

# A Discussion on the Modeling of the Residual Clock Behavior and Atmosphere Effects in the Astrometric and Geodetic VLBI Data Analysis

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## Abstract

We checked the dependence of the estimation of scientific interest parameters on the choice of the time span of each piece in the continuous piecewise linear modeling of the residual clock behavior and atmosphere effects by single analysis of 27 randomly selected VLBI sessions in which Shanghai station (Seshan25) participated. Results show that (1) different choices of the time span of each piece lead to differences in the estimation of station coordinates up to *centimeters* and in the weighted root mean squares (*wrms*) of the delay residuals up to dozens of *picoseconds*; (2) since the conditions of clock and atmosphere are different from session to session and from site to site, the reasonable range of the time span of each piece should be tested and chosen for each session as well as for each site, which is really arduous work in routine data analysis.

We also checked the performance of this modeling by simulated data tests. By using the User Partial function of CALC/SOLVE we compared the delay residuals from the continuous piecewise linear modeling of the residual clock behavior with that from the periodic modeling.

## 1. Introduction

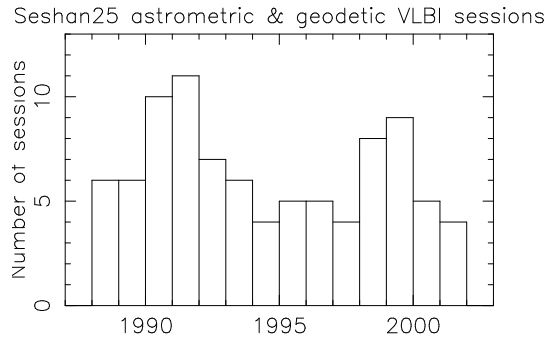
The clock behavior and the atmosphere effects are those of the principle errors in the astrometric and geodetic VLBI data analysis. A suitable modeling of these effects is accordingly important to the improvement in precision of estimations of scientific interest parameters such as site and source positions. While the ionosphere effects are computed from the differences in delay at two frequencies, the troposphere effects are relatively complicated and are usually modeled as the product of the surface meteorology dependent zenith delay and the observation elevation dependent mapping function [1]. There are residual atmosphere delays at each site and which exhibit as short-term variations. The modeling of the timing system also permits short, random clock variations while placing realistic physical constraints on the continuity and rates of change. The clock model is firstly a polynomial with parameters for offsets in clock epoch and rate and for frequency drift. Similar to the residual atmosphere effects there are residual short-term clock variations. Ma et al. [2] modeled these residual variations in clock behavior and atmosphere effects by continuous piecewise linear functions realized in CALC/SOLVE, Goddard's analysis software for astrometric and geodetic data. The estimated parameters are the initial residual zenith path delay and its rate of change in each linear section, the residual clock rates in each section with zero value at the first observation epoch. These parameters are considered as nuisance parameters to accommodate the residual variations and so are helpful to improve the estimation precision of scientific interest parameters.

We checked the dependence of the estimation of scientific interest parameters on the choice of the time span of each piece in the continuous piecewise linear modeling of the residual clock

behavior and atmosphere effects by single analysis of 27 randomly selected VLBI sessions related to Shanghai station (Seshan25). We also checked the performance of this modeling by simulated data tests. By using the User Partial function of CALC/SOLVE we compared the delay residuals from this modeling of the residual clock behavior with that by periodic modeling.

## 2. Data and Software

Seshan25 began to participate in regular astrometric and geodetic VLBI experiments from April of 1988. As of April of 2001 about 90 sessions are accumulated and the number of sessions each year is shown in Fig.1. From these sessions 27 are randomly selected and the single analysis of these sessions are performed by using CALC (9.12) and SOLVE (released on June 5 of 2000). The 1996 conventions of the International Earth Rotation Service [3] and the New Mapping Function [4] are adopted. The cut-off elevation angle is  $7^\circ$  and the type of data is group delay.



