

# Vienna Mapping Functions in VLBI Analyses

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## Abstract

In recent years numerical weather models (NWM) have been applied to improve mapping functions which are used for tropospheric delay modeling in VLBI and GPS data analyses. The Vienna Mapping Functions VMF (Boehm and Schuh, 2004, [1]) are based on direct raytracing through NWM, and allow exploitation of the full information provided in the NWM. On the other hand, the Isobaric Mapping Functions IMF (Niell, 2001, [4]) use intermediate parameters calculated from the NWM. In this study, pressure level data from the ECMWF (European Centre for Medium-Range Weather Forecasts) are applied to determine the coefficients of the VMF and the IMF. Used for the analyses of IVS-R1 and IVS-R4 VLBI sessions, both mapping functions improve the repeatability of baseline lengths (by ~10% for IVS-R1 and ~5% for IVS-R4) compared to the Niell Mapping Functions NMF (Niell, 1996, [3]).

## 1. Introduction

Raytracing through radiosonde data has often been applied to develop and validate mapping functions which are needed for tropospheric modeling in VLBI and GPS data analyses. For example, the Niell Mapping Functions NMF (Niell, 1996, [3]), which require station height, station latitude and day of the year as input parameters, were developed using radiosonde data over a wide range of latitudes. In recent years, much effort has been put into the development of mapping functions based on data from numerical weather models. Niell (2001, [4]) proposed the Isobaric Mapping Functions (IMF) which apply as input parameters the height of the 200 mbar pressure level ( $z_{200}$ ) and the ratio of the wet path delay along a straight line at  $3.3^\circ$  elevation and zenith delay ( $smfw3$ ). The equations relating these two parameters to the coefficients of the continued fraction form (see Eq. 1) are based on raytracing through radiosonde data. The tilting of the 200 mbar pressure level also provides information for the correction of hydrostatic north-south and east-west gradients before estimating the remaining wet gradients.

$$mf_{h,w} = \frac{1 + \frac{a}{1 + \frac{b}{1 + c}}}{\sin(e) + \frac{a}{\sin(e) + \frac{b}{\sin(e) + c}}} \quad (1)$$

When working on the implementation of these mapping functions with pressure level data from the ECMWF, it became evident that the NWM could be exploited more rigorously by discarding intermediate parameters like  $z_{200}$  and  $smfw3$ . The main idea behind the Vienna Mapping Functions VMF is to directly use the raytracing through the NWM instead of taking intermediate steps.

## 2. Determination of the Vienna Mapping Functions (VMF)

The continued fraction form for the hydrostatic and wet mapping function for an elevation angle  $e$  is shown in Eq. 1. This form is also used in the NMF (Niell, 1996, [3]) and in the IMF (Niell, 2001, [4]). Three coefficients  $a$ ,  $b$  and  $c$  are sufficient to map zenith delays down to elevations of  $3^\circ$ . In the case of VMF, these coefficients are determined from raytracing through NWM.

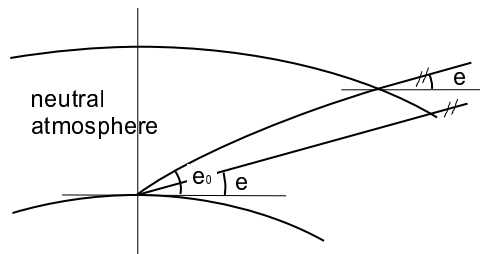


Figure 1. Raytracing through NWM

Input parameters for the raytracing program are an initial elevation angle  $e_0$ , and values for height, temperature and water vapor pressure at distinct pressure levels in the neutral atmosphere (e.g. 15 levels from 1000 to 10 hPa total pressure). The raytracing yields the outgoing (vacuum) elevation angle  $e$ , and the values for the hydrostatic and the wet mapping function. The hydrostatic mapping function includes the geometric bending effect.

Instead of raytracing at 10 different elevation angles from  $90^\circ$  to  $3.3^\circ$  elevation and estimating the coefficients  $a$ ,  $b$  and  $c$  in a least-squares adjustment (rigorous approach VMF(rig), Boehm and Schuh, 2004, [1]), the fast approach VMF(fast) requires the raytracing for only one initial elevation angle of  $3.3^\circ$  (see Fig. 1). This yields one value for the hydrostatic mapping function, one value for the wet mapping function, and the vacuum elevation angle  $e$  ( $\sim 3^\circ$ ). Then, pre-defined formulas are used for the  $b_h$ ,  $c_h$ ,  $b_w$ , and  $c_w$  coefficients, and the coefficients  $a_h$ ,  $a_w$  can be determined by simply inverting the continued fraction form (Eq. 1). To check the quality of VMF(fast), the values of the mapping functions have been compared to VMF(rig) for CONT02, a continuous 15-day VLBI campaign with eight stations in October 2002. For an elevation angle of  $5^\circ$  the RMS differences are about 5 mm for the hydrostatic and 1 mm for the wet part when the hydrostatic and wet zenith delays are assumed to be 2000 mm and 200 mm, respectively. This would imply that the corresponding error in the station height is less than 2 mm when the cutoff elevation angle is set to  $5^\circ$ . Moreover, the RMS differences vanish at about  $3^\circ$  because VMF(fast) is tuned for this elevation angle, and above  $5^\circ$  elevation the RMS differences are also decreasing rapidly.

