Long-term Stability of Radio Sources in VLBI Analysis

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Introduction

- **Presently complete global geodetic VLBI solutions with nearly 30 years of data are possible**
- **Stable compact radio sources can be modeled with only one source position for the whole time span**
- **Unstable radio sources known by e.g. high-level source structure complexity are not suitable for this – modeling error**
- **Question: Which radio sources are stable?**

  Possible approaches for the answer to the question:

  - Estimation and analysis of time series for radio source positions

  - list of structure indices of radio sources by P. CHARLOT (see IERS Technical Note No. 35)

Time series for radio source positions (1)

- **Basis: complete global VLBI solution with 24 hours VLBI sessions from January 1984 to July 2007**
- **Main parameter types are global station coordinates and velocities, 183 preliminary stable radio source positions from former investigations (see "First Steps to Investigate Long-Term Stability of Radio Sources in VLBI Analysis" in IVS 2006 General Meeting Proceedings, Concepcion, Chile)**

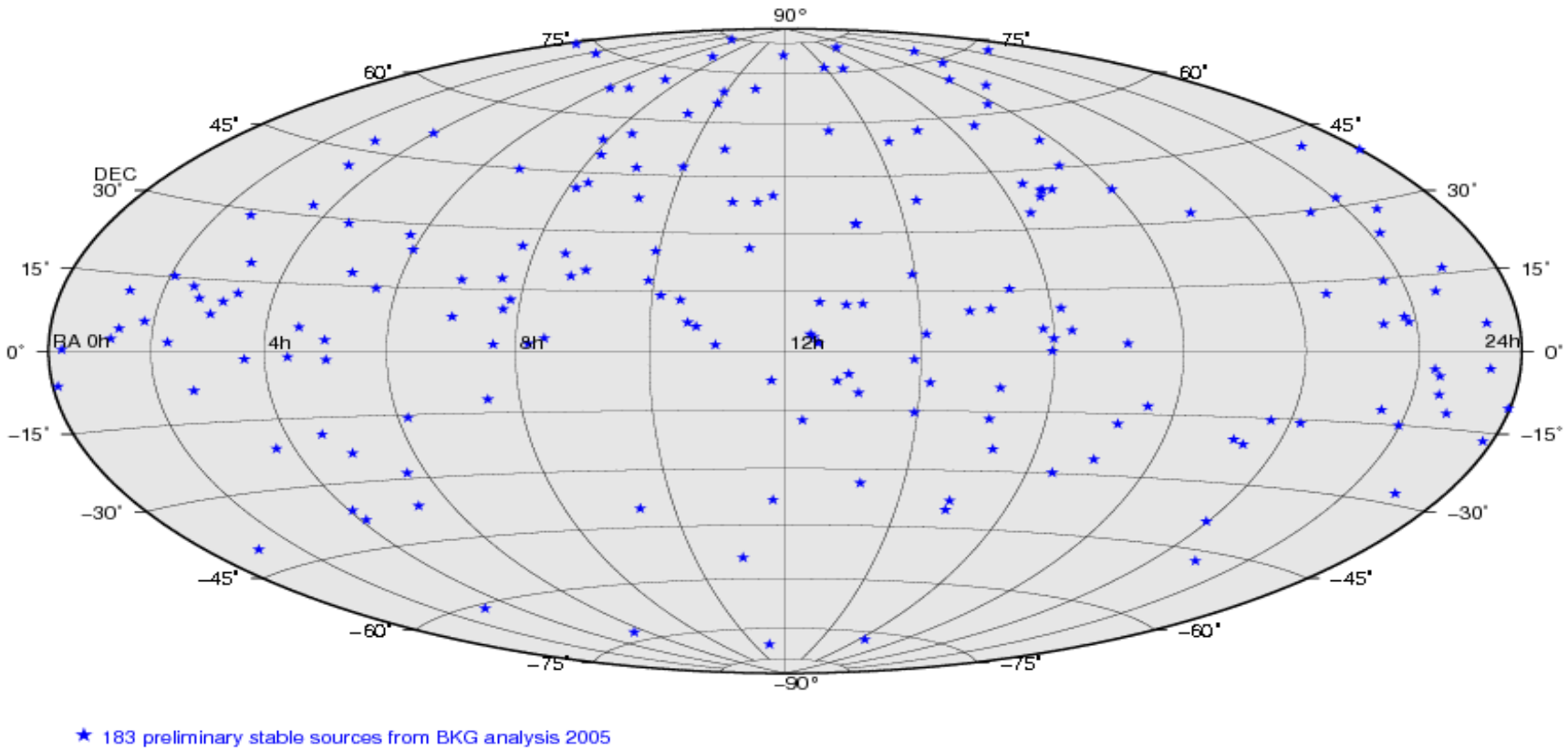
  - NNR/NNT conditions for the datum definition are applied to get:

