6th IVS General Meeting 2010

Reliability and Stability of VLBI-derived Sub-Daily EOP Models

Thomas Artz[1] Sarah Böckmann[1] Laura Jensen[1]
Axel Nothnagel[1] Peter Steigenberger[2]

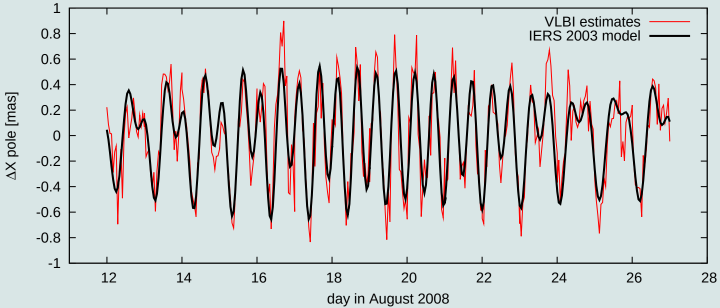
[1]Institut für Geodäsie und Geoinformation, Universität Bonn
[2]Institut für Astronomische und Physikalische Geodäsie, TU München

sub-daily EOP variations

- the IERS Conventions recommend a model based on ocean tidal models and nutation
- various origins of EOP variations due to tidal torques
  - tidal variations in oceans and atmosphere
  - non-tidal variations: thermally driven variations
  - tri-axiality of the Earth

empirical sub-daily models of the Earth's rotation (sdER-Models) that include the integral effect might be estimated from space geodetic techniques

CONT08: X pole

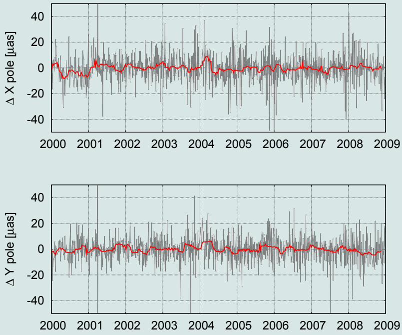


- IERS model explains the majority of sub-daily EOPs
- VLBI measures higher amplitudes

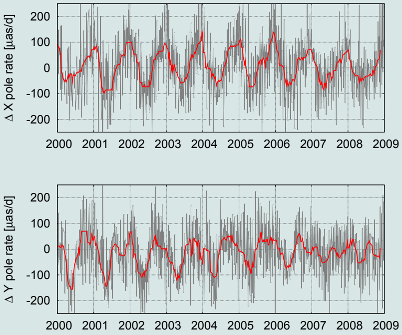
VLBI is capable to measure sub-daily EOPs

different sub-daily models: IERS 2003 vs. empirical VLBI

Polar motion

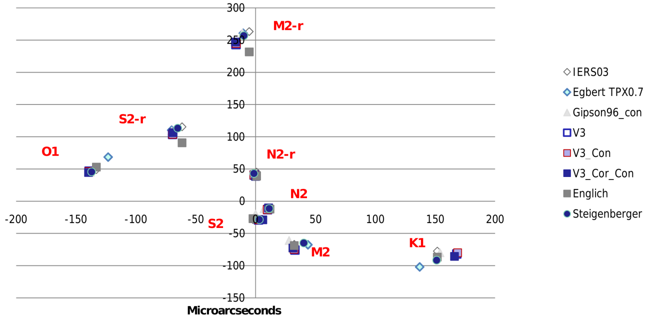


Polar motion rates



annual and semi-annual signal in EOP rates

PM comparison



http://www.cbk.waw.pl/EOPPW2009/contributions/session1/session1.2/mon11_Gipson_Ray.pdf

models depend on different data and solution strategies

deriving empirical sdER-Model

$$
\begin{aligned}
\Delta X(t) &= \sum_{j=1}^{n} -p_j^c \cos\psi_j(t) + p_j^s \sin\psi_j(t) \\
\Delta Y(t) &= \sum_{j=1}^{n} p_j^c \sin\psi_j(t) + p_j^s \cos\psi_j(t) \\
\Delta UT1(t) &= \sum_{j=1}^{n} u_j^c \cos\psi_j(t) + u_j^s \sin\psi_j(t)
\end{aligned}
$$

