

Precise Positioning of Spacecrafts by Multi-frequency VLBI

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Abstract

Multi-Frequency VLBI (MFV) is one of the most powerful methods for precise positioning of spacecraft. Radio sources for MFV transmit three carrier waves at S-band and one wave at X-band. These frequencies are set to resolve the cycle ambiguity of the carrier wave at X-band from the two group delays between carrier waves at S-band. A procedure to resolve the cycle ambiguity is proposed. Some conditions about frequency variation and prediction of position and the ionosphere are also summarized for resolving cycle ambiguities. The dedicated recording system for MFV is also developed. A preliminary observation of MFV is carried out with this system by using Lunar Prospector. As a result of the experiment, residual phases from predicted ones are within about 1000 degrees, and the RMS of the residual for the period of several seconds is about 4 degrees, which corresponds to 1.5 m in the positioning error around the Moon. It is confirmed that the hardware and software system have enough availability to achieve the expected accuracy in MFV.

1. Introduction

The study of orbital motion and the interior structure of the Moon and planets is one of the methods to approach for the revolution as a dynamical system and the origin of the solar system. It is a powerful method to measure the gravity fields obtained by the orbital motion of a spacecraft in order to estimate the inner layer and the density structure of the Moon and the planets. Orbits or positions of a lunar or planetary spacecraft have mainly been determined by range and Doppler measurements. These measurements provide only one-dimensional information about the position along the line of sight. On the other hand, differential VLBI (Very Long Baseline Interferometry) has the sensitivity of positioning in direction perpendicular to the line of sight, so that combining both range and Doppler measurements with differential VLBI enables us to measure three-dimensional position of a spacecraft.

VLBI methods have been used for positioning of spacecraft since 1960s. In these methods, carrier waves were transmitted from a spacecraft because of saving its transmitting power and obtaining high SNR. Unfortunately, the phase delay of a carrier wave, however, has cycle ambiguities, therefore only phase delay changes have been mainly used so far [1]. A new method, multi-frequency VLBI (MFV) was proposed [2, 3]. Radio sources for MFV transmits three frequency signals in S-band and one signal in X-band. These frequencies are set to resolve the cycle ambiguity of carrier wave at X-band from three group delays between carrier waves at S-band. A procedure to resolve the cycle ambiguity is proposed in this article. Some conditions about frequency variation and prediction of position and the ionosphere are summarized for resolving

the cycle ambiguity. The dedicated recording system for MFV is also developed. A preliminary observation of MFV is carried out with this system by using Lunar Prospector as a test of the whole system.

2. Cycle Ambiguity Resolving by MFV

In MFV method, two radio sources emit three carrier waves s_1, s_2, s_3 in S-band and one wave x in X-band. Cross correlation between the two received signals of the respective carrier waves at two ground stations produces four \times two fringe phases for each unit integration time. By differencing the fringe phases between two radio sources, four differenced fringe phase ϕ_i are obtained for the carrier wave i ($=s_1, s_2, s_3$ and x). Our final observable is the phase delay of the wave x without cycle ambiguity from the four phases. The phase delay $\tau_{pd,x}$ of the wave x is expressed as follows,

$$\tau_{pd,x} = \frac{\phi_x}{2\pi f_x} + \frac{kd}{f_x^2} + \frac{\sigma_x}{2\pi f_x} + \frac{N_x}{f_x}, \quad (1)$$

where f_i, σ_i, N_i is the frequency, the phase noise, the integer ambiguity of the wave i , respectively, and k is the ionospheric constant ($= 1.34 \times 10^{-7} [m^2 el.^{-1} s^{-1}]$), d is the differenced total electron content (TEC) [$el.m^{-2}$] in the ionosphere. In order to resolve the cycle ambiguity N_x , uncertainty of the left side $\tau_{pd,x}$ and the second and third term in Equation (1) must be less than $1/2f_x$. Conditions for the second and third term are summarized in the next section. The term $\tau_{pd,x}$ should be predicted within the accuracy of $1/2f_x$ by the phase delay τ_{pd,s_1} of the carrier wave s_1 as follows,

$$\tau_{pd,s_1} = \frac{\phi_{s_1}}{2\pi f_{s_1}} + \frac{kd}{f_{s_1}^2} + \frac{\sigma_{s_1}}{2\pi f_{s_1}} + \frac{N_{s_1}}{f_{s_1}}. \quad (2)$$

