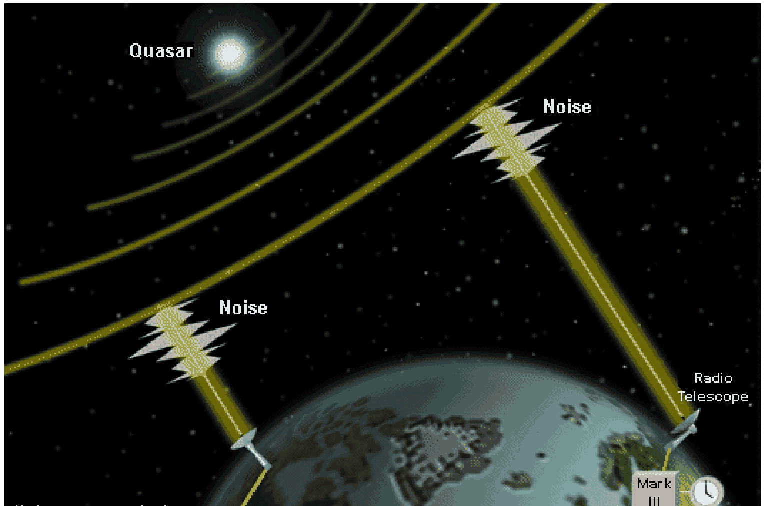
Correlator
τ,σ

Analysis Centers

AC1 AC2 AC3 AC4 AC5 AC6

Identical observations

Motivation



Correlator
τ,σ

Analysis Centers

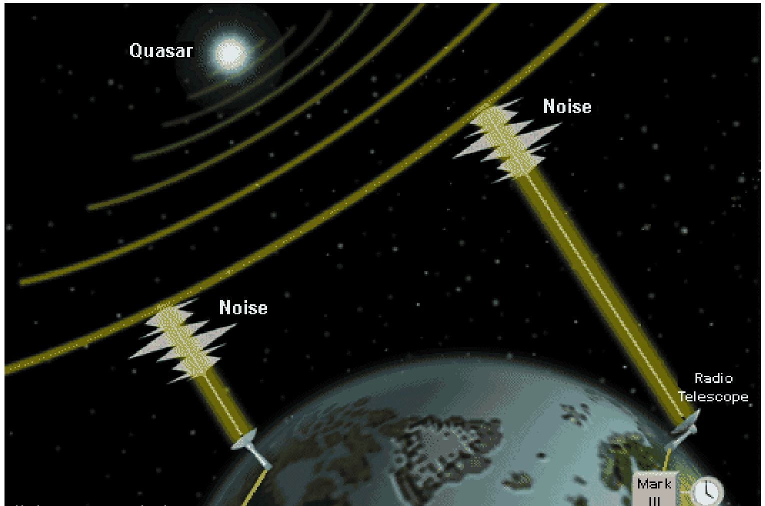
AC1 AC2 AC3 AC4 AC5 AC6

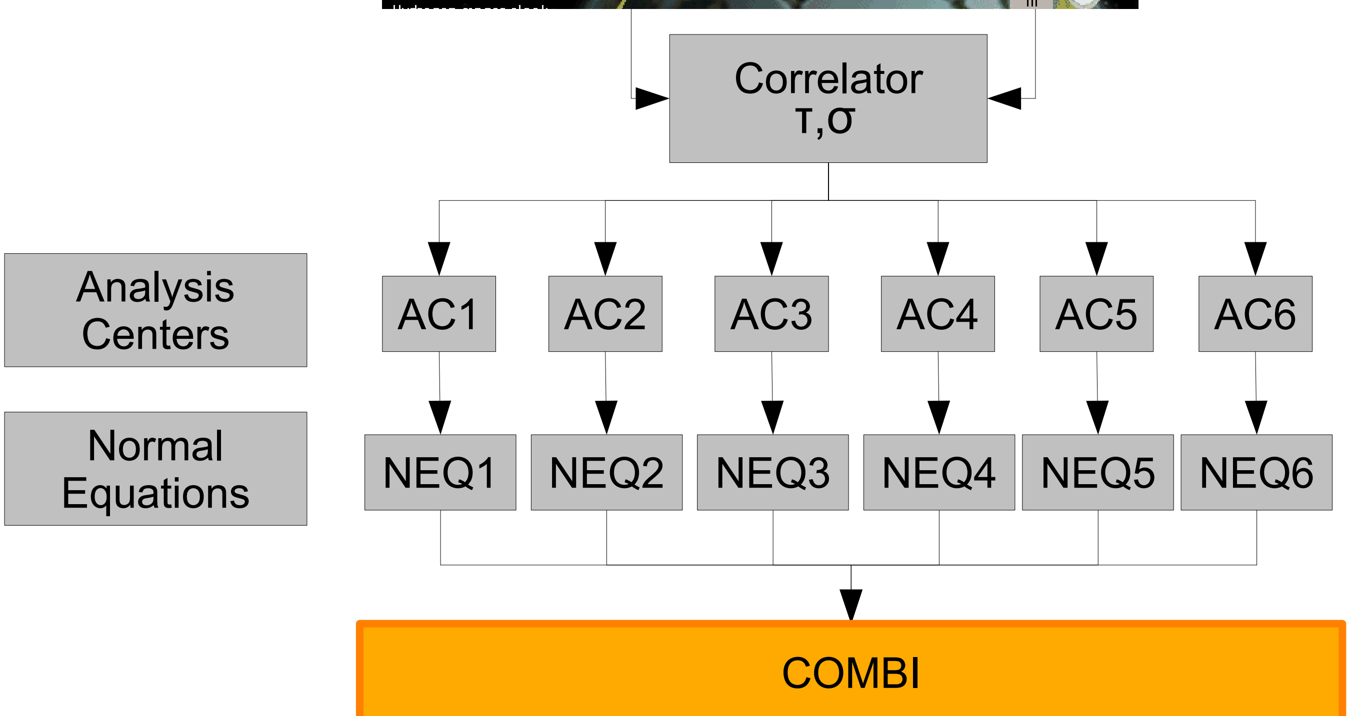
Normal Equations

NEQ1 NEQ2 NEQ3 NEQ4 NEQ5 NEQ6

Different analysis strategies

Motivation


Quasar
Noise
Noise
Radio
Telescope
Mark
III


Correlator
τ,σ
Analysis
Centers
AC1
AC2
AC3
AC4
AC5
AC6
Normal
Equations
NEQ1
NEQ2
NEQ3
NEQ4
NEQ5
NEQ6
COMBI