deriving empirical sdER-Model

$$\Delta X(t) = \sum_{j=1}^{n} -p_j^c \cos\psi_j(t) + p_j^s \sin\psi_j(t)$$

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Observation

new parameters

$$\mathbf{A} = \left(\ldots, \frac{\partial\tau}{p_j^c}, \frac{\partial\tau}{p_j^c}, \ldots\right)$$

$$\mathbf{N} = (\mathbf{A}^{\mathbf{T}}\mathbf{\Sigma}^{-\mathbf{1}}\mathbf{A})$$

$$\mathbf{n} = \mathbf{A}^{\mathbf{T}}\mathbf{\Sigma}^{-\mathbf{1}}\mathbf{\Delta}\tau$$

$$\mathbf{x} = \mathbf{N}^{-\mathbf{1}}\mathbf{n}$$

deriving empirical sdER-Model

$$\Delta X(t) = \sum_{j=1}^{n} -p_j^c \cos\psi_j(t) + p_j^s \sin\psi_j(t)$$

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new parameters

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$$\mathbf{N} = (\mathbf{A^T\Sigma^{-1}A})$$
$$\mathbf{n} = \mathbf{A^T\Sigma^{-1}}\boldsymbol{\Delta}\tau$$
$$\mathbf{x} = \mathbf{N^{-1}n}$$

Solution

pseudo observations
($\mathbf{y}$: PM & UT)

$$\mathbf{B} = \left(\frac{\partial\mathbf{y}}{p_j^c}, \frac{\partial\mathbf{y}}{p_j^s}, \ldots\right)$$
$$\mathbf{x} = (\mathbf{B^TWB})^{-1}\mathbf{B^TWy}$$

deriving empirical sdER-Model

$$
\begin{aligned}
\Delta X(t) &= \sum_{j=1}^{n} -p_j^c \cos\psi_j(t) + p_j^s \sin\psi_j(t) \\
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Observation

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Solution

pseudo observations ($\mathbf{y}$: PM & UT)

$$\mathbf{B} = \left(\frac{\partial\mathbf{y}}{p_j^c}, \frac{\partial\mathbf{y}}{p_j^s}, \ldots\right)$$
$$\mathbf{x} = (\mathbf{B^T W B})^{-1}\mathbf{B^T W y}$$

Normal Equation

$\mathbf{N}, \mathbf{n}$: highly resolved PM & UT

$$\bar{\mathbf{N}} = \mathbf{B^T N B}$$
$$\bar{\mathbf{n}} = \mathbf{B^T n}$$
$$\mathbf{x} = \bar{\mathbf{N}}^{-1}\bar{\mathbf{n}}$$

solution set-up

parameterization

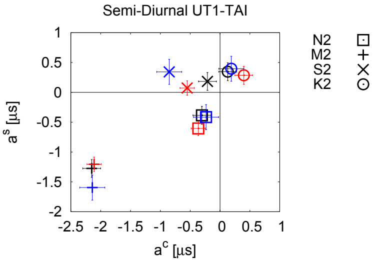
- CRF w.r.t. ICRF2
- TRF w.r.t. ITRF2005
- B-spline (Gc, Pt, Hr)
- 58 axis offsets
- ZWD (20 min)
- gradients (6 h)

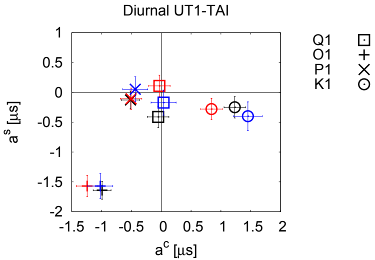
modelling

- APLO
- harmonic site position
- mean gradients
- nutation fixed: IAU2000A + VLBI
- sdER: IERS 2003

sdER-model

1. observation level: model coeff. + daily EOPs
2. solution level: hourly PM & UT
3. NEQ level: 15 min PM & UT (CRF, TRF, axis fixed)



