Instrumental Errors of Geodetic VLBI

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Abstract

A model of VLBI observables includes effects of geometry, environment of signal propagation and delays in the data acquisition system itself. Imperfection of the model of instrumental effects can be a considerable source of errors in the estimates of the targeted parameters. Two effects are considered in detail: spurious signals in phase calibration and instrumental polarization. Methods of their detection, evaluation and calibration are outlined. Influence of these effects on the final results is shown.

1. Introduction

A model of VLBI observations includes contributions to time delay due to 1) geometry, 2) source structure, 3) effects of medium of signal propagation from the source to the point of injection of phase calibration impulses, 4) signal propagation from the point of injection of phase-cal impulses to a formatter. The latter term we will call instrumental delay.

Mismodeling instrumental delay spawns instrumental errors. Two effects are investigated:

- errors in phase-cal phases,
- instrumental polarization.

The signal enters the data acquisition system after passing a point of injection of phase-cal impulses. Then it goes through a variety of different radioelectronic devices such as amplifiers, mixers; it propagates through waveguides and cables. Ideally, the phase calibration system should calibrate all delays which the signal acquires after passing through the point of injection of phase-cal impulses. The fringed phases become referred to that point when phase-cal phases are subtracted from raw fringed phases. In practice the phase-cal system itself is affected by spurious signals which distort phase calibration phases. Mismodeling this effect may give a noticeable contribution to an error budget [1]. It will be shown that in many cases it is possible to considerably reduce the influence of spurious signals on group delay by performing a special analysis.

Another source of instrumental delay is related to the transformation of a circularly polarized wave falling on the antenna’s aperture into the signal propagating through cables to the formatter. Instrumental polarization of radiotelescopes causes an additional delay.

2. Investigation of Phase-cal Phases and Phase-cal Amplitudes

A phase-cal antenna unit generates a rail of short impulses which are injected in a feed horn every microsecond. The phase calibration signal travels the same path as the signal from a source, and it is affected by the same transformations. Phases of the phase-cal signal at each channel are extracted during correlation and subtracted from fringe phases. The amplitude of the phase-cal
signal is not used in routine data analysis but it provides valuable information for calibration of the phase-cal system itself.

Since the amplitude of the phase-cal signal which is injected by the antenna unit is known and is constant, we can expect that the amplitude of the recorded signal should also be constant. Variations in recorded phase-cal amplitudes indicate that recorded phase-cal phases would have similar variations. However, variations in path length themselves do not change the amplitude of a signal. One of the reasons for variations in the recorded amplitude is the presence of narrow-band signals which are coherent with the true phase-cal signal. B. Corey proposed [2] to call them “spurious signals” or spur for brevity.

If we could determine the amplitude and phase of spurious signals we could subtract their contribution from the recorded phase of the phase-cal signal and improve measurements of instrumental delay [3].

The correlator computes sine and cosine components of the phase-cal signal as a fraction of the amount of correlating bits to the total amount of recorded bits. The measured phase-cal amplitude is expressed as a fraction of the total system noise. Therefore, the amplitude of the extracted and injected phase-cal signal are tied as

\[ A_e \sim \frac{A_s}{\sqrt{T_{sys}}}. \]

The measured phase \((\phi_m)\) and amplitude \((A_m)\) of phase-cal are related to the true phase and amplitude of phase-cal \((\phi_o, A_o)\) and spurious signal \((A_s, \phi_s)\):

\[
\sqrt{\frac{T_{sys}}{T_o}} A_m - A_o = A_s \cos(\phi_o - \phi_s) + O\left(\frac{A_s}{A_o}\right)^2
\]

\[
\phi_m - \phi_o = -\frac{A_s}{A_o} \sin(\phi_o - \phi_s) + O\left(\frac{A_s}{A_o}\right)^2
\]

where \(T_o\) is an averaged system temperature.

