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Viscosity of the Earth's Fluid Core from VLBI Data

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Abstract

Differential equations of the Earth's rotation are developed parameterizing dissipative perturbations with the help of the lag δ of the tides in the Earth as a whole, and the lag δ_c of those in its fluid core. The equations generalize the SOS model which is the basis of Nutation IAU 2000. Preliminary estimates of δ_c from analysis of the VLBI based obliquity rate and out-phase nutational amplitudes are presented.

1. Nutation IAU 2000 and Its Deficiencies

Nutation IAU 2000, recently adopted as an international standard, has significantly improved fitting to positions of the Celestial Pole obtained by the VLBI technique. However this theory is semi-empirical when modelling the effects of dissipation of energy in the Earth's rotation and that is why it is hard, if possible, to derive physically meaningful conclusions concerning the Earth's interior with such a theory. The dynamical model behind it is based on the work by Sasao, Okubo, and Saito, developed for the case of the Earth with the elastic mantle and the ideal fluid of its core (so called SOS model; see [4]). When applying this model to construct the new nutation theory, the dissipative effects of the Earth's rotation are usually treated in a formal way assuming that some of constants of the SOS model (for instance Love numbers involved) have imaginary parts and estimating them from the observed positions of the Celestial Pole (see, for instance, Shirai & Fukushima, 2001, [6]). Such semi-empirical approach is equivalent to incorporation of empirical terms into the differential equations of the SOS model. In fact at least five empirical terms must be incorporated to reach a good fit to observations.

The second defficiency of the SOS model (and thus of Nutation IAU 2000) is that only a part of perturbations caused by the non-rigidity of the Earth is accounted for, namely those from the tidal variations of the matrices of inertia of the mantle and the core, the rigid body approximation still being used to model the perturbing torques. In this approximation so called method of transfer function may be applied. However the omitted perturbing terms cannot be accounted for by any transfer function and thus perturbations from them are absent in Nutation IAU 2000.

2. Conventional and Revised SOS Model of the Earth's Rotation

Parameters of the SOS model are so called compliances κ, γ, β. The compliances κ, γ may be expressed in terms of the static k_2 and dynamic k_2^d Love numbers, relatively, while the compliance β is connected with the Love number k_2^c of the fluid core. The compliances (or the corresponding Love numbers) may be obtained either theoretically making use of constants of the adopted up-to-date models of the Earth's interior, or from analysis of VLBI data by fitting the rotation theory. Instead of compliances τ, γ, β we prefer to use the normalized Love numbers σ, ν, μ defined by the relations

$$\kappa = e\sigma,\ \gamma = e\frac{\nu}{\alpha},\ \beta = e\frac{\mu}{\alpha},$$

in which the constant α is the ratio of the main moment of inertia of the fluid core to that of the Earth as a whole, and e is the dynamical flattening.

If the "secular" Love number k_s is defined in the standard way by the expression

$$k_s = \frac{3Gm_E J_2}{R^3\omega^2} \approx 0.93831,$$

in which G, m_E, J_2, R, ω are the gravitational constant, the mass, the coefficient of the main zonal harmonics, the mean radius and the rotational rate of the Earth, then the parameters $\sigma,\ \nu,\ \mu$ may be presented in terms of the Love numbers k_2, k_2^d, k_2^c by the relations

$$\sigma = \frac{k_2}{k_s},\ \nu = \frac{k_2^d}{k_s},\ \mu = \frac{k_2^c}{k_s}.$$

In these notations the standard SOS equations of the Earth's rotation (see Moritz & Mueller, 1987, [4]) may be written in the form

$$\dot u(1+e\sigma) - ie\omega(1-\sigma)u + (\alpha + e\nu)(\dot v + i\omega v) = L + i\frac{\sigma}{\omega}\frac{\partial L}{\partial t},$$
$$\dot u + \dot v + iv\omega\left(1 + e_c - \mu\frac{e}{\alpha}\right) = \frac{\nu}{\alpha}\left[L\left(1-e\right) - \frac{i}{\omega}\frac{\partial L}{\partial t}\right],$$

where the term L at the right parts is the normalized rigid body torque, $u = \omega_1 + i\omega_2, v = v_1 + iv_2$ are the complex combinations of the components of the vectors of the angular velocities $\overline{\omega} = (\omega_1, \omega_2, \omega_3)$ of the mantle, and $\overline{v} = (v_1, v_2, 0)$ of the differential rotation of the core.