  > Zero net rotation and zero net translation for 26 selected station positions and velocities with respect to VTRF

  > Zero net rotation for 183 preliminary stable sources with respect to ICRF

- **Estimation of local radio source positions in each session for generating the time series – basis solution 1**

Distribution of 183 preliminary stable sources from BKG analysis 2005



★ 183 preliminary stable sources from BKG analysis 2005

Time series for radio source positions (2)

- **Problem: basis solution 1 without time series for 183 preliminary stable radio sources**
- **Reduce set of preliminary stable radio sources for the datum definition by one source**
- **Separate solution with no other changes referring to basis solution 1 for estimation of local source positions for this selected radio source**
- **Advantage: nearly no change in datum definition for generating of time series for all radio source positions**
- **Disadvantage: separate calculation for each stable radio source**
- **Result: time series for all radio sources**

Weighted mean for each radio source position

- **Assumption: all radio sources are stable**
- **Use the time series with locally determined source positions (L) and their standard deviations (s) to estimate a weighted mean (x) for each radio source component**
- **Definition of the weights: $p_i = 1 / (s_i)^{**2}$ for i = 1,2, …, n and n = number of values in the time series of the source component**
- **Weighted mean: x = [pL] / [p]**
- **Residuals: $v_i = x - L_i$**
- **Result: Weighted mean and residuals for all radio sources in both components (right ascension and declination)**

Outlier detection

- **Purpose: increase the reliability of the used data**
- **Basis: outlier test by GRUBBS (e.g. in HELMUT WOLF (1975), Ausgleichungsrechnung), test of the maximum residual v_{max}**
- **Test statistic: $g = |v|_{max} / SQRT ([vv] / n)$**
- **Confidence level g_s from a table with the number of data points and the significance level of 1 percent**
- **Decision rule: $g < g_s$ or $g = g_s$ no outlier detected, no action necessary**
- **Decision rule: $g > g_s$ outlier detected, corresponding data point not used for new estimation of the weighted mean**

Test for normal distribution (1)

- **Starting point: based on time series of radio source positions residuals to the weighted mean of a radio source component can be tested for normal distribution (at least 12 sessions)**
- **Purpose: uncovering of systematic errors**
- **Basis: smooth test between the empirically determined distribution and the theoretical normal distribution (e.g. in G. REISSMANN (1978), Die Ausgleichungsrechnung)**
- **Parameters of the normal distribution: mean value and variance, computed from the sample (residuals)** $v_{mean} = [v] / n$ **and** $s^2 = [(v - v_{mean})^2] / n-1$
- **Test statistic:** $\chi^2 = \sum_{m=1}^{r} \frac{(h_m - np_m)^2}{np_m}$

r: number of classes (r=10), class width = s/2
m: current number of classes
h_m: empirical absolute frequency of class m
p_m: theoretical probability of class m

Test for normal distribution (2)

- **Confidence level χ_s^2 from a table of the chi-square distribution with the significance level of 1 percent $(\chi_s^2 = 18.5)$**
- **Decision rule: $\chi^2 < \chi_s^2$ distribution of the sample (residuals) in no contradiction with assumption that it comes from a statistical universe**