Motivation

Problem IVS (intra-technique) combination:

- multiple use of the same set of original observations
- different analysis options, but a number of identical models

=> Contributions of the ACs **cannot be completely independent**
BUT: treated as independent

Goal:

Introduce correlations to account for the dependence of the individidual contributions

Correlations:

Assumption 1: *n* independent contributions

=> correlation = 0

Assumption 2: *n* identical contributions

=> correlation = 1

Negligence of correlations

=> **estimated parameters**: no effect

=> **formal errors**: too optimistic by $\sqrt{n}$

IVS combination:

contributions not identical, not completely independent

* quantify the **level of correlations**

* investigate influence of **neglecting / considering correlations** on estimated parameters & formal errors

Combining Normal Equation Systems

functional model:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \hat{x}_c$$

stochastic model:

$$\Sigma\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} N_1^{-1} & 0 \\ 0 & N_2^{-1} \end{bmatrix}$$

adjustment:

$$(N_1 + N_2)\,\hat{x}_c = n_1 + n_2$$

Combining Normal Equation Systems

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adjustment:

$$(N_1 + N_2)\,\hat{x}_c = n_1 + n_2$$

=> Correlations cannot be included

Combining Observation Equation Systems

functional model:

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \hat{x}_c$$

stochastic model:

$$\Sigma\left(\begin{bmatrix} l_1 \\ l_2 \end{bmatrix}\right) = \begin{bmatrix} \sigma_1^2 \Sigma_{11} & \sigma_{12} \Sigma_{12} \\ \sigma_{12} \Sigma_{12} & \sigma_2^2 \Sigma_{22} \end{bmatrix}$$

adjustment:

$$\begin{bmatrix} A_1^T & A_2^T \end{bmatrix} \Sigma^{-1} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \hat{x}_c = \begin{bmatrix} A_1^T & A_2^T \end{bmatrix} \Sigma^{-1} \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

Combining Observation Equation Systems

functional model:

$$\begin{bmatrix} l_1 \\ l_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \hat{x}_c$$

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=> Correlations can be included

=> Observation equations are not available

Modification of Calc/Solve

=> extract observation equations

Test dataset: CONT02

Simulation of two solutions

- **different databases** of the same session (IVS data server & BKG)
  => different selection of outliers, clock breaks, calibration, ...
- **different analysis options**
  => different parameterization of ZWD, gradients, different weighting, a priori values for ZWD and gradients

Combination at the level of observation equations

=> combined parameters, correlations

Combining Observation Equations

Input

A_i ... jacobian matrix

l_i ... vector of observations

$\Sigma(l_i)$ ... covariance matrix of obs.

$\sigma^2_{0i}, \sigma_{0ij}$ ... a priori (co)variance components

Output

$\hat{x}_c$ ... combined estimates

$\Sigma(\hat{x}_c)$ ... covariance matrix

σ^2_i, σ_{ij} ... a posteriori (co)variance components

ρ_{ij} … correlations

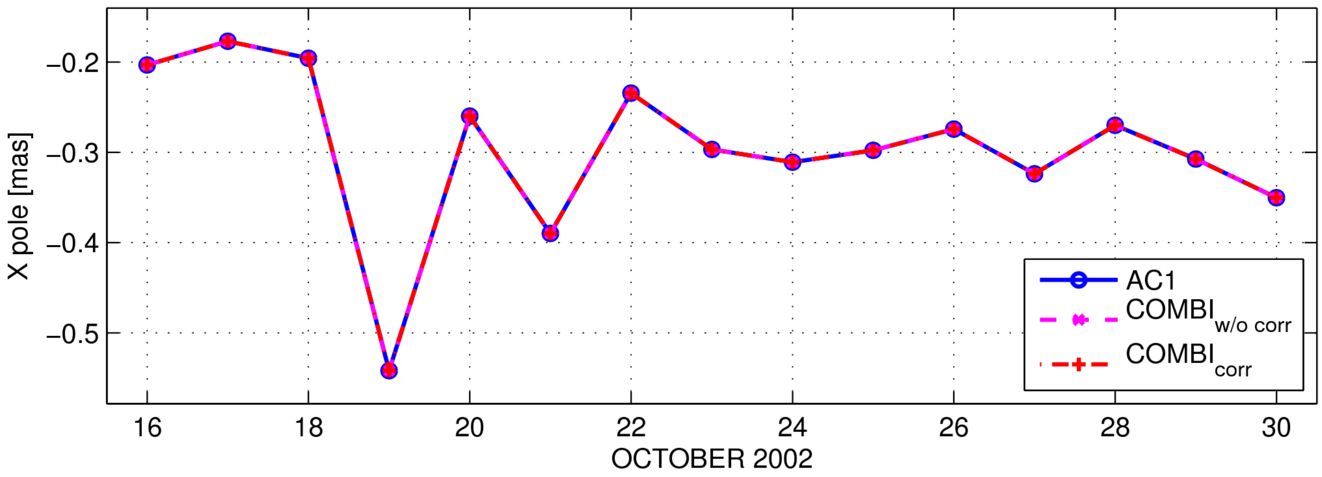
Variance- / Covariance Component Estimation

Combining 2 equal contributions (correlated by 1)