We see that the measured amplitude of the phase-cal can be affected by the variations caused by changes in system temperature and spurious signals. Normally, system temperature is measured before each scan and it is recorded in log files. The amplitude of a spurious signal can be determined by LSQ fitting provided we know how to model the phase of the spurious signal \(\phi_s\).

The phase of the spurious signal depends on its origin. B. Corey proposed a list of possible sources of spurious signals [2]. Some types of spurs can be parameterized as \(\phi_s = f \phi_0 + \phi_i\) where \(\phi_i\) is a constant and \(f\) is a parameter.

Then spur can be classified according to the following scheme:

- **A-spur.** \(f_s = 0\) — Additive spur. It is generated by the electronics of the data acquisition system and is not related to the signal.
- **B-spur.** \(f_s = -f_0\) — Image spur. It occurs due to an admixture of the signal at the image of the intermediary frequency.
- **C-spur.** \(f_s = \frac{\omega_{10}}{\omega_{1p}}\) — Cross-band spur. It occurs due to an admixture of the signal from the opposite band.
- **D-spur.** \(f_s = -\frac{\omega_{10}}{\omega_{1p}}\) — Image cross-band spur. It occurs due to an admixture of the signal at the image band of the intermediary frequency of the opposite band.
• L-spur. $A_m = A_o + L \phi_o, \quad \phi_m = \phi_o + L \phi_o$ — Linear spur. The origin is unclear.

Here $\omega_{LO}$ and $\omega_{LO}^{op}$ are the frequencies of local oscillator (LO) at the current and opposite bands. The presence of a spurious signal results in a harmonic dependence of the measured amplitudes and phases on the total phase of the phase-cal signal. The frequency of this dependence is $1 - f$ for all spurs, except L-spur, and it can be represented in this table (units are 1/cycle):

<table>
<thead>
<tr>
<th>Spur</th>
<th>X-band</th>
<th>S-band</th>
</tr>
</thead>
<tbody>
<tr>
<td>A-spur</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>B-spur</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>C-spur</td>
<td>0.75</td>
<td>1.25</td>
</tr>
<tr>
<td>D-spur</td>
<td>3.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>

The following algorithm for determination of amplitudes and phases of spurs from the amplitudes of the phase-cal is proposed and implemented in the program Phase_Doctor:

• Modeling system temperatures as $T_m = T_o \cdot a(e) \cdot b(t)$, editing, outliers rejection. Here $a(e)$ is a function of elevation angle and $b(t)$ is a function of time. Then normalization of phase-cal amplitudes.

• Resolving ambiguities in phase-cal. (Assumptions: a dominating contribution is LO variations which are common at X- and S- bands; phase variations in the individual channels are smaller than one phase turn);

• LSQ fitting the model of harmonic variations in $A_m(\phi_m)$. Iterative refining for taking into account non-linear terms $O\left(\frac{A_m}{A_o}\right)^2$;

• Computation of contribution of spurs to group and phase delays.

Figure 1. Amplitude of phase-cal versus its phase at station MATERA, experiment 99APR26XA

An alternative way is to use the phase of the phase-cal signal in the individual channel itself for spurious signal detection. The phase of phase-cal $\phi_i$ in the i-th channel can be modeled as

$$\phi_i(t) = \omega_o \tau_p(t) + \omega_i \tau_g(t) + \phi_o(t) + \phi_s(t)$$

(2)
where \( \omega_0 \) is a reference frequency, \( \tau_p(t) \) is the phase delay of phase-cal, \( \omega_i \) is the frequency of the individual channel, \( \tau_g(t) \) is the group delay of phase-cal, \( \phi_0(t) \) is the phase offset at the channel and it may have a small number of breaks, \( \phi_r(t) \) is the residual phase of the phase-cal signal.

