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On Correlations Between Parameters in Geodetic VLBI Data Analysis

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Abstract

The study of correlations between parameters estimated in least squares adjustments helps to investigate observing geometries and permit a more thorough assessment of the parameters' formal errors. High correlations between atmospheric parameters represented as zenith atmospheric excess path lengths and the vertical component of the station coordinates have long been considered as established facts in geodetic and astrometric VLBI. In this paper we investigate this commonly held view on the basis of transforming the variance/covariance matrices from geocentric into local (topocentric) coordinates. We find that correlations between the estimated relative clock offsets and the local vertical components are much higher than those between the atmospheric parameters and the vertical components. This result may be used as an argument to invest more in the stability of atomic clocks for further improvements in geodetic and astrometric VLBI.

1. Introduction

In the statistical literature correlations are mainly treated in the context of correlations between observations and in cases where parameters from a first adjustment are used as inputs for a second one. In contrast to that the consequences of the existence of correlations between estimated parameters are seldom discussed or interpreted.

Correlations between parameters are expressed as correlation coefficients r_{xy} computed from the variance/covariance matrix, i.e. from the main diagonal elements (standard deviations $\sigma_x = \sqrt{Q_{xx}}$ and $\sigma_y = \sqrt{Q_{yy}}$) and the corresponding off-diagonal element, i.e. the covariance Q_{xy}:

$$r_{xy} = \frac{Q_{xy}}{\sigma_x \cdot \sigma_y}. \tag{1}$$

In the adjustment of VLBI observations a commonly held view is that the topocentric height component and the atmospheric excess zenith path parameter are highly correlated (e.g. [1], [2]). The reason given is that in both cases the delay observable is affected by the sine of the elevation (ϵ) at the observing site as expressed in (2) and (3):

$$\Delta\tau_{Atm} = -\frac{1}{c} \cdot \frac{1}{\sin\epsilon} \cdot \Delta L^Z_{Atm} \tag{2}$$

$$\Delta\tau_U = -\frac{1}{c} \cdot \sin\epsilon \cdot \Delta U \tag{3}$$

where ΔL^Z_{Atm} is the change in atmospheric zenith path length, ΔU the change in the vertical component of the station coordinates, and $\Delta\tau$ are the resulting delay changes while c is the speed of light.

Generally, the vertical component ΔU is not readily available in least squares adjustments of VLBI observing sessions since the model is normally represented in geocentric x,y,z coordinates. Hence, the variance/covariance matrix is also expressed geocentrically. In order to investigate correlations which are related to the topocentric station components, i.e. vertical (U), East (E) and North (N) components, the error propagation law has to be applied to the covariance matrix Q in the form (see [3]):

$$\mathbf{Q}^{UEN} = \mathbf{B} \cdot \mathbf{Q}^{XYZ} \cdot \mathbf{B}^T \qquad (4)$$

B is the matrix of partial derivatives with respect to the individual parameters or the so-called design matrix (here we only display **B** for a single site, for all other sites this pattern repeats):