## 3. Validation of the VMF

Improved mapping functions are expected to improve geodetic accuracies. Good measures for the quality of geodetic results are the “baseline length repeatability” and the difference in baseline length when changing the cutoff elevation angle (“elevation angle cutoff test”). For the geodetic VLBI analysis, the classical least-squares method of the OCCAM 5.1 VLBI software package (Titov et al., 2001, [6]) is used. Free network solutions with a minimum of squared station coordinate residuals are calculated for the 24h sessions with five Earth orientation parameters being estimated (nutation, dUT1 and pole coordinates). Atmospheric loading parameters are obtained from Petrov and Boy (2004, [5]), and total gradient offsets are estimated every 6 hours using the model by Davis et al. (1993, [2]).

Baseline length repeatabilities are determined for CONT02 (2002, October 16-31), and for all IVS-R1 and IVS-R4 sessions from January 2002 till August, 2003. For the following investigations all baselines which include the station TIGO/Concepcion (Chile) are excluded, because due to the small antenna dish (6 m diameter), the low SNR does not allow a reliable validation of the tropospheric mapping functions. Also, the baselines with station Gilmore Creek (Alaska, U.S.A.) are not considered before the Earthquake on November 3, 2002. The cutoff elevation angle was set to 5°. Approximately 70% - 80% of the baselines got improved repeatabilities with the new mapping functions IMF and VMF compared to NMF. The mean relative improvement is ~5% for IVS-R4 and CONT02, and it is even ~10% for IVS-R1 (Boehm and Schuh, 2004, [1]), mainly due to the enormous improvement for baselines with station Tsukuba (Japan) (Fig. 2) which does not take part in IVS-R4 and CONT02.

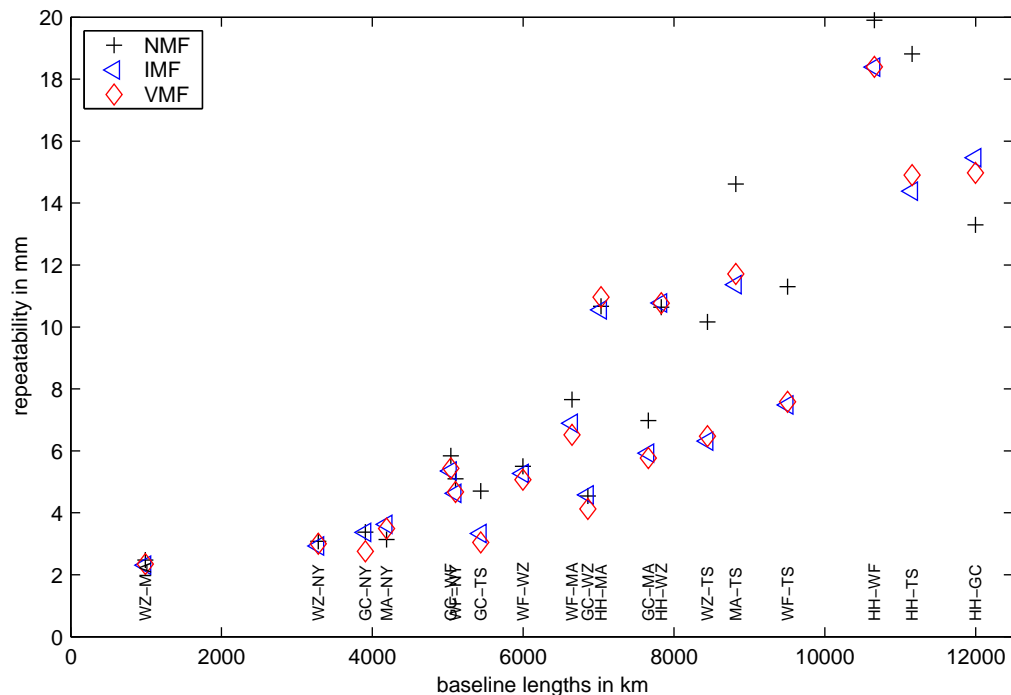
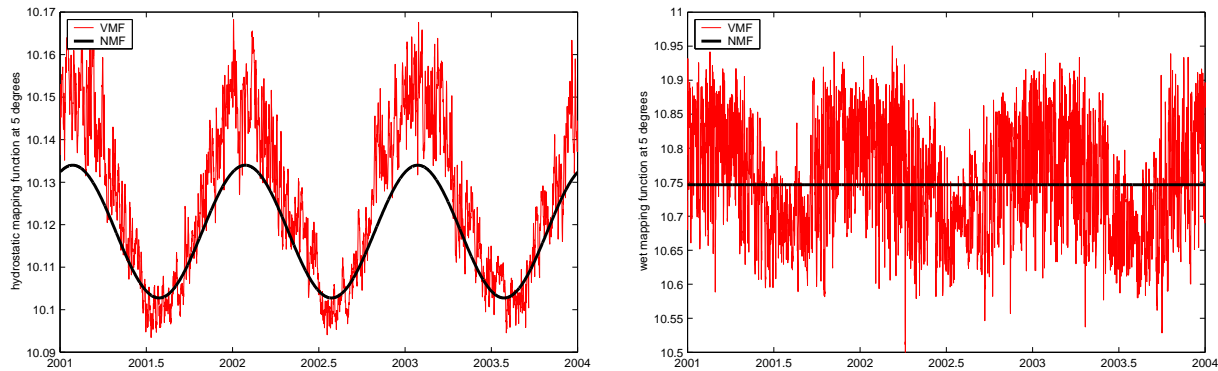