Having combined equations (2) for all scans and for each band we can adjust a series of \( \tau_p(t) \), \( \tau_g(t) \) and find estimates of \( \phi_0 \) at the initial epoch and at the epochs of phase breaks by using LSQ method. Residual phases would have harmonic variations if spurious signals affected one or several channels. However, this approach does not allow us to detect spurs which have the same phase and amplitude at all channels of the band. An advantage of this approach is that the measurements of phase-cal phases are not affected by variations of system temperature and therefore the errors of measurements of system temperature do not affect residual phases.

The following algorithm for determination of amplitudes and phases of spurss from the residual phases of the phase-cal signal is proposed and implemented in the program Phase_Doctor:

- Resolving ambiguities in phase-cal phases;
- Building a model of phase-cal phases:
  - determination of phase break epochs in the individual channels;
  - determination of which spurs affect the phases.
- Adjustments of parameters of the phase-cal phase model using LSQ:
  - Values of a phase offset and phase breaks for each channel;
  - Amplitudes and phases of spurs;
  - An array of phase delays in phase-cal over a band for each scan, each band;
  - An array of group delays in phase-cal over a band for each scan, each band.
- Iterations for taking into account non-linearity of the problem.
- Computation of contribution of spurss to group and phase delays.

Figure 2. Residual phase in phase-cal (in rad) versus full phase at station MEDICINA, experiment 99AUG16XA
3. Investigation of Effects of Instrumental Polarization

Antennas receive not only wanted right circular polarization (RCP), but some amount of unwanted left circular polarization (LCP). Fringe phases of RR and LL scans are not the same:

\[ \phi_{RR} - \phi_{LL} = k \sin \left( 2(\psi_1 - \psi_2) + \phi_0 \right) \]

\[ k \approx \frac{\text{Amp}_{LL}}{\text{Amp}_{RR}} \]

where \( \phi \) is the fringe phase, and \( \psi \) is the parallactic angle and Amp is the fringe amplitude. The coefficient \( k \) is a parameter of the radiotelescope and may depend on frequency and elevation.

Special polarization experiments were conducted in order to investigate the effects of instrumental polarization. Some antennas swapped polarization and observed LCP in one scan and RCP in the next scan while other antennas observed RCP only. The ratios of fringe amplitudes of RL cross scans to the amplitudes of normal RR scans for the same source were investigated. In the case of observations of unpolarized sources \( \frac{\text{Amp}_{LL}}{\text{Amp}_{RR}} \approx \left( \frac{\text{Amp}_{RL}}{\text{Amp}_{RR}} \right)^2 \) and therefore this method is rather more sensitive to instrumental polarization. Fringe amplitude of RL scans should be zero if a system is perfect.

- Extension of euro46 experiment 1998.12.14: two RL and two RR scans were obtained.
  MEDICINA and CRIMEA swapped polarization, other stations observed RCP-polarization.
  Result: \( \frac{\text{Amp}_{RL}}{\text{Amp}_{RR}} \approx 0.05 - 0.2 \) for European VLBI stations.

- brd01 experiment 1999.07.01: RL and RR scans were obtained. MEDICINA swapped polarization 96 times during 24-hours experiment. Some results are presented in the figure 4.

4. Conclusions

- Phase-cal phases can be cleaned up to the 0.01 rad level. Typically root mean square of contribution of a phase-cal noise to group delay is about 5–10 psec. It can be reduced to a level below 2 psec by LSQ adjustment of a parametric model.
Figure 4. Ratios of RL to RR fringe amplitudes for three sources observed in brd01 experiment

- Spurious signals in phase-cal were detected at all stations.
- Removal of spurious signals reduces the scatter of the residual fringe phases significantly, while it improves solutions only marginally. Spurious signals affect a solution at about the same level as the cable calibration.
- Instrumental polarization of European VLBI antennas is 5–20%. It has a strong dependence on frequency and antenna orientation.

References


( Web: http://giub.geod.uni-bonn.de/vlbi/development/spur/spurs_bec.ps.gz )