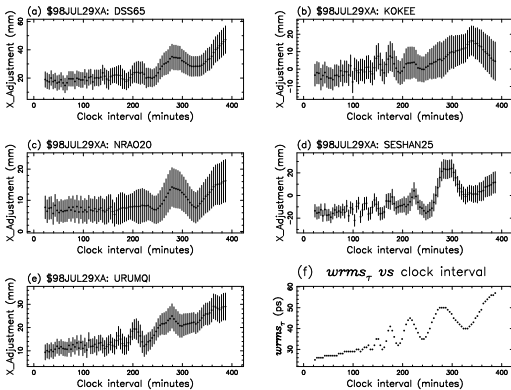
**Fig.1.** The distribution of the astrometric and geodetic VLBI sessions involving SESHAN25 during 1988 to 2001

## 3. Analysis and Discussion

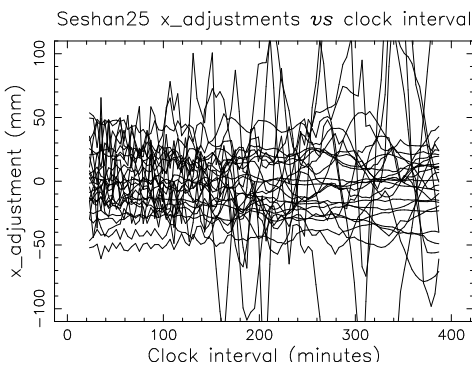
We tested the performance of the continuous piecewise linear modeling of the residual clock behavior and atmosphere effects by setting the time span of each piece as  $391 - 4i$  (min,  $i = 1, \dots, 97$ ). The calculation stops when the observation equation becomes singular. When we check the modeling of the clock behavior the time span of each piece for the modeling of the atmosphere effects is 20 min. When we check the modeling of the atmosphere effects that for clock behavior is 60 min. We extracted from the results of the single analysis the delay residual *wrms*, the adjustment and its formal error ( $\sigma_x$ ) of the  $x$ -component of site coordinates in order to survey the effects of the (time span) length of each piece on the parameter estimation.

Take the session on July 29 of 1988 (\$98JUL29XA) as an example. Fig.2 and Fig.3 show respectively the distribution of the  $x$ -component adjustment, its formal error and the delay residual *wrms* versus the length of each piece in the modeling of the residual clock behavior and the atmosphere effects. Fig.4 shows the  $x$ -component adjustments of SESHAN25 in all the selected 27 sessions versus the piece length in the modeling of the clock behavior. Fig.5 is for the modeling of the atmosphere effects. The situations for  $y$  and  $z$ -components are similar to that for  $x$ . From these figures the following can be deduced. (1) Different choices of the piece length leave significant differences in the adjustments of coordinates and in the delay residual *wrms* up to *centimeters* and dozens of *picoseconds* respectively. (2) When the piece length is relatively short, the distribution of the adjustment of coordinates and the delay residual *wrms* are scattering, while as the piece length increasing the distribution exhibit some fluctuation. (3) The effects of different piece lengths on the adjustment of coordinates are different from session to session as well as from site to site and up to *centimeters* in difference. (4) Generally speaking, the parameter estimations are stable with relatively short piece lengths. But the length cannot be too short in order to provide enough

degrees of freedom. With the piece length for atmosphere effects at 20 min it is preferable to set the piece length between 20 min to 100 min for the clock behavior (refer to Fig.4). With the clock at 60 min it is preferable to set between 10 min to 40 min for the atmosphere (refer to Fig.5). However, even if these settings are chosen the coordinate adjustments in some sessions are fluctuated by several *centimeters*.



**Fig.2.** The distributions of the adjustment and formal error of  $x$ -component versus the piece length in the modeling of clock

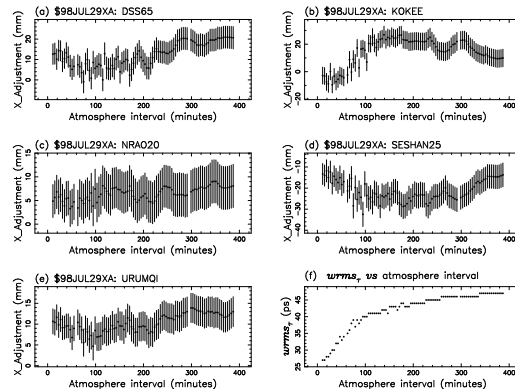


**Fig.4.** The adjustments of  $x$ -component of SESHAN25 versus the piece length in the modeling of clock for all the analyzed 27 sessions