The normalized rigid body torque L is given by the expression

$$L = -ip\omega\xi\zeta,$$

where p is the parameter of the lunar or solar precession

$$p = \frac{3}{2}\frac{mG}{r^3}e,$$

$\xi = \rho_1 + i\rho_2,\ \zeta = \rho_3$ are the ecliptical coordinates of the tide arousing body, and e is the dynamical flattening. (In fact the rigid body torque is the sum of the lunar and solar components: $L = L^1 + L^2$ where $L^k = -ip_k\xi^k\zeta^k$, $p = p_1 + p_2$, and p_1, p_2 are parameters of the lunar and solar precession.)

In the more rigorous formulation the SOS equations have to be replaced by the following ones

$$\dot u\left(1 + \frac{2}{3}e\sigma\right) - ie\omega(1-\sigma)u + \left(\alpha + \frac{2}{3}e\nu\right)(\dot v + i\omega v) + iv\sum_{k=1,2}(1-3\zeta_k^2)p_k =$$
$$= L + (\delta + i)\frac{\sigma}{\omega}\frac{\partial L}{\partial t} + L^d + L_c^d,$$
$$\dot u + \dot v + iv\omega\left[1 + e_c - \frac{\mu e}{\alpha}(1 + i\delta_c)\right] =$$
$$= \frac{\nu}{\alpha}\left[L\left(1 - \frac{2}{3}e\right) - \frac{i}{\omega}\frac{\partial L}{\partial t}\right] + i\delta\frac{\nu}{\alpha}\left[L(1-e) + i\left(\frac{2}{\omega}\right)\frac{\partial L}{\partial t}\right] = 0,$$

in which the normalized dissipative torque L^d consists of the lunar L_1^d and solar L_2^d components caused by the dissipation in the lunar and solar tides, and of the cross interaction torque $L_{1,2}^d$ of these tides

$$\begin{aligned} L^d &= L_1^d + L_2^d + L_{1,2}^d, \\ L_k^d &= -4p_k\epsilon_k\sigma\delta\left[\omega\xi_k\zeta_k + i\left(\zeta_k\frac{\partial}{\partial t}\xi_k - \xi_k\frac{\partial}{\partial t}\zeta_k\right)\right]\ (k=1,2), \\ L_{1,2}^d &= 2\sigma\delta\omega(p_1\epsilon_2 + p_2\epsilon_1)(\xi_2\zeta_1 + \xi_1\zeta_2), \end{aligned}$$

while L_c^d includes the terms due to the dissipation in the fluid core

$$L_c^d = \nu\delta_c\left[\frac{1}{2}pv(3\cos^2\theta - 2\cos\theta - 1) + ie\left(1 - \frac{\nu}{\alpha}\right)L\right],$$

θ being the mean obliquity (see Krasinsky, 2003, [3]; a minor error in L_c^d is here corrected).

These equations explicitly depend on the two dissipative parameters δ and δ_c. The parameter δ is the well-known tidal lag of the Earth as a whole that strongly affects the orbital motion of the Moon and is responsible for the evolution of the Earth-Moon system. The parameter δ_c is the phase lag of the tides caused by the differential rotation of the fluid core and as we show further, it plays important part in the Earth's rotation.

Setting the tidal lags δ, δ_c equal to zero one could expect that the standard and revised systems of the differential equations become equivalent. However there is no full equivalence: in the revised version of the equation for the variable u the factor $1 + 2e\sigma/3$ stands in place of the factor $1 + e\sigma$ in the standard SOS system. The origin of this small but not negligible discrepancy has been traced as arising due to the incomplete form of the centrifugal tidal potential commonly used to account for effects of elasticity (only the tesseral components of this potential have been taken into consideration).

3. Contribution from Viscosity of the Fluid Core to the Obliquity Rate

From the geophysical point of view it seems interesting to interpret the observed value $\dot{\theta}_{obs} = -24.08 \pm 0.017$ mas/cy of the obliquity rate based on VLBI data (see Shirai & Fukushima, 2001, [6]). The main part of this effect may be explained in the frame of the rigid body model (Williams, 1994, [7]). In fact it is not a secular but long periodic term broken to the time series. After Williams this rigid body obliquity rate is equal to -26.8 mas/cy which value may be compared with results of the more recent rigid body models of nutation: -26.5 in SMART97 (Bretagnon et all. 1998, [1]) and -27.2 in RDAN97 (Roosbeck & Dehant, 1998, [5]). Removing the rigid body effect the remaining discrepancy of the obliquity rate $\dot{\theta}_{obs}$ with observations due to dissipative effects is as follows:

$$\dot{\theta}_{obs} = 2.7 \div 3.1 \text{ mas/cy},$$

depending on the applied rigid body model.