Consider / neglect correlations

=> **estimated parameters**: no effect

X-pole estimates



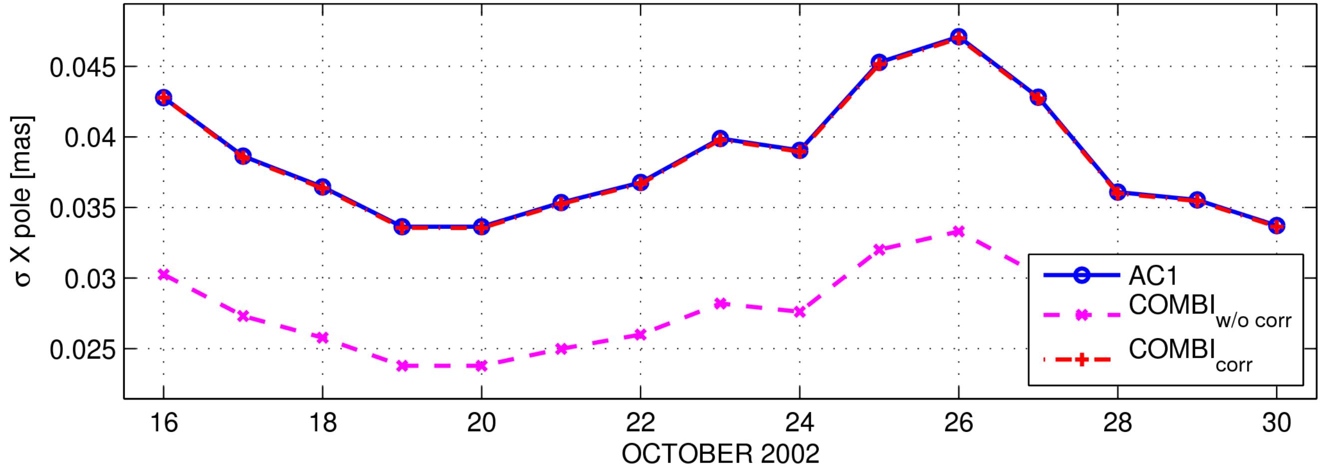
Combining 2 equal contributions (correlated by 1)

Consider / neglect correlations

=> **estimated parameters**: no effect

=> **formal errors**: too optimistic by $\sqrt{2}$

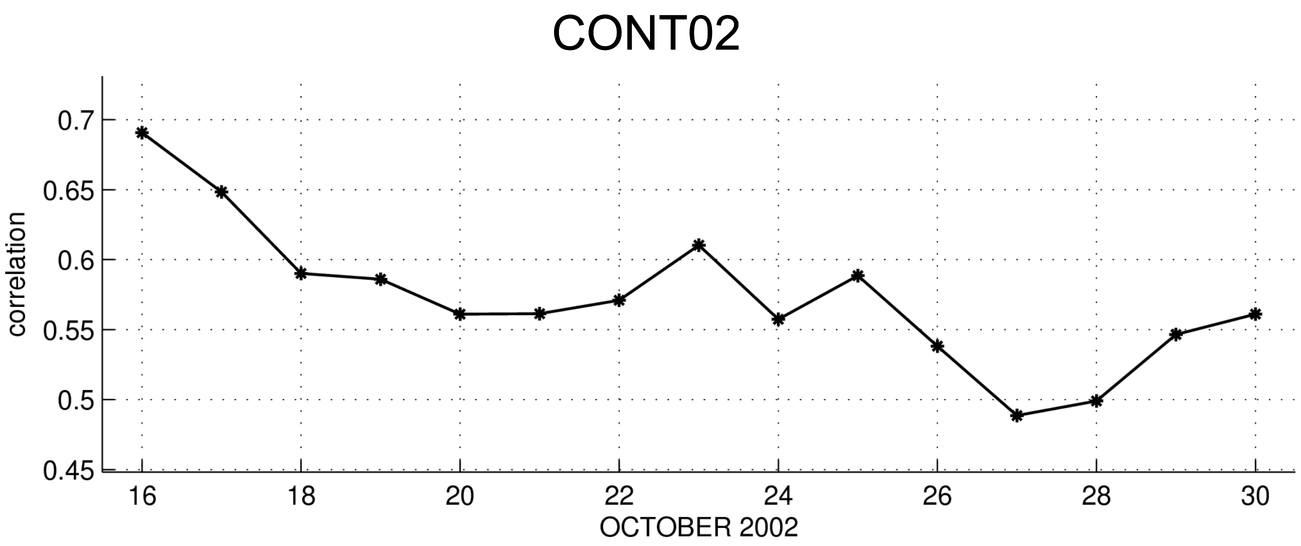
X-pole formal errors



Correlations

Combining 2 different contributions (correlated by ?)

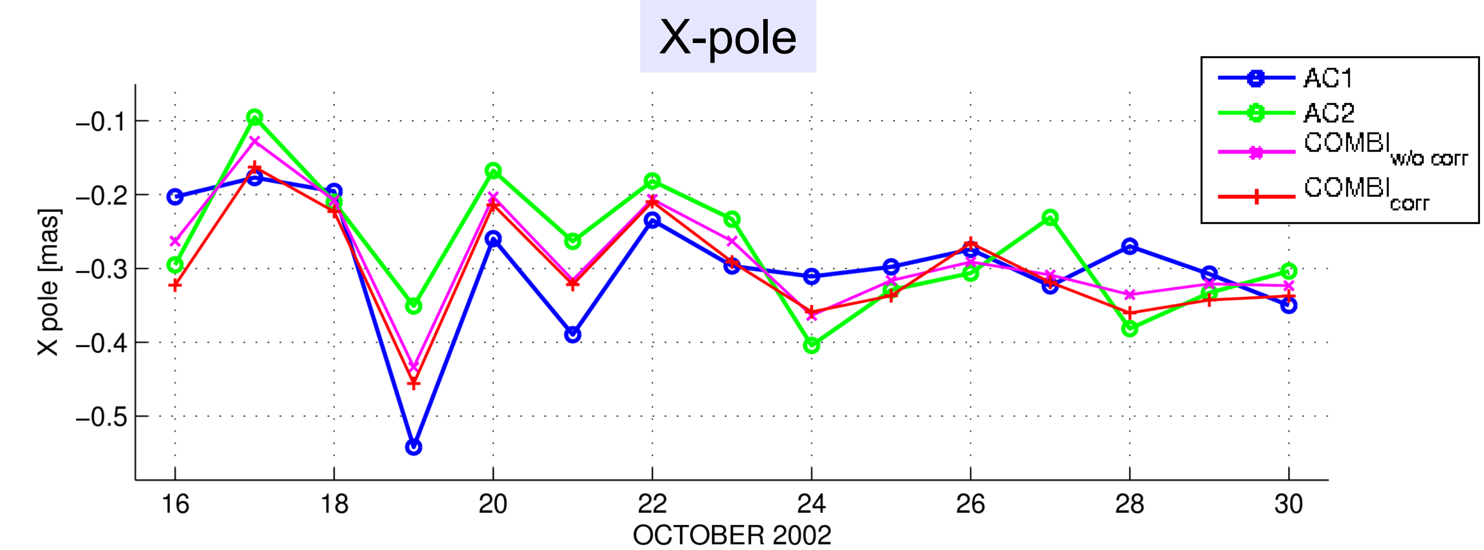
Correlations $\rho = \frac{\sigma_{ij}}{\sqrt{(\sigma_i^2 \cdot \sigma_j^2)}}$

