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Modelling Vertical Total Electron Content from VLBI Observations

Thomas Hobiger, Harald Schuh

Institute of Geodesy and Geophysics, University of Technology, Vienna

Contact author: Thomas Hobiger, e-mail: thobiger@mars.hg.tuwien.ac.at

Abstract

The vertical total electron content (VTEC) can be understood as the sum of electrons in a column ranging in zenith direction from the ground through the ionosphere with a footprint size of one square meter. Although VLBI is a differential technique it is possible to derive absolute TEC values for each station from VLBI observations as shown in prior papers and presentations. At the Institute of Geodesy and Geophysics, Vienna, investigations of the functional and stochastical model have been made. An approach dealing with trigonometric functions that allows direct conclusions on amplitudes and phases of the sub-daily periods is presented. Other strategies using piece-wise linear functions and an extended piece-wise linear approach with adaptive interval widths are shown, too. The usage of kernel functions, in this case of Gaussian type, as a very general approach for modelling the ionosphere, is illustrated. The weights of a delay observable used for the stochastical model should also consider the zenith distance on each station and a corresponding weighting function is applied.

1. Theory

VLBI observations are performed at two different frequencies (X- and S-band) in order to correct for the ionospheric delay. This information can be used to model the ionosphere above each station, although only the differences between the two stations are measured. Instrumental offsets at each station bias these measurements (eq. 1).

$$\tau_{model}(t) = \tau_{ion,1}(t) - \tau_{ion,2}(t) + \tau_{offset,1} - \tau_{offset,2} \quad (1)$$

The ionospheric delay $\tau_{ion,i}(t)$ at X-band over station i at time t can be modeled by equation (2) with an appropriate mapping function (eq. 3) (e.g., Schaer 1999, [1]).

$$\tau_{ion,i}(t) = \frac{1.34 \cdot 10^{-7}}{f_x^2} \cdot M_f(\varepsilon_i) \cdot VTEC_i(t) \quad (2)$$

$$M_f(\varepsilon_i) = \frac{1}{\cos\left\{\arcsin\left[\frac{R\cos\varepsilon_i}{R+h}\right]\right\}} \quad (3)$$

VTEC represents the vertical total electron content at the intersection point of the ray path in zenith direction with the infinitesimally thin ionospheric layer assumed to be at height h (=450 km) (fig. 1), R is the mean radius of the Earth, ε_i is the elevation angle at station i and f_x the effective ionospheric frequency at X-band.

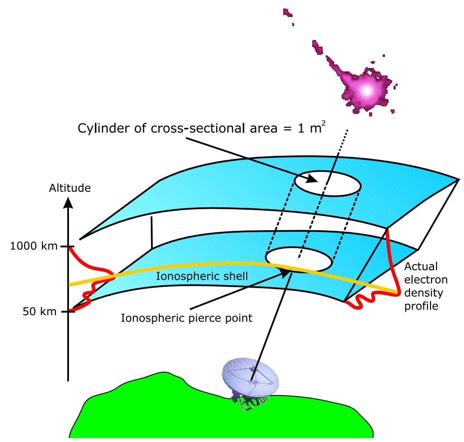

Figure 1. Modelling the ionosphere by means of VLBI

2. Functional Model

Three different approaches for modelling the VTEC have been investigated.

- Fourier components (eq. 4), first used by Kondo (1991), [2]
- piece-wise linear function (PLF) with adaptive interval lengths (eq. 5), 8 observations per interval
- Kernel-functions (eq.6)

$$VTEC_{FOURIER,i}(t) = a_{i0} + \sum_{k=1}^{4} \left[a_{ik} \cos\left(\frac{kt\pi}{12}\right) + b_{ik} \sin\left(\frac{kt\pi}{12}\right) \right] + c_i t \qquad (4)$$

$$VTEC_{PLF,i}(t) = offset_i + rate_{i,1}(t_1 - t_0) + rate_{i,2}(t_2 - t_1) + \ldots + rate_{i,n}(t_n - t) \qquad t \leq t_n \quad (5)$$

$$VTEC_{KERNEL,i}(t) = \sum_{i=1}^{n_{max}} A_i \cdot KF_i(t) \qquad KF_i(t) = \exp(-C^2 \cdot (t - t_i)^2) \qquad (6)$$

The different functional models are compared for the IVS-R4 session of Sept. 4th, 2003. VTEC values derived for station Fortaleza are shown in the left plot of figure 2 (Fourier-approach), center plot of figure 2 (PLF-approach), and right plot of figure 2 (Kernel-approach) in comparison to the official IGS combined solution (from [4]). The absolute correlation coefficients $|r_{ik}|$ between all unknown parameters of the above-mentioned IVS-R4 session are plotted in figure 3 according to the different modelling strategies.