  - systematic errors could not be proven
- **Decision rule: $\chi^2 > \chi_s^2$ distribution of the sample (residuals) in contradiction with assumption that it comes from a statistical universe**

  - systematic errors could be proven, e.g.

  instabilities of radio source components exist

Further inspections

- **Check located sources with $\chi^2 < \chi_s^2$ concerning the following criteria empirically determined:**
  - **Rate less or equal | 0.1 | mas/year**
  - **Rate/Sigma Rate less | 3.5 |**
  - **WRMS less or equal 1 mas**
  - **Total time span of observations per source greater than 3 years**

Results

- **1189 radio sources with position time series available**
- **482 radio sources investigated with at least 12 sessions**
- **226 stable radio sources could be determined with normal distribution not rejected ($\chi^2 < \chi_s^2$) i.e.**

  194 sources in both components

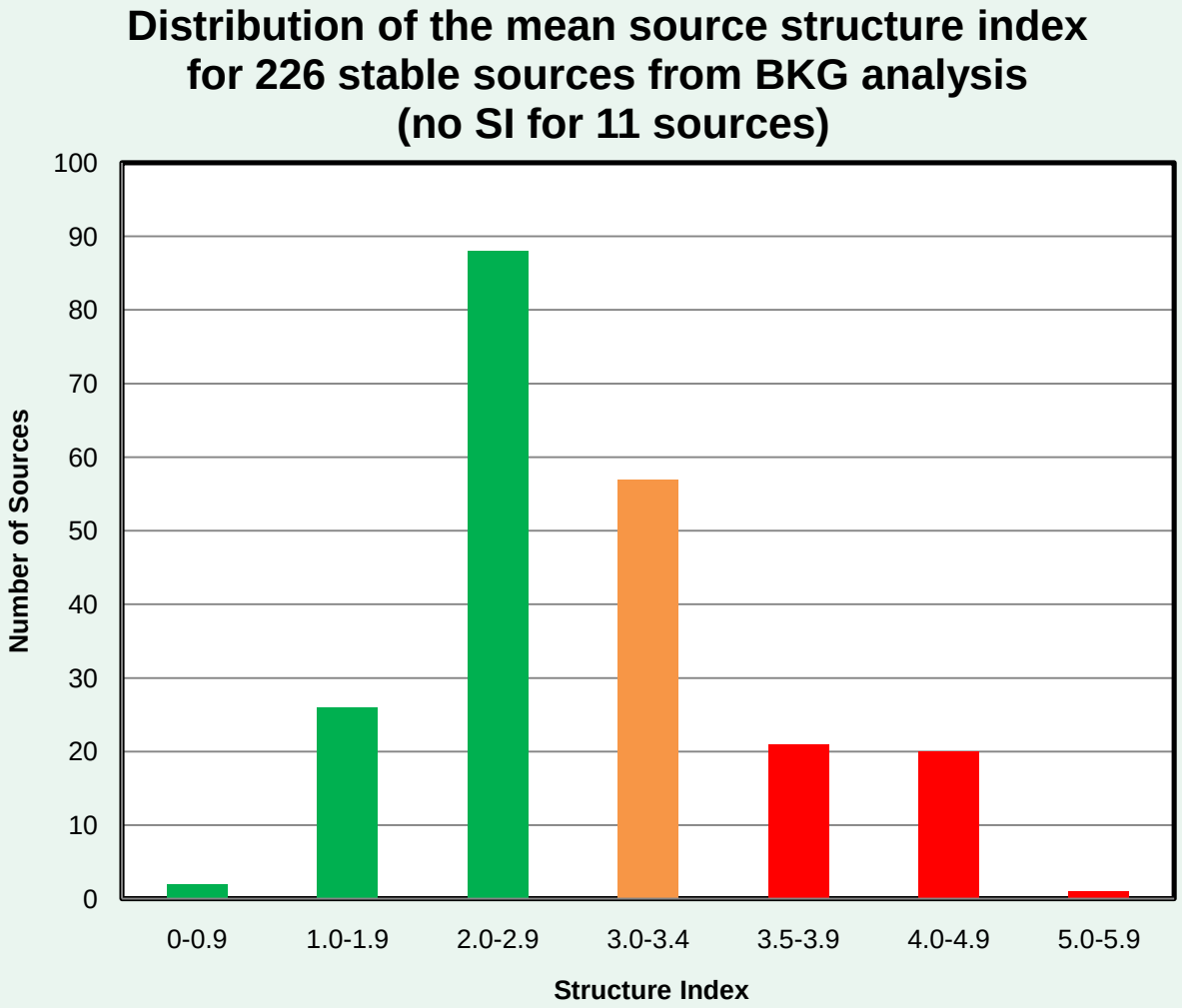
  32 sources in only one component,

  and successful check of rate, sigma of rate, WRMS, and total time span per source
- **Comparison with mean source structure index (SI) values from list in IERS Technical Note 35, Chapter 5 (P. Charlot et al.) available for 215 sources**

  116 sources with SI < 3.0 (limit for selection of ICRF2 defining sources)

  99 sources with SI > or = 3.0

Results


Distribution of the mean source structure index
for 226 stable sources from BKG analysis
(no SI for 11 sources)
Number of Sources
100
90
80
70
60
50
40
30
20
10
0
0-0.9
1.0-1.9
2.0-2.9
3.0-3.4
3.5-3.9
4.0-4.9
5.0-5.9
Structure Index