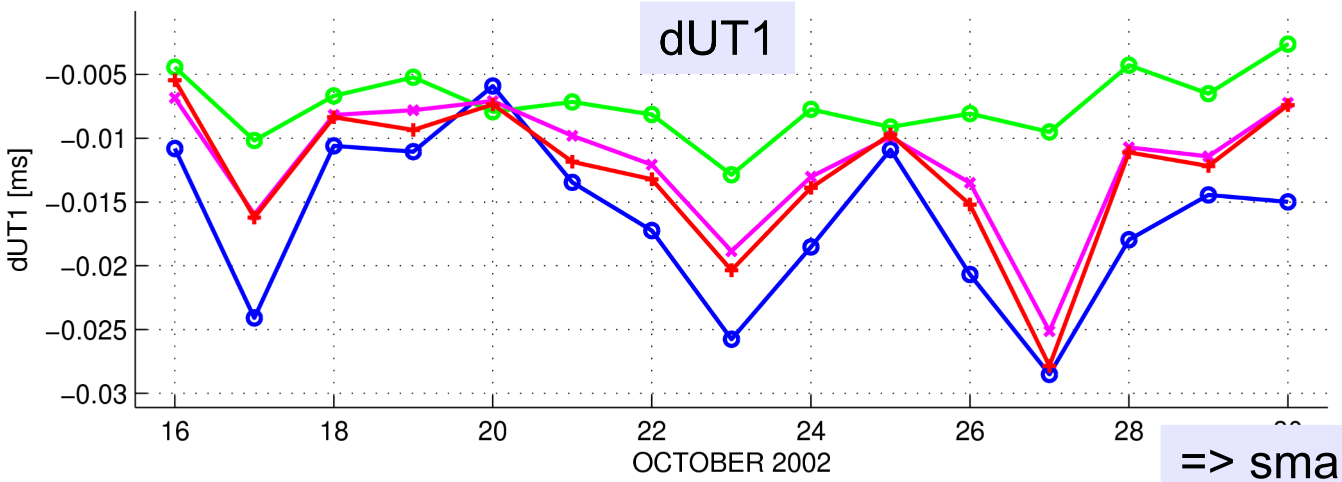


=> Correlations 0.5 - 0.7

=> Level of correlations not constant

Influence of correlations on **estimated parameters**

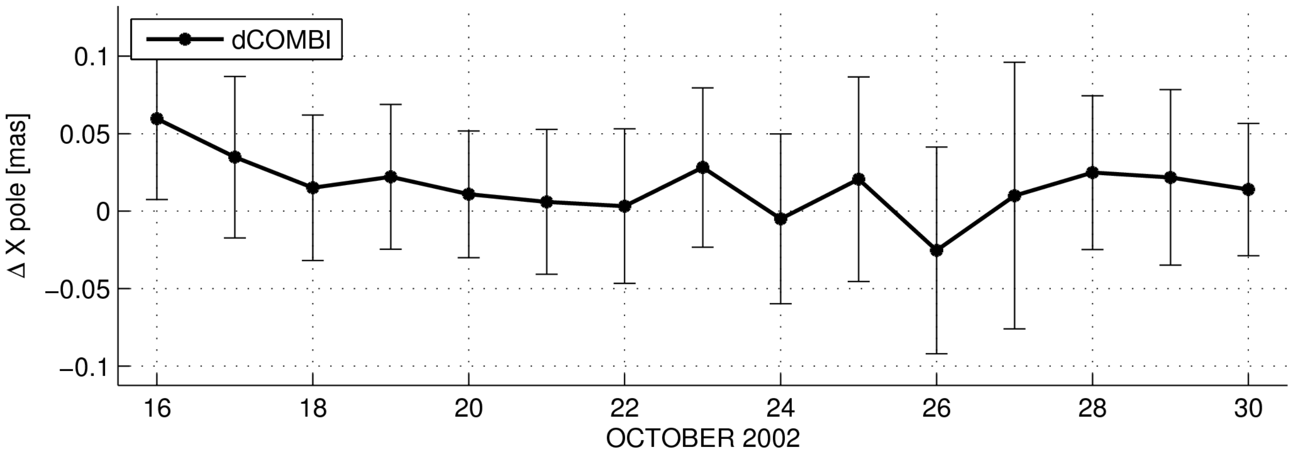


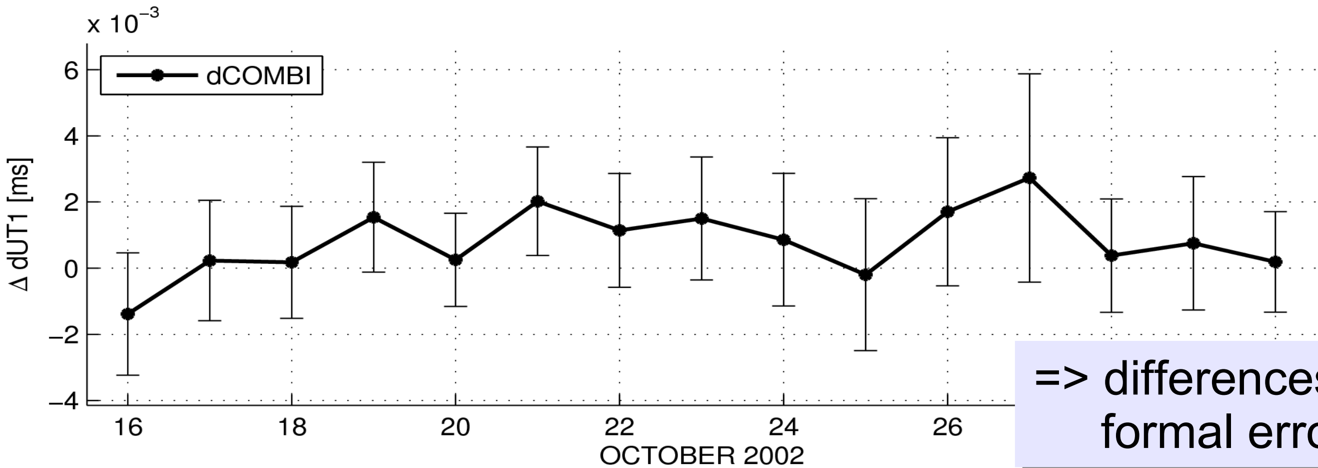


=> small differences

Influence of correlations on **estimated parameters**

COMBI$_{corr}$ – COMBI$_{w/o\ corr}$

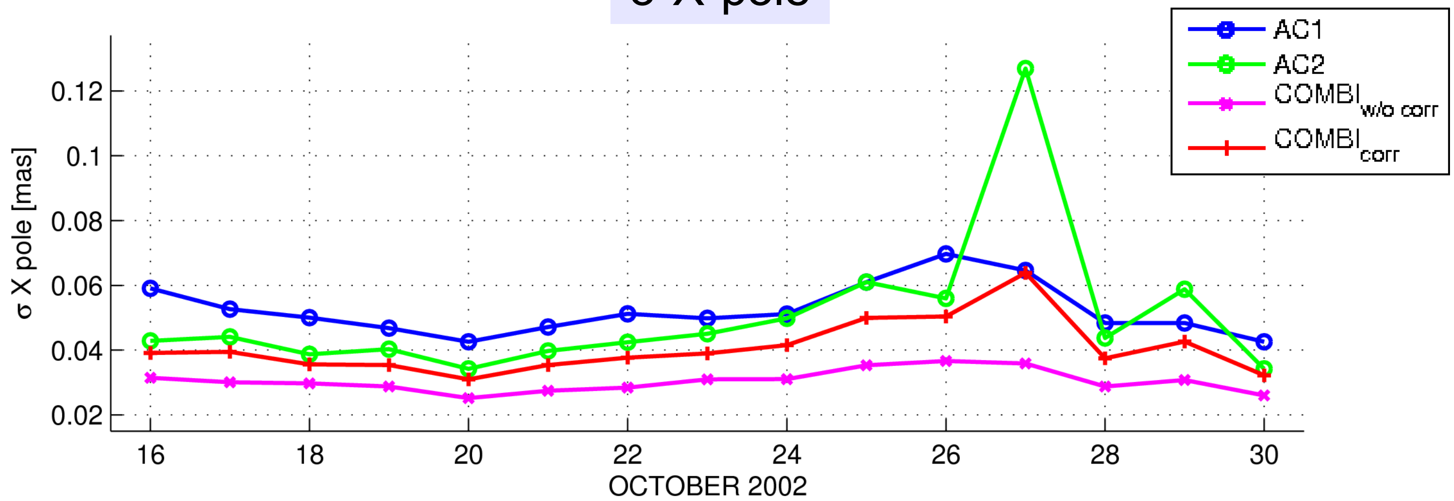




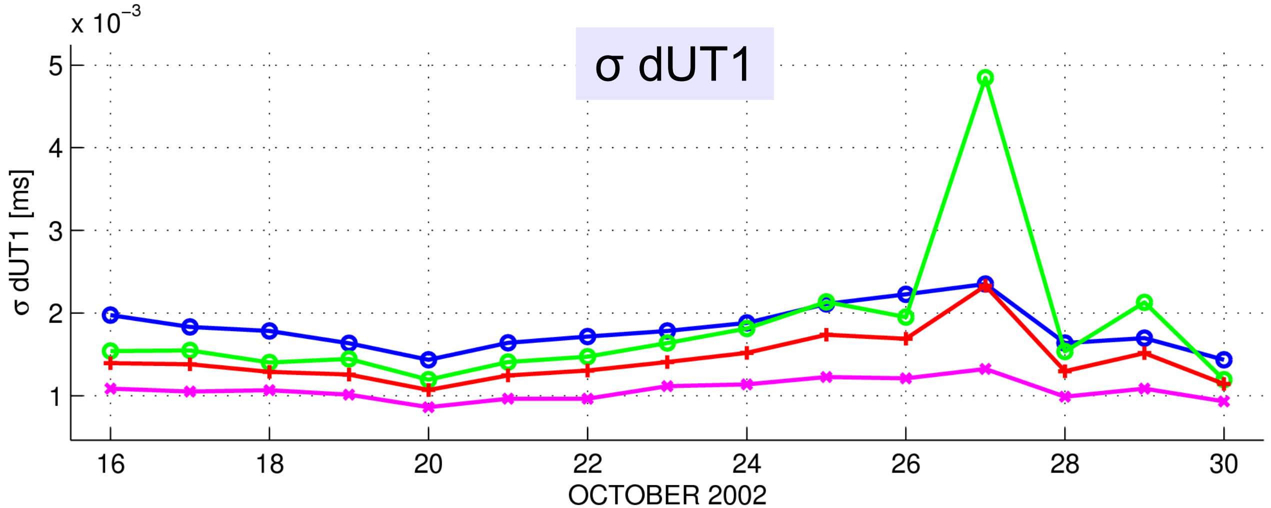
=> differences within formal errors

Influence of correlations on **formal errors**