Because the delay ambiguity interval $1/f_{s_1}$ of carrier wave s_1 in Equation (2) is wider than that of wave x in Equation (1), it is easier to resolve the delay ambiguity of s_1 than that of x . In order to resolve the cycle ambiguity N_s , uncertainty of the left side τ_{pd,s_1} and the second and third term in Equation (2) must be less than $1/2f_{s_1}$. Conditions for the second and third term are summarized in the next section. The term τ_{pd,s_1} should be predicted within the accuracy of $1/2f_{s_1}$ by the group delay τ_{gd,s_3s_1} of s_3-s_1 as follows,

$$\tau_{gd,s_3s_1} = \frac{\phi_{s_3} - \phi_{s_1}}{f_{s_3} - f_{s_1}} + \frac{kd}{f_{s_3}f_{s_1}} + \frac{\sigma_{s_3} - \sigma_{s_1}}{f_{s_3} - f_{s_1}} + \frac{N_{s_3} - N_{s_1}}{f_{s_3} - f_{s_1}}. \quad (3)$$

It is easier to resolve the delay ambiguity of s_1-s_3 than that of s_1 . In order to resolve the cycle ambiguity $N_{s_1}-N_{s_3}$, uncertainty of the left side τ_{gd,s_3s_1} and the second and third term in Equation (2) must be less than $1/2(f_{s_3}-f_{s_1})$. Conditions for the second and third term are summarized in the next section. The term τ_{gd,s_3s_1} should be predicted within the accuracy of $1/2(f_{s_3}-f_{s_1})$ by group delay τ_{gd,s_2s_1} of $s_2 - s_1$ as follows,

$$\tau_{gd,s_2s_1} = \frac{\phi_{s_2} - \phi_{s_1}}{f_{s_2} - f_{s_1}} + \frac{kd}{f_{s_2}f_{s_1}} + \frac{\sigma_{s_2} - \sigma_{s_1}}{f_{s_2} - f_{s_1}} + \frac{N_{s_2} - N_{s_1}}{f_{s_2} - f_{s_1}}. \quad (4)$$

It is easier to resolve the delay ambiguity of s_1-s_2 than that of s_1-s_3 . In order to resolve the cycle ambiguity $N_{s_2}-N_{s_1}$, uncertainty of the left side τ_{gd,s_2s_1} and the second and third term in Equation (2) must be less than $1/2(f_{s_2}-f_{s_1})$. Conditions for the second and third term are summarized in the next section. The term τ_{gd,s_2s_1} should be predicted within the accuracy of $1/2(f_{s_2}-f_{s_1})$ by another tracking method. The condition of the accuracy is summarized in the next section.

2.1. Conditions for Resolving Cycle Ambiguities

In order to resolve the cycle ambiguity N_x mentioned above, the frequencies are selected as, $f_{s_1}=2212\text{MHz}$, $f_{s_2}=2218\text{MHz}$, $f_{s_3}=2287\text{MHz}$ and $f_x=8456\text{MHz}$. The condition of the phase noise, the delay accuracy of the prediction, the TEC, the frequency difference, frequency stability are summarized in Table 1. Radio transmitters on spacecraft should be designed to satisfy also these conditions. These conditions are discussed in more detail by some works [4, 5].

Table 1. Conditions for resolving cycle ambiguities

Phase noise, σ_{ϕ_i}	less than 4.2	degrees
Accuracy of the predicted delay	better than 8.3×10^{-8}	second
TEC, d	less than 2.3×10^{15}	el./ m^2
Frequency difference between two radio sources	less than 1.7×10^5	Hz
Frequency stability $y(\tau)$ $10 < \tau < 100$ sec.	better than 2.8×10^{-5}	Hz/Hz

3. Ground System for MFV

MFV method will be realized under the Japanese lunar exploring program, SELENE. The radio sources will be installed in the relay satellite and the vrad satellite [6]. We developed a new ground VLBI system, which consists of hardware to sample and record the carrier waves and software to estimate phase delays. The hardware samples and digitizes the video signals at sampling rate of 200 sps and records them on an 8 mm tape. The narrow bandwidth sampler and recorder is shown in Figure 1. The new software cross-correlates the recorded signals and produces the phase delay. The product is input to software to estimate orbits and gravity fields [7].