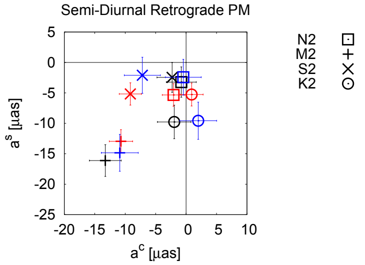


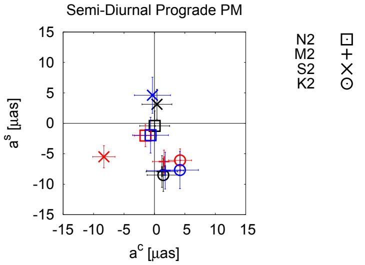
RMS of amplitude differences [μs] (33 terms)

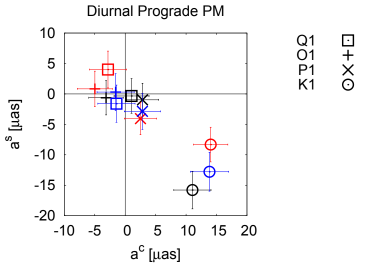
	Obs. level	Sol. level	NEQ level
IERS 2003	0.42	0.50	0.45
Obs. level		0.24	0.21
Sol. level			0.30

diff. $> 3 \cdot \sigma$
at S_2, Q_1 & K_1

PM w.r.t. IERS 2003





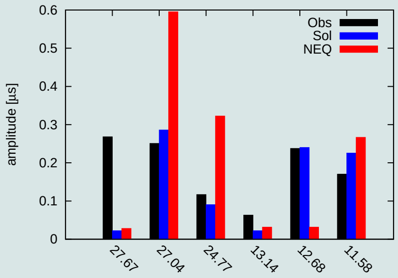


RMS of amplitude diff. [μas] (44 terms)

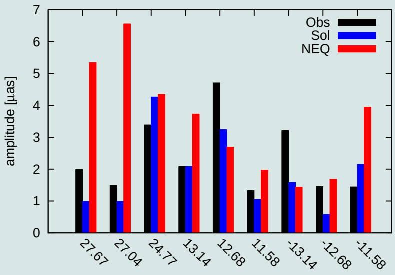
	Obs.	Sol.	NEQ
IERS 2003	4.87	5.06	4.97
Obs. level		1.90	3.12
Sol. level			2.93

zero terms: periods where no amplitude is expected

UT: 6 zero terms



PM: 9 zero terms



noise floor: below 0.6 μs (UT); 7 μas (PM)

RMS amplitude differences w.r.t. IERS 2003 [μas]

	std.	TRF, CRF Axis fix[a]	ZWD 60, GRAD 24	no har. pos.	nutation est.	no apr. sdER
UT	0.42	0.41	0.42	0.46	0.42	0.40
PM	4.87	4.98	5.06	5.29	5.00	4.89

[a] Hr, Gc, Pt: B-spline pos and velocities estimated

RMS amplitude differences w.r.t. IERS 2003 [μas]

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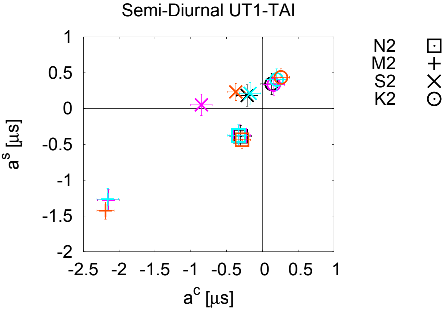
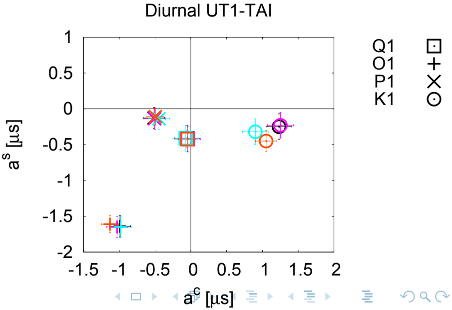
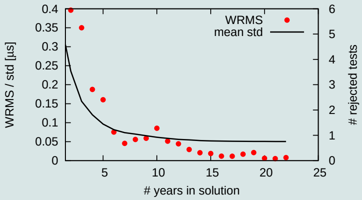




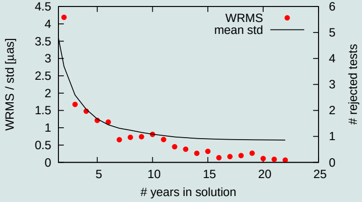
cummulative yearly solutions:

- 2009, 2008 + 2009, ..., $\sum_{i=1984}^{2009} y_i$
- WRMS of coeff. diff. & hypothesis test (propability 95%)

dUT



PM



cummulative yearly solutions:

- 2009, 2008 + 2009, . . . , $\sum_{i=1984}^{2009} y_i$
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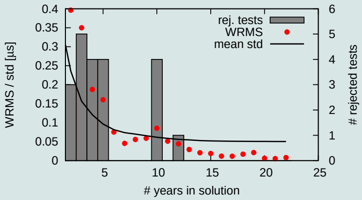
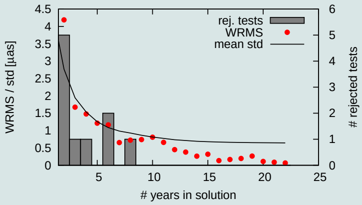
dUT



PM

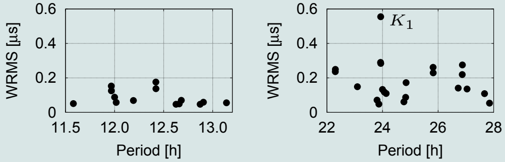


data of at least 12 years leads to a stable solution

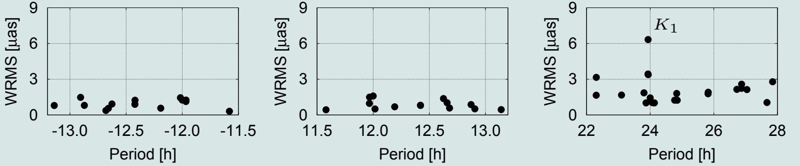
amplitude repeatability

- 12-year solutions starting in 1986
- WRMS w.r.t. linear regression for each tidal term

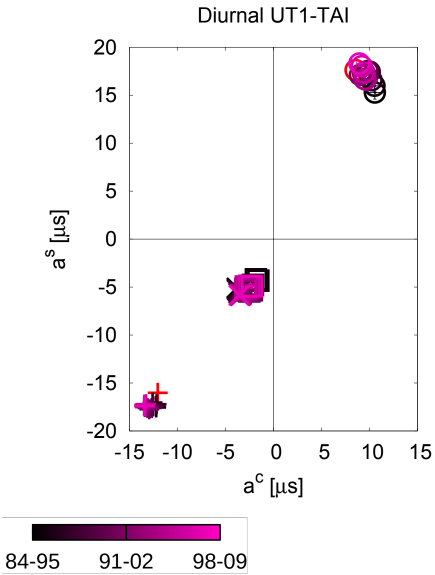
dUT

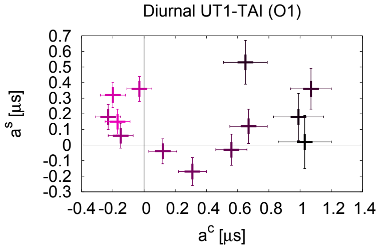


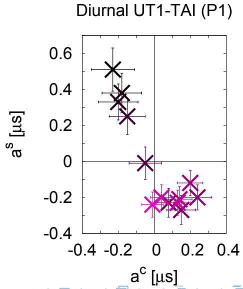
PM



temporal evolution?







Conclusions

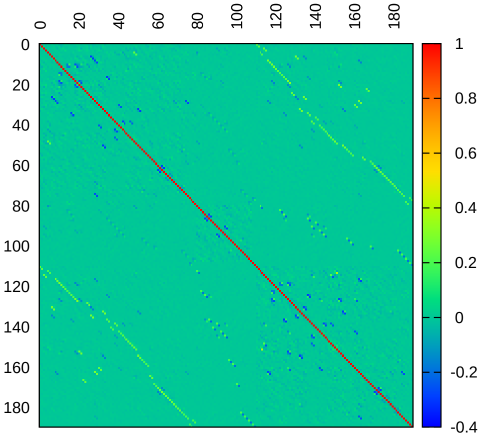
- 3 methods to derive sdER-Model
  - agreement not perfect
  - noise floor and formal errors comparable
- significant impact of analysis options
  - station position handling
  - troposphere parameterization
- with data of 12 years almost no significant change after adding one more year of data
- some terms might not be constant in time