The differences for all IVS-R4 stations between the individual VLBI approaches and the IGS solution during the 24h IVS-R4 session were analyzed, too (figure 4 and table 1).

3. Stochastical Model

A stochastical model was developed, that takes the different elevation angles $(\varepsilon_1, \varepsilon_2)$ on each station into account (eq. 7 and 8). Taking this function to the i-th power allows increasing the weight

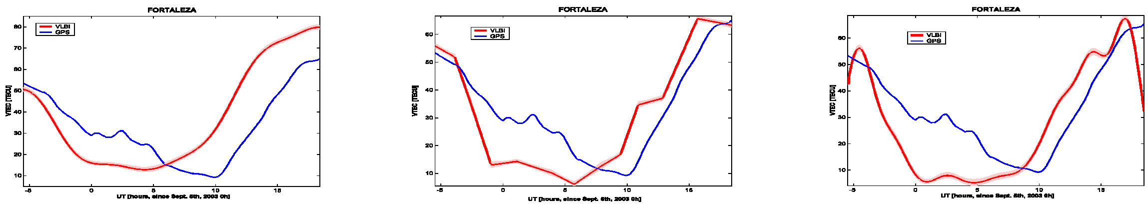

Figure 2. VLBI results for station Fortaleza from the Fourier (left), PLF (central) and Kernel (right) approach compared to the IGS solution.

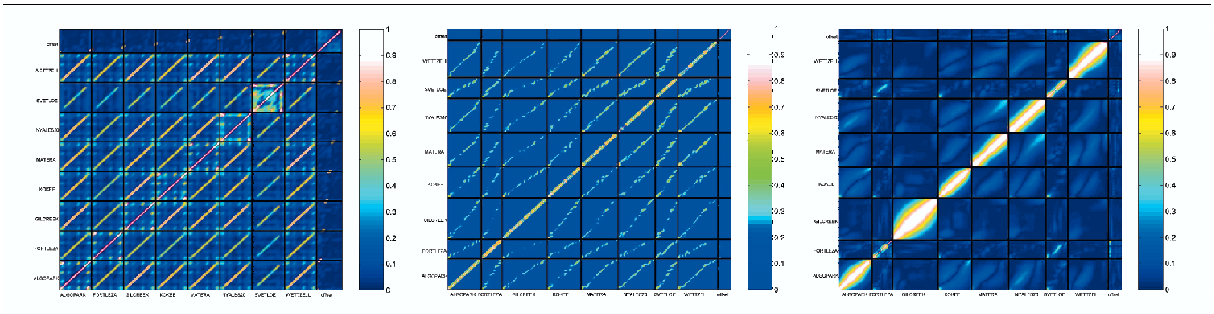

Figure 3. Absolute correlation coefficients between all unknown parameters for the Fourier (left), PLF (central) and Kernel (right) approach.

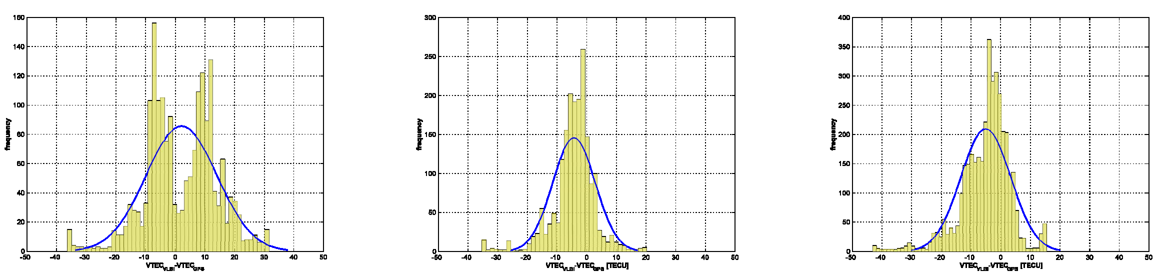

Figure 4. Histogram of the differences between VLBI and GPS (IGS) for the Fourier (left), PLF (central) and Kernel (right) approach.