Figure 1. Narrow bandwidth sampler for MFV

4. Preliminary Experiment of MFV by Using Lunar Prospector

As a preliminary test of the whole system, an experiment of Lunar Prospector was carried out from 08:00 to 17:00 in Sep. 21, 1998. The antennas involved in the observation were the Kashima 34m-diameter antenna of CRL, the Mizusawa 10m-diameter antenna of NAO and the Tsukuba 3.8m-diameter antenna of GSI. The developed ground system was used to record and estimate the fringe phases. Unfortunately it transmitted only one carrier wave, so that the test for resolving the ambiguity of the fringe phase could not be achieved. The residual fringe phases after the correction for the ionospheric delay have systematic variations of about 1000 degrees with period of about one hour (Figure 2). These variations are supposed to be caused by the errors in the initial orbital elements and model errors of the lunar gravity fields used. Although these long period variations cannot be estimated in this experiment, they will be estimated in SELENE mission by MFV. The short period variations of the residual fringe phases will remain even if the two variations are estimated. The rms of the residual for the period of several tens of seconds is about 4.4 degrees (Figure 3). The rms residual for several seconds means that we can determine the position of Lunar Prospector within the error of 1.5 m around the Moon if we can correct the variations with long period by using the MFV.

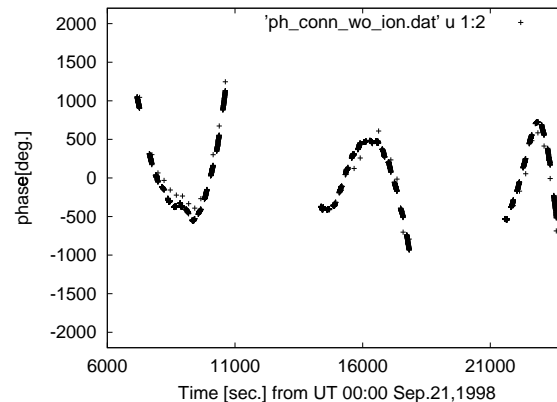


Figure 2. Residual fringe phases. Linear components are removed.

4.1. Conclusion

MFV method, which enables us to obtain the phase delay of RF signals at S- and X-band without cycle ambiguity, is proposed. The several conditions of the realization of this method are summarized. The new ground VLBI system, which consists of hardware to sample and record the received signals and software to estimate the phase delay, has been developed.

The preliminary experiment was carried out by using Lunar Prospector. Unfortunately it transmitted only one carrier wave, so that the test for resolving the ambiguity of the fringe phase could not be achieved. The residual fringe phases after the correction for the ionospheric delay have systematic variations of about 1000 degrees with period of about one hour. These variations are supposed to be caused by the errors in the initial orbital elements and model errors of the lunar gravity fields used. The rms of the residual fringe phases averaged for several tens of seconds

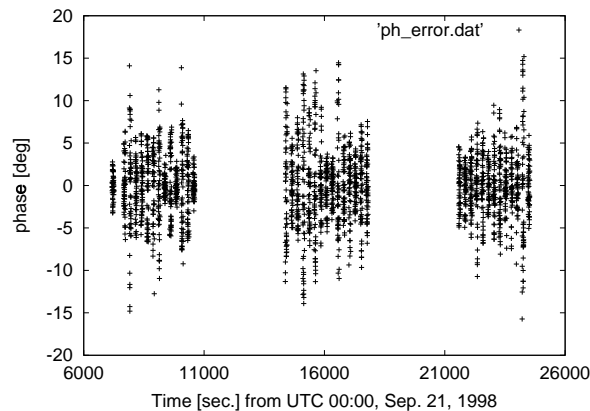


Figure 3. The short period variation of the residual fringe phase

reaches about 4 degrees, which corresponds to 1.5 m in the positioning error around the Moon. It is confirmed that the hardware and software system have enough availability to achieve the expected accuracy in MFV.

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