From the revised version of the SOS equations one can derive the simple analytical expressions for the two components of the obliquity rate: $\dot{\theta}_\delta$ induced by dissipation of the Earth as a whole and $\dot{\theta}_{\delta_c}$ due to the dissipation caused by the differential rotation of the fluid core:

$$
\begin{aligned}
\dot{\theta}_\delta &= 2p\sigma\delta\sin\theta(\epsilon\cos\theta - 2\tilde{\epsilon}),\\
\dot{\theta}_{\delta_c} &= p\nu e\left(1-\frac{\nu}{\alpha}\right)\delta_c\sin\theta\cos\theta,
\end{aligned}
$$

where $\epsilon = (p_1\epsilon_1 + p_2\epsilon_2)/p = 2.04\times10^{-5}, \tilde{\epsilon} = (p_1 n_1\epsilon_1 + p_2 n_2\epsilon_2)/\omega p = 6.27\times10^{-7}$.

The component $\dot{\theta}_\delta$ of the obliquity rate may be reliably evaluated making use of the estimate $\delta = 0.0376$ based on LLR data, that gives the positive rates 0.675 mas/cy and 0.153 mas/cy for perturbations from the Moon and Sun, relatively, with the total value 0.928 mas/cy. Then the remaining part of the observed obliquity rate must be attributed to the effect of the fluid core:

$$\dot{\theta}_{\delta_c} = 1.8 \div 2.2 \text{ mas/cy}.$$

Applying the given above theoretical expression for $\dot{\theta}_{\delta_c}$ we obtain the estimate of δ_c:

$$\delta_c \approx 0.01,$$

that may be interpreted as an evidence of quite unsignificant viscosity of the fluid core.

4. Amplitudes of Out-Phase Nutations and FCN Tidal Damping

From the revised SOS equations it is easy to derive that the out-phase amplitudes of the main nutations (18.6 and half year periods) with the sufficient accuracy may be written in the form:

$$
\begin{aligned}
d\theta^f &= -R_f^{out}\sin\theta_0 d\phi_0^f,\\
\sin\theta d\phi^f &= R_f^{out} d\theta_0^f,
\end{aligned}
$$

where

$$
\begin{aligned}
R_f^{out} &= \left[-\nu\delta + \sigma_f(\alpha-\nu)\delta_c\right]\frac{f}{f+f_c}(1-\alpha)^{-1},\\
f_c &= \omega e_c(1-\sigma_f)(1-\alpha)^{-1}
\end{aligned}
$$

f^c being the frequency of Free Core Nutation (FCN), and $\sigma_f = \beta/e_c = e\mu/\alpha e_c \approx 0.239$ is its correction for tides in the fluid core.

In Table 1 the observed in-phase and out-phase amplitudes (in mas) are reproduced from the work (Shirai & Fukushima, 2001, [6]) for the two main nutations. In this table there are also given the corresponding estimates of δ_c, obtained with the help of the given above analytical expression for the coefficient R_f^{out}. Note that the amplitudes presented in the table are not really observed quantities but the theory-dependent ones because they are obtained by the mentioned above formal method estimating the imaginary parts of the coefficients of the transfer function as solve-for parameters from which the "observed" out-phase amplitudes have been derived.

Table 1. Observed main nutations and estimates of δ_c

Period	$d\phi_{in}$	$d\phi_{out}$	δ_c	$d\theta_{in}$	$d\theta_{out}$	δ_c
6798.38	17206	3.341	0.38	9205	-1.506	0.47
182.62	-1317	-1.717	0.46	579	-0.570	0.44

One can see that the four independent estimations of δ_c presented in Table 1 are in a good accordance. The rather large values of δ_c are marginally out the boundary of the physically meaningful range $\delta_c < \delta/\alpha \approx 0.35$ (obtained assuming that the dissipation takes place only in the fluid core) and are inconsistent with the small values δ_c derived above from the obliquity rate. It is noteworthy that the resulting out-phase amplitudes are small differences between the large contributions caused by the phase lags δ and δ_c. So the calculations are very sensitive to the numerical values of the parametres involved. Modelling the obliquity rate is sensitive to the ratio ν/α and probably uncertainty of the parameters is one of the reasons of this inconsistency. From the other hand in the work (Dehant & Defraigne, 1997, [2]) it is shown that the indirect action of the oceanic tides through the fluid core contributes half of the observed out-phase amplitudes of both the 18.6 year and semi-annual nutations. Thus we may suppose that the estimated δ_c is an effective parameter that includes both these effects (viscosity of the fluid core and the ocean tides). Probably in this way one could explain the inconsistency of δ_c in Table 1 with the value derived from the observed obliquity rate which is not affected by the ocean tides.

The developed theory shows that the parameter $\delta_f = \sigma_f \delta_c$ in the expression for the out-phase amplitudes has meaning of the parameter of FCN damping. For the corresponding quality factor Q_{FCN} we obtain

$$Q_{FCN} = \frac{1}{2\delta_f} \approx 20 \div 200.$$

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