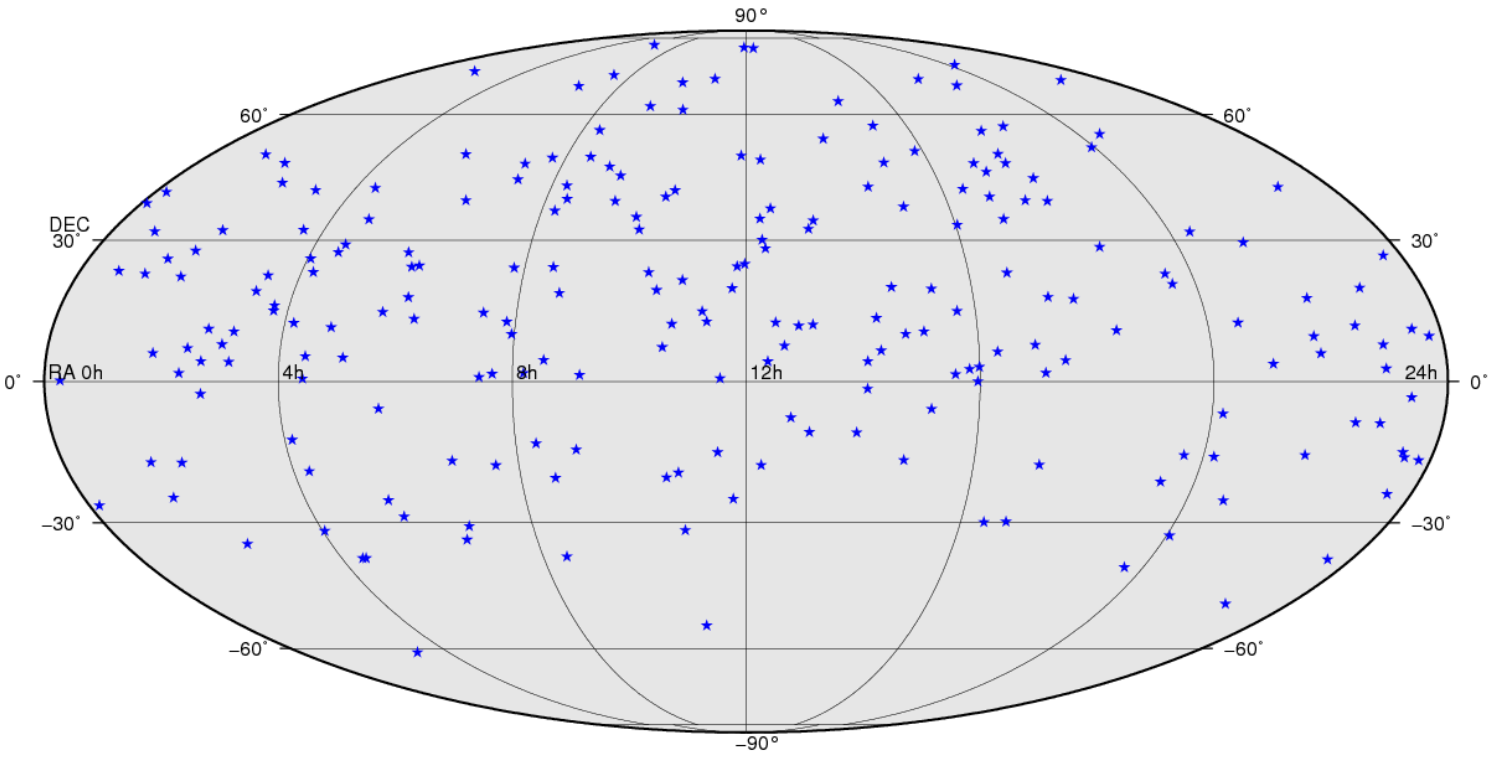
Results

- **Comparison of 226 stable sources from BKG analysis with official 295 ICRF2 defining sources:**
  - **Number of identical sources: 100**
  - **Number of sources with too little data in BKG analysis: 46**
  - **Number of sources with no data in BKG analysis: 31**