NEQ transformation

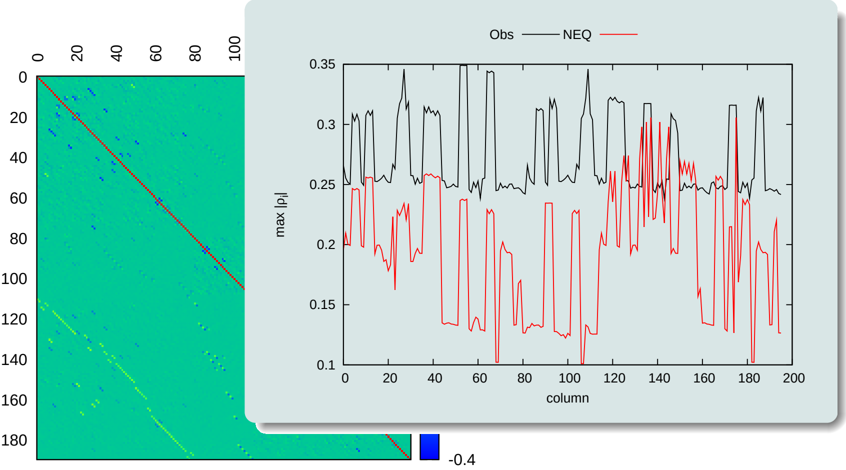
$$\mathbf{B} = \begin{pmatrix} -c_1(t_1) & s_1(t_1) & \cdots & -c_n(t_1) & s_n(t_1) & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & & & \\ -c_1(t_m) & s_1(t_m) & \cdots & -c_n(t_m) & s_n(t_m) & & & \\ s_1(t_1) & c_1(t_1) & \cdots & s_n(t_1) & c_n(t_1) & & & \\ \vdots & \vdots & \vdots & \vdots & \vdots & & & \\ s_1(t_m) & c_1(t_m) & \cdots & s_n(t_m) & c_n(t_m) & & & \\ & & & & & c_1(t_1) & s_1(t_1) & \cdots \\ & & & & & \vdots & \vdots & \vdots \\ & & & & & c_1(t_m) & s_1(t_m) & \cdots \end{pmatrix}$$

$$c_j(t_i) = \cos\psi_j(t_i) \qquad s_j(t_i) = \sin\psi_j(t_i)$$

correlations

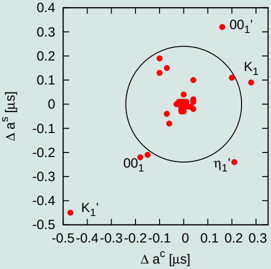


correlations

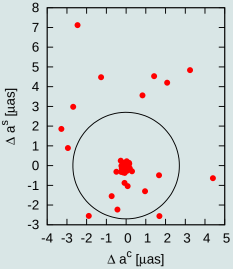


no significant correlations

dUT


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.5 -0.4 -0.3 -0.2 -0.1 0 0.1 0.2 0.3
Δ a^s [µs]
Δ a^c [µs]
00_1'
K_1
00_1
η_1'
K_1'

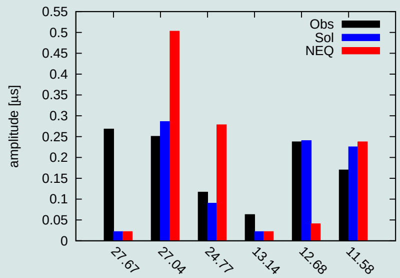
PM


8
7
6
5
4
3
2
1
0
-1
-2
-3
-4 -3 -2 -1 0 1 2 3 4 5
Δ a^s [µas]
Δ a^c [µas]

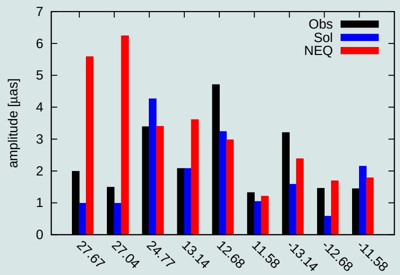
zero terms

- stable solution: 12 years ⇒ **noise level?**
- 12-year solutions shifted by 1 year (starting in 1984 ⇒ 15)

dUT



PM



noise level of solutions almost constant (starting in 1986)