$$\mathbf{B} = \begin{vmatrix} \frac{\partial U}{\partial X} & \frac{\partial U}{\partial Y} & \frac{\partial U}{\partial Z} & \frac{\partial U}{\partial CL0} & \frac{\partial U}{\partial CL1} & \frac{\partial U}{\partial CL2} & \frac{\partial U}{\partial AT1} & \cdots & \frac{\partial U}{\partial ATn} & \frac{\partial U}{\partial NG} & \frac{\partial U}{\partial EG} \\ \frac{\partial E}{\partial X} & \frac{\partial E}{\partial Y} & \frac{\partial E}{\partial Z} & \frac{\partial E}{\partial CL0} & \frac{\partial E}{\partial CL1} & \frac{\partial E}{\partial CL2} & \frac{\partial E}{\partial AT1} & \cdots & \frac{\partial E}{\partial ATn} & \frac{\partial E}{\partial NG} & \frac{\partial E}{\partial EG} \\ \frac{\partial N}{\partial X} & \frac{\partial N}{\partial Y} & \frac{\partial N}{\partial Z} & \frac{\partial N}{\partial CL0} & \frac{\partial N}{\partial CL1} & \frac{\partial N}{\partial CL2} & \frac{\partial N}{\partial AT1} & \cdots & \frac{\partial N}{\partial ATn} & \frac{\partial N}{\partial NG} & \frac{\partial N}{\partial EG} \\ \frac{\partial CL0}{\partial X} & \frac{\partial CL0}{\partial Y} & \frac{\partial CL0}{\partial Z} & \frac{\partial CL0}{\partial CL0} & \frac{\partial CL0}{\partial CL1} & \frac{\partial CL0}{\partial CL2} & \frac{\partial CL0}{\partial AT1} & \cdots & \frac{\partial CL0}{\partial ATn} & \frac{\partial CL0}{\partial NG} & \frac{\partial CL0}{\partial EG} \\ \frac{\partial CL1}{\partial X} & \frac{\partial CL1}{\partial Y} & \frac{\partial CL1}{\partial Z} & \frac{\partial CL1}{\partial CL0} & \frac{\partial CL1}{\partial CL1} & \frac{\partial CL1}{\partial CL2} & \frac{\partial CL1}{\partial AT1} & \cdots & \frac{\partial CL1}{\partial ATn} & \frac{\partial CL1}{\partial NG} & \frac{\partial CL1}{\partial EG} \\ \frac{\partial CL2}{\partial X} & \frac{\partial CL2}{\partial Y} & \frac{\partial CL2}{\partial Z} & \frac{\partial CL2}{\partial CL0} & \frac{\partial CL2}{\partial CL1} & \frac{\partial CL2}{\partial CL2} & \frac{\partial CL2}{\partial AT1} & \cdots & \frac{\partial CL2}{\partial ATn} & \frac{\partial CL2}{\partial NG} & \frac{\partial CL2}{\partial EG} \\ \frac{\partial AT1}{\partial X} & \frac{\partial AT1}{\partial Y} & \frac{\partial AT1}{\partial Z} & \frac{\partial AT1}{\partial CL0} & \frac{\partial AT1}{\partial CL1} & \frac{\partial AT1}{\partial CL2} & \frac{\partial AT1}{\partial AT1} & \cdots & \frac{\partial AT1}{\partial ATn} & \frac{\partial AT1}{\partial NG} & \frac{\partial AT1}{\partial EG} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ \frac{\partial ATn}{\partial X} & \frac{\partial ATn}{\partial Y} & \frac{\partial ATn}{\partial Z} & \frac{\partial ATn}{\partial CL0} & \frac{\partial ATn}{\partial CL1} & \frac{\partial ATn}{\partial CL2} & \frac{\partial ATn}{\partial AT1} & \cdots & \frac{\partial ATn}{\partial ATn} & \frac{\partial ATn}{\partial NG} & \frac{\partial ATn}{\partial EG} \\ \frac{\partial NG}{\partial X} & \frac{\partial NG}{\partial Y} & \frac{\partial NG}{\partial Z} & \frac{\partial NG}{\partial CL0} & \frac{\partial NG}{\partial CL1} & \frac{\partial NG}{\partial CL2} & \frac{\partial NG}{\partial AT1} & \cdots & \frac{\partial NG}{\partial ATn} & \frac{\partial NG}{\partial NG} & \frac{\partial NG}{\partial EG} \\ \frac{\partial EG}{\partial X} & \frac{\partial EG}{\partial Y} & \frac{\partial EG}{\partial Z} & \frac{\partial EG}{\partial CL0} & \frac{\partial EG}{\partial CL1} & \frac{\partial EG}{\partial CL2} & \frac{\partial EG}{\partial AT1} & \cdots & \frac{\partial EG}{\partial ATn} & \frac{\partial EG}{\partial NG} & \frac{\partial EG}{\partial EG} \end{vmatrix} . \qquad (5)$$

Since in different coordinate systems there are only interdependencies in the coordinate components but not between other parameters, the design matrix can be simplified to read (again for one site only):

$$\mathbf{B} = \begin{vmatrix} \frac{\partial U}{\partial X} & \frac{\partial U}{\partial Y} & \frac{\partial U}{\partial Z} & 0 & \cdots & 0 \\ \frac{\partial E}{\partial X} & \frac{\partial E}{\partial Y} & \frac{\partial E}{\partial Z} & 0 & \cdots & 0 \\ \frac{\partial N}{\partial X} & \frac{\partial N}{\partial Y} & \frac{\partial N}{\partial Z} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 1 & & 0 \\ \vdots & \vdots & \vdots & & \ddots & \\ 0 & 0 & 0 & 0 & & 1 \end{vmatrix} . \qquad (6)$$