σ X-pole

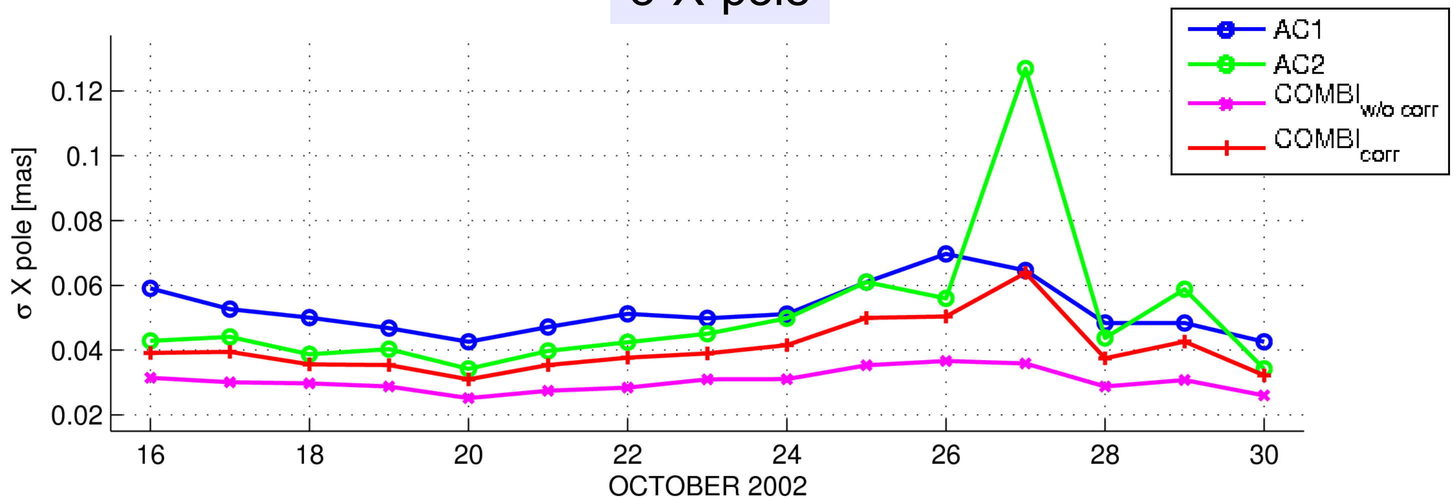


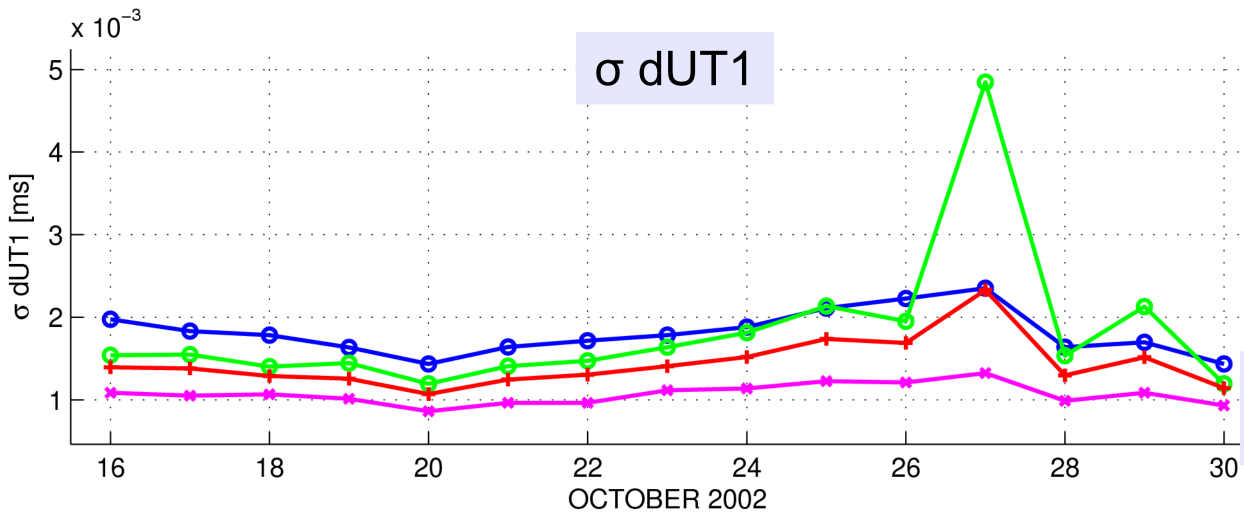
σ dUT1



Influence of correlations on **formal errors**

σ X-pole





=> formal errors bigger by ~1.22

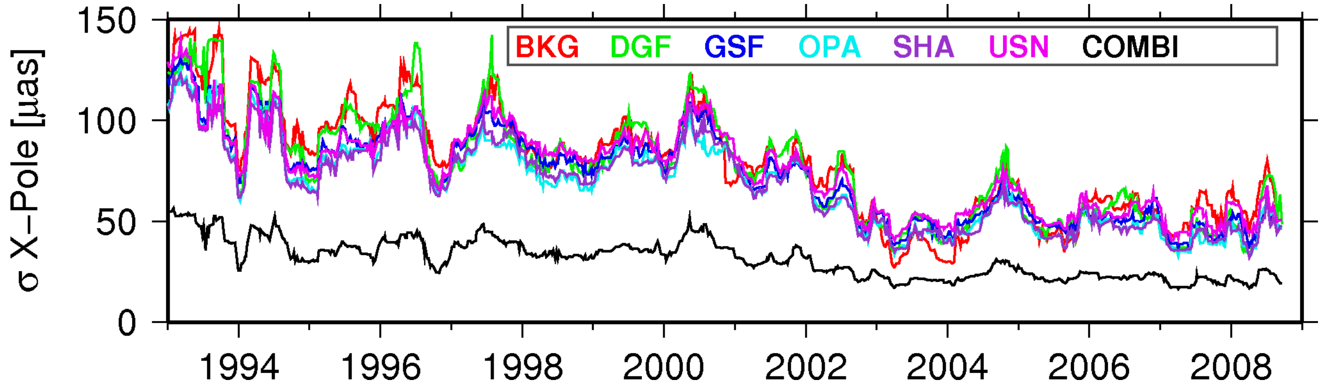
IVS combination

based on normal equations

=> Correlations cannot be included

=> too optimistic formal errors

formal errors



Determination of scaling factors

Assumption:

- combination = calculation of average
- Influence of correlations on formal errors (error propagation):