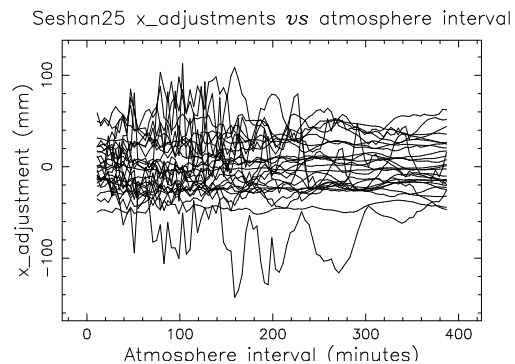
With the continuous piecewise linear modeling of the residual clock behavior and atmosphere effects it is good to choose the suitable piece length for each session as well as for each site. This is arduous work and accordingly it is necessary to test other modeling ways.

#### 4. Simulated Data Tests

Let the simulated data be in the form  $\sum_i A_i \sin(2\pi t_j/T_i + \phi_i) + N(0, \sigma_n)$ , where  $i = 1, \dots, n$  denotes the sequence number of the periodic terms,  $t_j$  is the observation epoch,  $N(0, \sigma_n)$  is white noise with zero value mean and standard deviation  $\sigma_n$ . Here we take the periods in hours as 0.5, 1.0, 4.0, 5.0, 12.0 and 24.0, the corresponding amplitudes (in any default units) as 22.0, 20.0, 90.0, 999.0, 50.0 and 50.0, the phases in degrees as 10.0, 13.0, 34.0, 80.0, 80.0 and  $-80.0$  and  $\sigma_n$  as



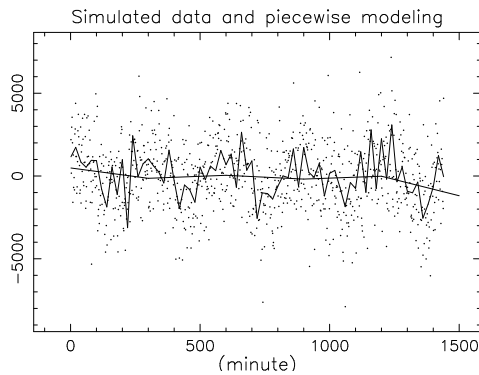
**Fig.3.** The distributions of the adjustment and formal error of  $x$ -component and the delay residual  $wrms$  versus the piece length in the modeling of atmosphere



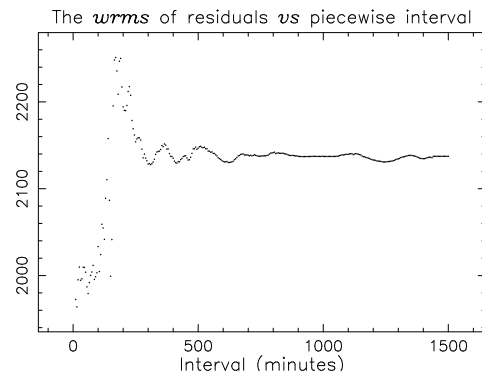
**Fig.5.** The adjustments of  $x$ -component of SESHAN25 versus the piece length in the modeling of atmosphere for all the analyzed 27 sessions

1999.0. The simulated data are generated with 90 s sampling interval and with the assumed first and last observation epoch at 0 h and 24 h.

Set the piece length in the continuous piecewise linear modeling in the form  $5.0 + 5.0k$  (min,  $k = 1, 2, \dots$ ). Fig.6 shows the simulated data (points) and their modeling with piece length as 20 min (short broken line) and 300 min (long broken line). It is clear that the short pieces can track the data better than the long pieces. Fig.7 shows the distribution of the residual *wrms* versus the piece length. When the pieces are very short the residual *wrms* (about 2000) is on the same level of  $\sigma_n$  (1999.0), which indicates that the periodic variations in the simulated data are well modeled. As the piece length increasing the *wrms* increases rapidly and exhibits some fluctuation around the standard deviation of the simulated data (note that within a whole period the standard deviation of a periodic signal is  $A/\sqrt{2}$ ,  $A$  is the amplitude), which indicates that the simulated data are not properly modeled. In conclusion, the piece length in the continuous piecewise linear modeling cannot be arbitrarily chosen. There should be some reasonable range and it should be determined by real case based on observation analysis.