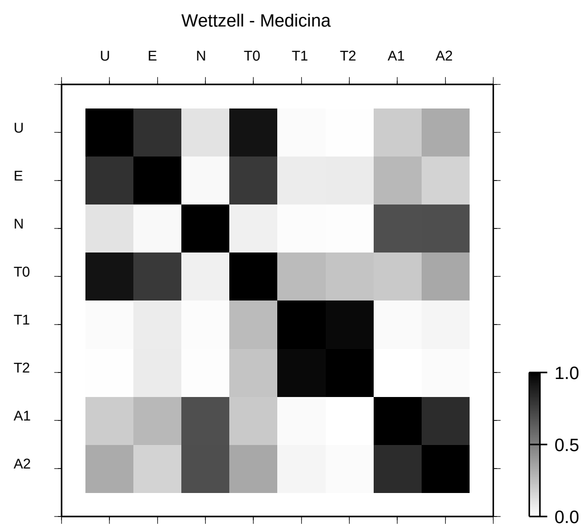
The correlation coefficients can then be computed according to (1) forming the matrix of correlation coefficients.

2. Results

For better insight we will display the correlation coefficients in a graphical form using shades of grey between black and white (correlation coefficient magnitude of 1 and 0, respectively) as displayed in a single baseline example in figure 1 (observations between Wettzell and Medicina carried out in a EUROPE session using a minimal parameterization). This consists of just the three topocentric station components (U,E,N) of Medicina, a second order clock polynomial (T_0, T_1, T_2), and one atmosphere parameter for each station (A1, A2).

At first glance we see that the two atmosphere parameters are highly correlated due to the fact that the elevation angles for the two stations are very similar. The clock rate (T1) and acceleration (T2) parameters are correlated likewise due to the fact that T2 is always the square of T1. What we do not see, however, is a high correlation between any of the atmosphere parameters and Medicina's vertical component. On the contrary, there is hardly any correlation between these parameters. More striking, the clock offset is highly correlated with the vertical component. In addition, a lesser degree of correlation with the East component is apparent leading to a prominent correlation between the East and the vertical component. The atmospheric parameters are correlated significantly only with the North component.

In order to study this phenomenon on a broader basis we transformed the covariances of a number of European and global 24-hour sessions in various configurations into the local systems. As an example, figure 2 displays the correlations of a five-station European session.



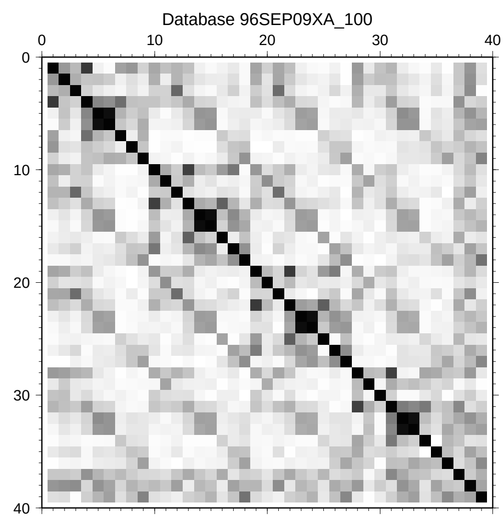


Figure 1 (left). Correlation coefficients. See text for explanation. Figure 2 (right). Correlation coefficients of 5-station EUROPE session. Parameter sequence: DSS65 (U,E,N, $T_0, T_1, T_2, Atm_1, Atm_2, Atm_3$), Matera (as DSS65), Noto (as DSS65), Onsala (as DSS65), Wettzell ($T_0, T_1, T_2, Atm_1, Atm_2, Atm_3$)

Here, as in the single baseline example, the correlations between the clock offsets with the corresponding vertical components are clearly visible. Of course, there are the typical correlations between clock rate and acceleration or within the atmosphere gradients but they are of minor interest here.

A similar situation also prevails in a series of test computations we made with different parameterizations. In global networks we found the same trend in correlations between the vertical components and the clock offsets. However, here the magnitude of the individual correlation coefficients depended to a large extent on the observing schedule and the distribution and number of the observations on each baseline.

3. Discussion

The surprisingly low level of correlations between the atmosphere parameters and the vertical components we found in our study contradicts earlier investigations (e.g. [1], [2]). However, it is quite obvious to assume strong correlations between parameters for an observation at zenith where one can hardly distinguish between an error being caused by the atmosphere or by the local vertical.