  - **Number of sources with failure to comply with BKG stability criteria: 118**
- **29 additional stable sources from BKG analysis with SI < 3.0 referred to 295 ICRF2 defining sources**

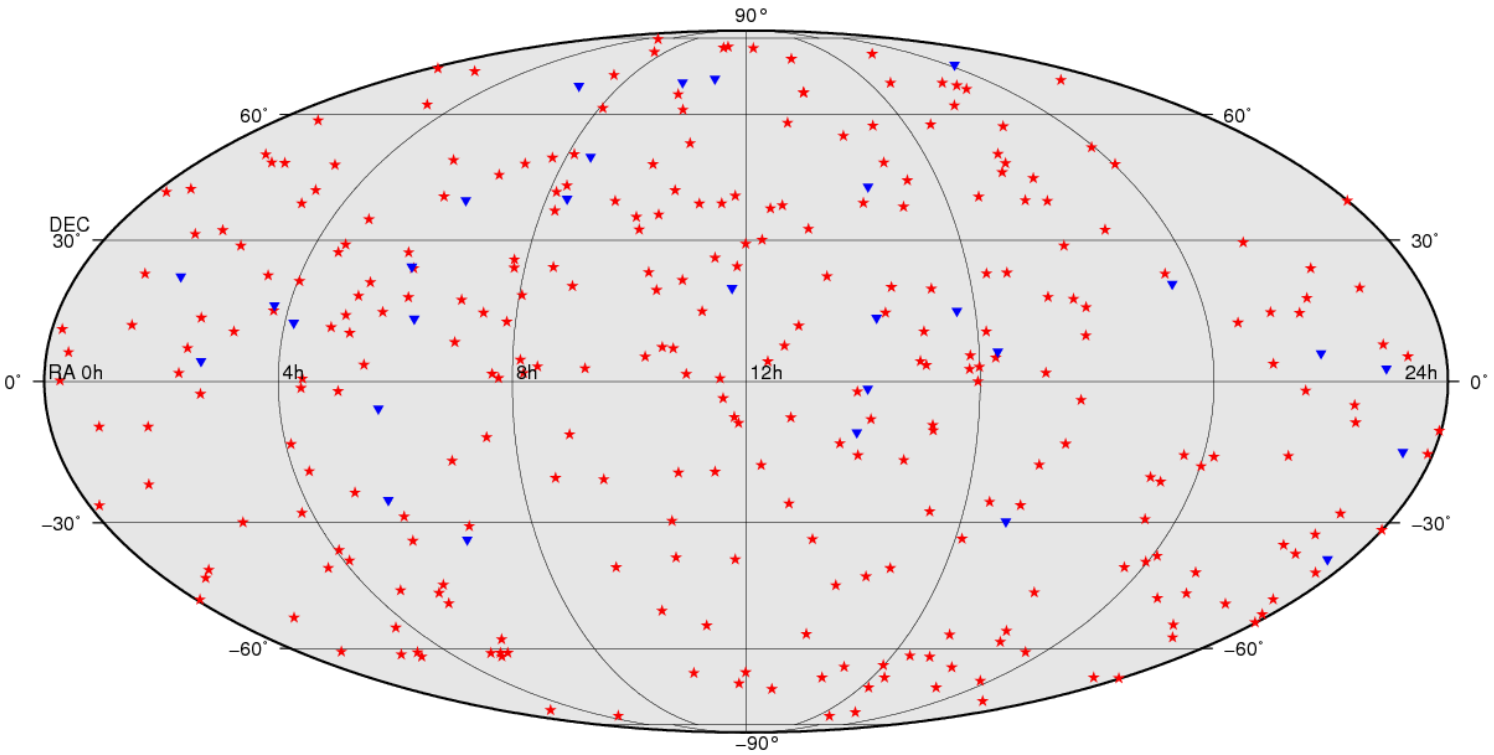
➢ **Time series and results are available at:**