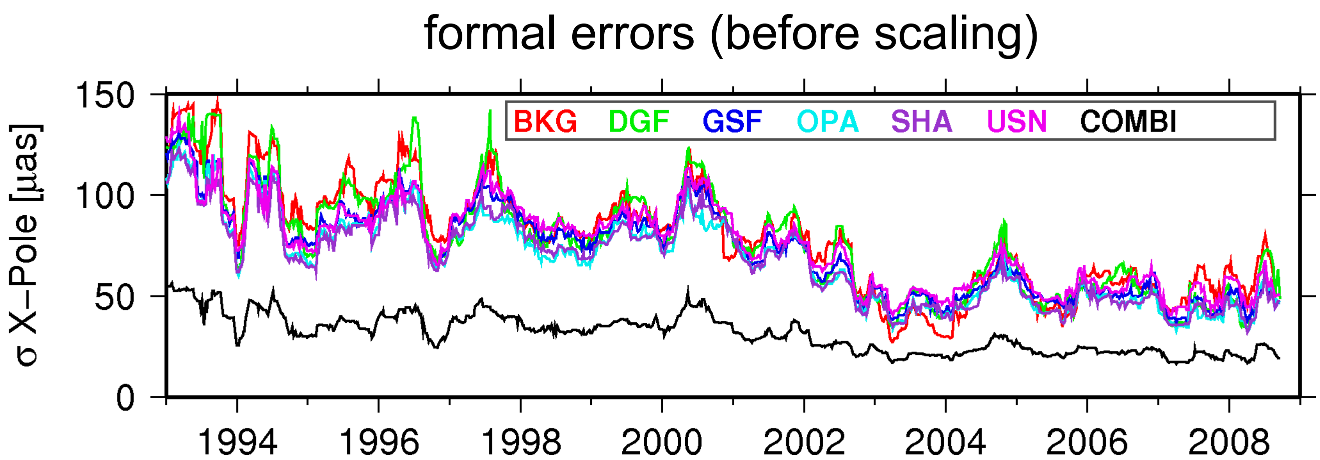
$$\sigma^2_{corr} = \left[\frac{1}{n}, \cdots, \frac{1}{n}\right] \cdot \begin{bmatrix} \sigma^2_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{1n} & \cdots & \sigma^2_{nn} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}, \quad \sigma^2_{w/o\,corr} = \left[\frac{1}{n}, \cdots, \frac{1}{n}\right] \cdot \begin{bmatrix} \sigma^2_{11} & & \\ & \ddots & \\ & & \sigma^2_{nn} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{n} \\ \vdots \\ \frac{1}{n} \end{bmatrix}$$