**Fig.6.** Simulated data and the continuous piecewise linear modeling



**Fig.7.** The residual *wrms* of the simulated data versus the piece length

Set the periods in hours of the periodic modeling as 0.5, 1.0, 2.0, 3.0, 4.0, 5.0, 6.0, 8.0, 12.0 and 24.0. Through a least squares fitting of the above mentioned simulated data the residual *wrms* is found to be 1981.9, which is very near to the standard deviation of the white noise and which demonstrates a good modeling of the simulated data. By comparing the situation with short piece length in the continuous piecewise linear modeling, the periodic modeling can track the variation of the data with rather small numbers of parameters.

We also tested the periodic modeling by using simulated broad frequency band noise as shown in Fig.8 (dots). The periodic and continuous piecewise linear modeling of the simulated noise are shown in Fig.8, Fig.9 and Fig.10. It is shown that the periodic function tracks the variation of data well.

## 5. Real Data Tests

By applying the User Partial function in CALC/SOLVE, we performed single analysis of the 90 sessions in Fig.1. Fig.11 shows the delay residual *wrms* from the periodic and from the piecewise modeling of the residual clock behavior. It is shown that the periodic modeling can reduce the delay residual *wrms* by 2 ps to 10 ps compared with that of the piecewise modeling.

## 6. Concluding Remarks

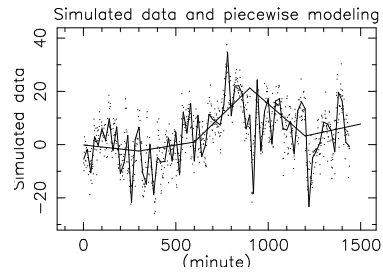
From the above analysis it is deduced that the choice of the piece length in the continuous piecewise linear modeling of the residual clock behavior and atmosphere effects leaves effects on the estimation of scientific interest parameters for instance up to *centimeters* in coordinates and up to several *picoseconds* in delay residual *wrms*. This effect can be different from session to session as well as from site to site. It is accordingly necessary to determine the suitable piece length in real data analysis. Simulated data tests and real data analysis show that the periodic modeling is preferable in modeling the residual short-term variations.

Our work is still in its preliminary stage. We will perform full tests of the periodic modeling of residual clock behavior and realize the periodic modeling of the residual atmosphere effects in data analysis software.

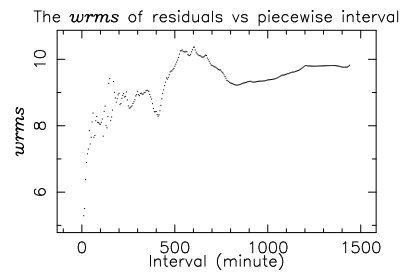
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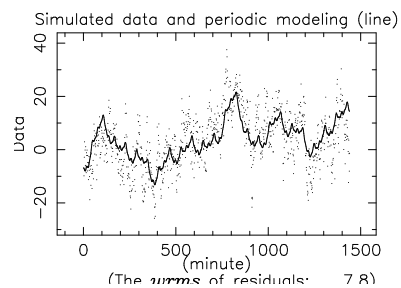
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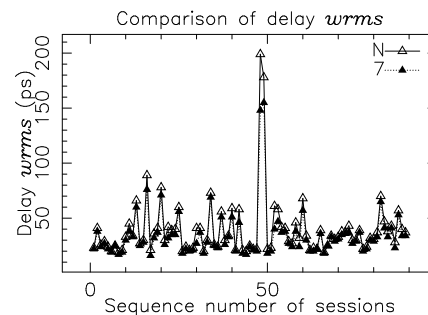
**Fig.8.** Simulated broad frequency band data and the piecewise modeling



**Fig.9.** The residual *wrms* of the simulated broad frequency band data versus the piece length



**Fig.10.** The periodic modeling of the simulated broad frequency band data



**Fig.11.** Comparison of the delay residual *wrms* from periodic (“7”) and piecewise (“N”) modeling of the residual clock behavior