In order to explain the phenomenon we return to the initial equations (2) and (3) which emphasize the importance of the elevation angles in this scenario. The estimation process is based on partial derivatives which in turn can easily be deduced from (2) and (3) resulting in

$$\frac{\partial \tau_{obs}}{\partial U} = -\frac{1}{c} \cdot \sin \epsilon \tag{7}$$

$$\frac{\partial \tau_{obs}}{\partial Atm^Z} = -\frac{1}{c} \cdot \frac{1}{\sin \epsilon}. \tag{8}$$

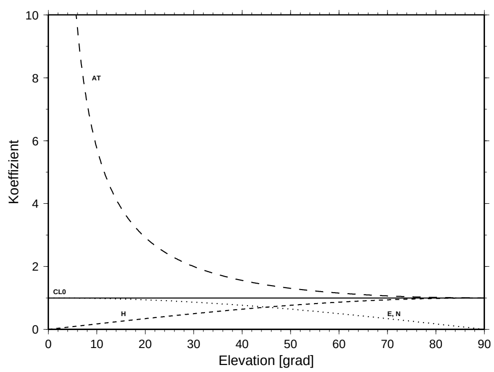
If we only had a few observations more or less near the zenith these would generate very similar coefficients close to 1 (ignoring the $\frac{1}{c}$ here). The consequence would be that we would find a very high correlation coefficient between the vertical component and the atmosphere parameter in a least squares adjustment with only these observations.

But the observing schedules of VLBI sessions are constructed in a way that over the full 24-hour period the sky is sampled in as many different directions as possible generating a multitude of observing geometries with many different elevation angles. In recent years the elevation limits have been reduced to as low as 4°.

If we display the values of the partial derivatives w.r.t. the elevation (see figure 3) we see that at low elevations the values of the local vertical (U) and of the atmosphere parameter diverge considerably, a situation which leads to low correlation between atmosphere parameters and the vertical component. At the same time we see that the clock offset coefficient, which is a constant, always stays very close to the vertical or horizontal components (N,E). Hence, the observing geometry can never be varied in a way that the partial derivatives of coordinate components are very different from the clock offset and a high correlation has to be expected here.

In order to explain the high correlation between clock offset and vertical component we may also approach the problem from a geometrical perspective. Let's assume that we have only three different observations on a single short baseline (Fig. 4) where two observations are in opposite directions close to the horizon (elevation 0°, #1 and #2) and a third one is at zenith (#3).

The first two observations are necessary to determine the clock offset (T_0) which, in the case of a low elevation observation, could otherwise not be distinguished from a horizontal shift of station B. The effect of the clock offset is identical in all directions (displayed as a semi circle about station B). If we now want to determine the height of station B we need the value of the clock offset as determined by the first two observations. If we had not determined the clock offset with the first two observations, the height of station B would be undetermined. Since the clock offset has an identical effect in all directions or elevations, its correlation with other parameters and particularly with the vertical coordinate component is much higher than that of the atmosphere which has a very strong elevation dependent signature.



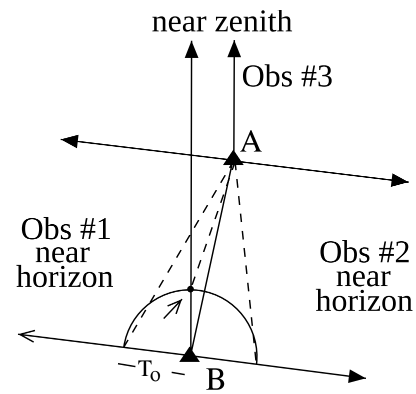


Figure 3 (left). Partial derivatives w.r.t. to elevation. Figure 4 (right). Three-dimensional geometry of 2 observations near horizon in opposite directions and one observation towards zenith with stations A and B. Clock offset T_0 represented as distance from station.

4. Conclusion

Although the high correlation coefficient between the clock offset and the topocentric vertical component comes as a surprise it can be explained from a geometric perspective as well as from a detailed analysis of the partial derivatives used in the least squares adjustment. At least from the standpoint of correlation, the atmosphere parameters do not seem to present the main problem for the accuracy of the vertical component. There does not appear to be a correlational mechanism by which the atmospheric errors are directly mapped into the vertical component. However, we should note the fact that the correlations increase when the elevation mask is lifted.

As a consequence of our investigations more attention should be paid to the stability of the atomic clocks used in VLBI observations. Our analysis shows that the clock errors are indeed mapped directly into the vertical components (correlation levels $>$ 80%). As another test it would be interesting to investigate whether stations with stable hydrogen masers show a better repeatability in the height component than other stations. Nevertheless, we feel that clock stability aspects and clock parameterization in the least squares adjustments need further investigations in order to improve the overall accuracy of geodetic VLBI results.

References

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