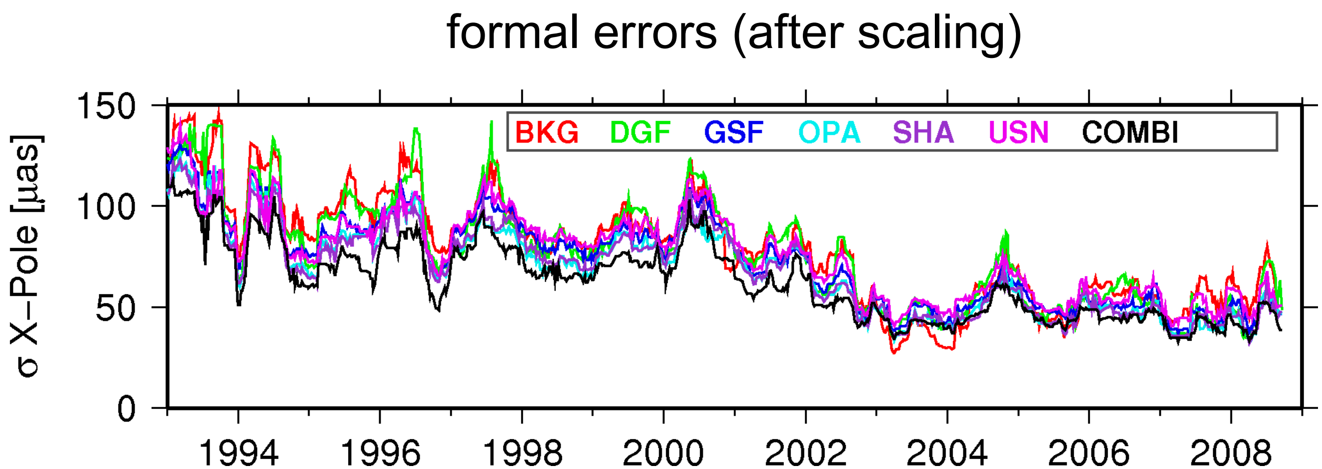
$$\Rightarrow scaling\ factor = \frac{\sqrt{\sigma^2_{corr}}}{\sqrt{\sigma^2_{w/o\,corr}}}, \qquad n = No\ ACs$$

- 6 ACs, equal precision for each AC, correlated by 0.6

=> scaling factor = 2

IVS combination





Conclusions

Simulations:

- quantify the level of correlations between contributions to IVS combination
  => significant correlations between 0.5 and 0.7

- influence of correlations on
  * estimated parameters: => small differences, but within formal errors
  * formal errors: => too optimistic formal errors if correlations are neglected

IVS combination:

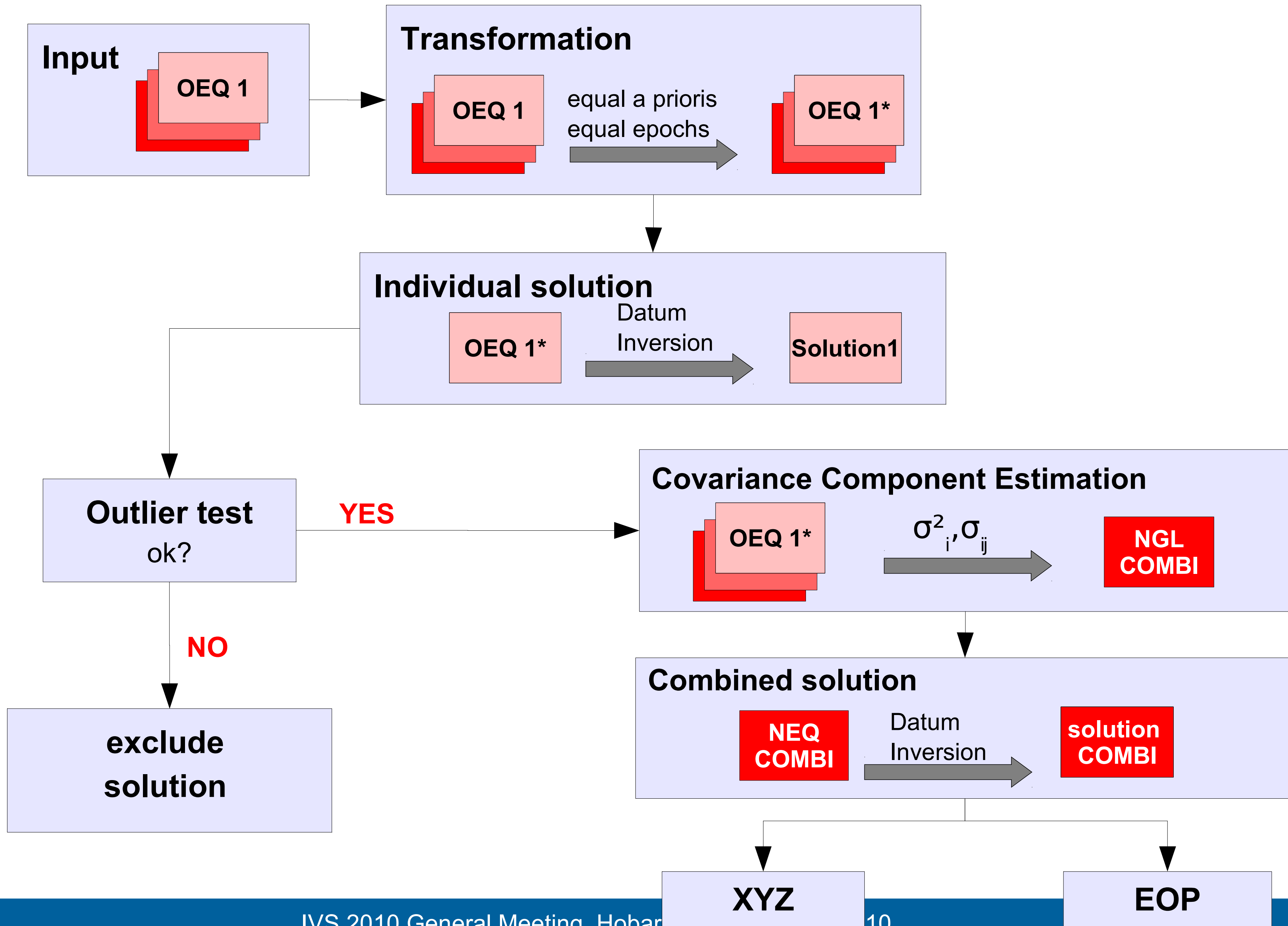
- based on normal equations
  => correlations cannot be included within the combination itself
- scale formal errors => factor of 2