Figure 2. Baseline length repeatabilities for IVS-R1. Especially, the repeatabilities for all baselines with station Tsukuba (TS) in Japan are improved considerably. The repeatabilities are approximately the same for VMF and IMF.

Fig. 3 shows the hydrostatic and wet mapping functions for VMF and NMF at station Tsukuba (Japan). There is a significant bias between the hydrostatic mapping functions, which is caused by the fact that NMF is based on radiosonde data in the U.S.A. (Niell, 1996, [3]) and Tsukuba does not fit into this latitude-dependent model.

Another measure of the quality of mapping functions can be obtained by elevation angle cutoff tests. These tests show how baseline lengths change when the cutoff elevation angle is varied. They are a good measure for the absolute accuracy of the mapping functions, because they reveal systematic errors in the mapping functions as the baseline lengths will be influenced when the cutoff angle is changed. Supposing that NMF and VMF yield almost identical results when the



Hydrostatic mapping functions at 5° elevation for NMF and VMF. The time series are biased significantly, which again biases the station heights by  $\sim -3\text{mm}$  (see Tab. 1).

Wet mapping functions at 5° elevation for NMF and VMF. NMF does not account for seasonal variations.

Figure 3. Mapping functions at station Tsukuba for NMF and VMF. Unlikely to NMF, the Vienna Mapping Functions (as well as IMF) are able to model variations with periods down to 6 hours.

cutoff elevation angle is set to 10°, the elevation angle cutoff test can be taken to determine station height changes when using VMF (or IMF) instead of NMF for 5° cutoff (see Table 1). These results are confirmed by the rule of thumb saying that the station height error is  $\sim 1/3$  of the mapping function error for the lowest observations.

Table 1. Changes in mean station heights in mm for IVS-R1 and IVS-R4 when using VMF or IMF instead of NMF. These results are derived from elevation angle cutoff tests.

	gc	wz	ny	ma	wf	ts	hh	ft	kk	ap	sh
IVS-R1	+1.9	+0.4	+1.0	-1.0	-1.7	-3.0	-0.5				
IVS-R4	+1.8	-0.3	+0.6	-0.9				-0.6	-1.1	-0.3	-1.0

#### 4. Mapping Function Parameters Provided by IGG

The Institute of Geodesy and Geophysics (IGG) of the Vienna University of Technology gets regular access to data from the ECMWF (European Centre for Medium-Range Weather Forecasts). Derived parameters which are necessary to determine IMF and VMF for non-commercial purposes have been provided to the scientific community since September 2003. Table 2 gives an overview of the ECMWF datasets which are used for these computations. The parameters for IMF (z200 and smfw3) are provided on a global grid with a resolution of 2.5° x 2.0°. The coefficients  $a_h$ ,  $a_w$  for the hydrostatic and wet part of VMF(fast) are determined for all geodetic VLBI stations. This list will be extended to other selected sites, e.g. the IGS (International GPS Service) stations in near future. All parameters are given every six hours and they can be found at the webpage <http://www.hg.tuwien.ac.at/~ecmwf>.

Table 2. Specification of the ECMWF datasets which are used for the computation of IMF and VMF(fast)

	IMF	VMF(fast)
doy 1 in 1979 - doy 365 in 2001	2.5° x 2.0° ERA-40 Re-Analysis pressure level dataset (15 levels from 1000 to 10 hPa)	
doy 1 in 2002 - doy 238 in 2003	2.5° x 2.0° operational pressure level dataset (15 levels from 1000 to 10 hPa)	
doy 239 in 2003 - now	2.5° x 2.0° operational pressure level dataset (15 levels)	~0.3° operational pressure level dataset (21 levels)

## 5. Conclusions and Outlook

Recent mapping functions such as IMF and VMF based on data from numerical weather models like ECMWF provide better repeatabilities of baseline lengths. There is a mean improvement of 5% to 10% versus results obtained by NMF, but further investigations remain to be done. Especially, the quality of the mapping functions at certain stations has to be evaluated by means of nearby radiosonde data or different numerical weather models, and a closer look needs to be taken at systematic effects revealed by elevation angle cutoff tests. So far, baseline length repeatabilities are not significantly better with VMF compared to IMF, although VMF exploits the NWM more rigorously. But as the time series get longer, one might speculate that the advantages of VMF become visible, since starting with doy 239 in 2003 the vertical and horizontal resolution of the ECMWF data has increased significantly (to 21 levels and down to 0.3°, respectively), and the quality of ECMWF data will improve steadily in the future.

## References

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