- **http://ivs.bkg.bund.de/vlbi/ and click Radio Source Positions Time Series** or
- **ftp://ivs.bkg.bund.de/pub/analysis/radio_source_positions_time_series**

Distribution of 226 stable radio sources from BKG analysis 2009



Distribution of 295 ICRF2 defining sources and 29 additional stable sources from BKG analysis 2009



Axis stability

- **Assessment of axis stability by a set of radio sources:**

  - Computation of annual reference frames from radio source position time series (bkg000c.ts)

  - Estimation of relative orientation between annual reference frames and a reference catalog (gsf008a.cat)

  - Scatter of rotation parameters from annual reference frames (WRMS) is a measure for stability of axes

- **Equation for estimation of rotation parameters eps for one source position:**

with (Xi, Yi, Zi) from source position time series, and (Xref, Yref, Zref) from reference catalog after transformation from right ascension and declination to X, Y, Z

Axis stability

- **Least squares estimation in consideration of source position errors to get yearly rotation parameters**
- **Table with scatter of rotation parameters (WRMS) derived from different sets of sources:**

Set of sources	No. of sources used	WRMS epsX mas	WRMS epsY mas	WRMS epsZ mas	WRMS epsXYZ mas	Input file time series	Reference file catalog	mean DEC degree
212 ICRF1 def. sources	211	0.0152	0.0074	0.0243	0.0165	bkg000c.ts	gsf008a.cat	15
226 stable sources BKG	226	0.0034	0.0062	0.0146	0.0104	bkg000c.ts	gsf008a.cat	16
295 ICRF2 def. sources	264	0.0083	0.0088	0.0091	0.0088	bkg000c.ts	gsf008a.cat	5

Conclusions

- **Generating and statistical analyzing of time series for radio source positions are useful to get more information about long-term stability of radio sources**
- **After successful test for normal distribution of residuals to the weighted mean of both radio source components also inspection of rate, sigma of rate, WRMS, and total time span per source is necessary to identify stable sources**
- **A set of 226 stable radio sources could be identified by this method with axis stability of 0.01 mas**
- **100 ICRF2 axes-defining sources could be verified independently from method applied in the ICRF2 working group**
- **29 stable sources from BKG analysis with source structure index less than 3.0 can be used to increase the number of 295 ICRF2 defining sources (ftp://ivs.bkg.bund.de/pub/analysis/radio_source_positions_time_series/bkg_29stable_sou_add-icrf2def.txt)**