Disadvantage: Simulations are carried out with one software only

=> smaller correlations using the results of different software packages?

Thanks to IAG for Travel support!

Combination Observation Equations



Correlations

Level of correlations by using different solution setups

analysis options	correlation
parameterization of ZWD & gradients	~ 0.90
a prioris for ZWD & gradients	~ 0.95
reference clock	~ 0.99
weighting	~ 0.80
different databases & different analysis options	~ 0.6

Influence of correlations on **estimated parameters**

Validation of scaling factors
by empirical approach

Comparison with independ EOP series (e.g. Bulletin A)

$$diff_{single} = EOP_{VLBI\,single} - EOP_{BullA}$$

$$diff_{combi} = EOP_{VLBI\,combi} - EOP_{BullA}$$

Accuracy:

- WRMS of differences: $WRMS_{single}, WRMS_{combi}$
- formal errors of differences: $STD_{single}, STD_{combi}$

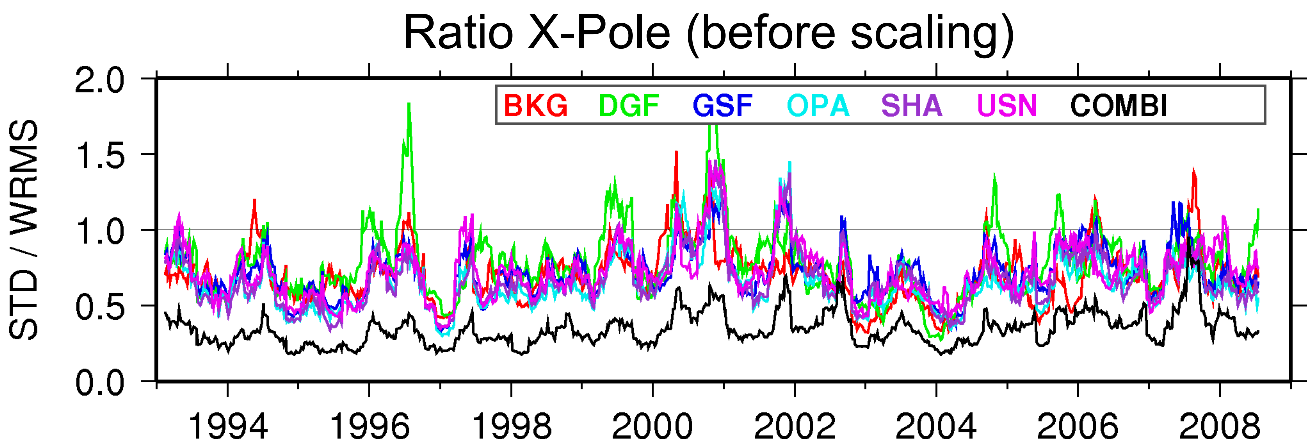
Ratios of combined and single solution should be equal:

$$\frac{X \cdot STD_{combi}}{WRMS_{combi}} \approx \frac{STD_{single}}{WRMS_{single}}$$

scaling factor → X

Validation of scaling factors
by empirical approach

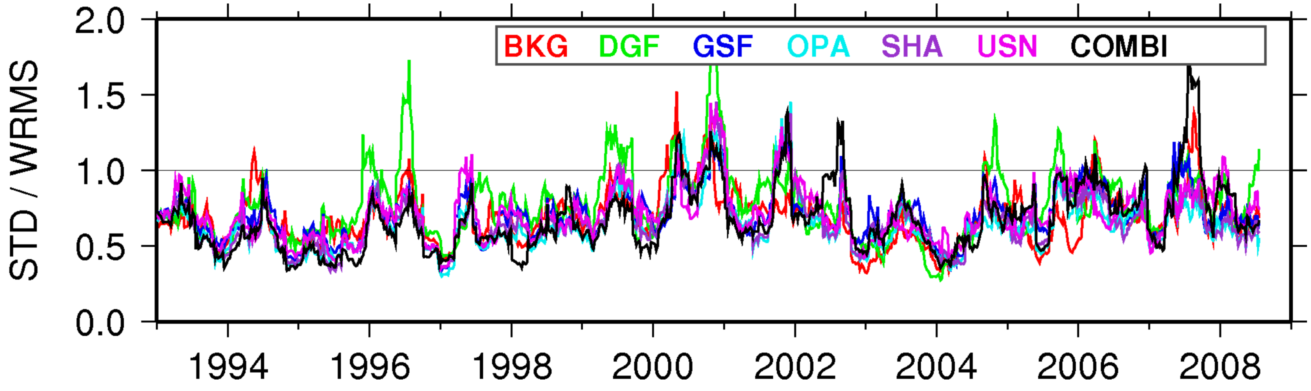
$$\frac{STD_{single}}{WRMS_{single}} \qquad \frac{STD_{combi}}{WRMS_{combi}}$$



Validation of scaling factors
by empirical approach

$$\frac{STD_{single}}{WRMS_{single}} \quad \frac{2 \cdot STD_{combi}}{WRMS_{combi}}$$

Ratio X-Pole (after scaling)



=> Assumptions to calculate scaling factor OK