Table 1. Mean differences of various VLBI approaches to GPS during the IVS-R4 session of Sept. 4th, 2003

VLBI (Fourier-approach) minus GPS:	2 TECU	$\pm$ 12 TECU
VLBI (PLF-approach) minus GPS:	-4 TECU	$\pm$ 8 TECU
VLBI (Kernel-approach) minus GPS:	-5 TECU	$\pm$ 10 TECU

either on low or on high elevation observations depending on the value of i (figure 5). Varying values of i and applying the PLF-approach for the IVS-R4 session of Sept. 4th, 2003 yields different results for the a posteriori sigma (σ_0), of the mean differences to GPS ($\overline{\Delta VTEC}_{VLBI-GPS}$) and the r.m.s ($\sigma\left(\Delta VTEC_{VLBI-GPS}\right)$) as shown in table 2. For $i = +4$ the a posteriori σ_0 is close to 1 (indicates that weights were chosen correctly, see Koch, 1997, [3]) and that the r.m.s. VLBI - GPS reaches the smallest value. However, when using $i = +4$ the mean difference to GPS is rather big (-4.95 TECU) which can be explained by the reduced ability to separate the instrumental offsets from the ionospheric parameters due to downweighting of elevation observations.

$$M_f(\varepsilon) = \frac{1}{\cos\left\{\arcsin\left[\frac{R\cos\varepsilon}{R+h}\right]\right\}} = \frac{1}{\sqrt{1-\left(\frac{R}{R+h}\cos\varepsilon\right)^2}} \tag{7}$$

$$w^i(\varepsilon_1,\varepsilon_1) = \left(\frac{M_f(90^\circ)}{\sqrt{\frac{1}{2}\left[M_f(\varepsilon_1)\right]^2+\frac{1}{2}\left[M_f(\varepsilon_2)\right]^2}}\right)^i = \tag{8}$$
$$= \left(\frac{2\cdot\left[1-\left(\frac{R}{R+h}\right)^2\cos^2\varepsilon_1\right]\cdot\left[1-\left(\frac{R}{R+h}\right)^2\cos^2\varepsilon_2\right]}{2-\left(\frac{R}{R+h}\right)^2\left(\cos^2\varepsilon_1+\cos^2\varepsilon_2\right)}\right)^{i/2}$$

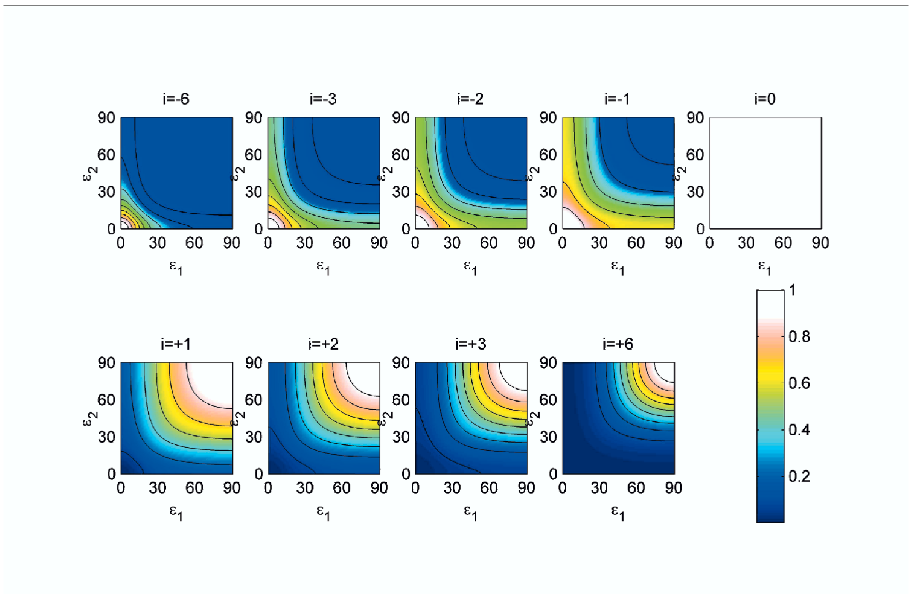

Figure 5. Weighting of VLBI measurements as a function of elevations (ε_1,ε_2), normalized to maximum.

4. Conclusions

It is possible to derive ionospheric parameters in terms of vertical total electron content exclusively from VLBI data, i.e. without any external information. Comparisons with GPS show

Table 2. Impact of different weighting of PLF-approach on a posteriori sigma (σ_0), on the mean difference to GPS $\overline{\Delta VTEC}_{VLBI-GPS}$), and on the r.m.s ($\sigma\left(\Delta VTEC_{VLBI-GPS}\right)$)

i	-6	-4	-3	-2	-1	0	1	2	3	4	6
σ_0	18,05	10,12	7,27	5,33	4,08	3,05	2,27	1,71	1,28	1,01	0,61
$\overline{\Delta VTEC}_{VLBI-GPS}$	-4,83	-4,81	-4,79	-4,72	-4,63	-4,58	-4,62	-4,72	-4,99	-4,95	-4,93
$\sigma\left(\Delta VTEC_{VLBI-GPS}\right)$	7,91	7,89	7,88	7,95	7,88	7,91	7,89	7,81	7,64	7,49	7,48

differences of up to a few TEC units. The piece-wise linear model and the weighting function shown in eq.(8) are recommended.

5. Acknowledgements

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