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A New Free Core Nutation Model with Variable Amplitude and Period

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Abstract

Three most long and dense VLBI nutation series obtained at the Goddard Space Flight Center, Institute of Applied Astronomy, and U.S. Naval Observatory were used for investigation of the Free Core Nutation (FCN) contribution to the celestial pole offset. Some recent studies have showed that the FCN period or/and phase does not remain constant, but varies in a rather wide range of about 410–490 days (for equivalent period). To implement this result in practice, a new FCN model with variable amplitude and period (phase) is developed. Comparison of this model with observations shows better agreement than existing one. After correction of the differences between observed VLBI nutation series and the IAU2000A model, they decreased to a level about 100 microarcseconds.

1. Introduction

Free Core Nutation (FCN) of the Earth is predicted more than a century ago as a common rotational mode of a body having an ellipsoidal solid shell and fluid core. Investigation of the FCN is an important scientific task. First, the FCN parameters determined from observations provide valuable, sometimes unique, information about processes in the Earth's interior. From the practical point of view, accurate modelling of the FCN term, including prediction, is necessary to compute celestial pole offset with accuracy compatible with modern VLBI observations.

The IAU2000A model based on the MHB2000 model developed in Mathews, *et al.*, (2002) and adopted as a new IAU standard can predict a regular part of the nutation with accuracy of about 100 μas. However, the FCN contribution is much larger, up to 400 μas, which yields degradation of accuracy in modelling celestial pole offset, if FCN not accounted for. It is well known also that the FCN contribution gives the prevailing contribution to the power spectrum of the differences between observed nutation series and modern models.

The IAU2000A model, like the previous ones, is constructed as a Poisson series, and does not include free mode terms, such as the FCN, which cannot be presented as a Poisson series term with predictable parameters. For this reason, FCN parameters are to be determined from the VLBI observations.

Historically, the FCN frequency is considered as a constant fundamental value included in transfer function expression describing the relationship between the amplitudes of nutation terms for real and rigid Earth. Many authors have made an effort to estimate the FCN period and possible reasons for its excitation (see *e.g.* Mathews and Shapiro, 1995; Brzeziński and Petrov, 1998; Shirai and Fukushima, 2001a; Herring *et al.*, 2002; Malkin and Terentev, 2003a, 2003b). They found the FCN period to be in the range of 425–435 solar days, with average value about 430^d.

Recently, it was found from a wavelet analysis of VLBI nutation series that the FCN period likely varies in a range of about 410–490 days (Malkin and Terentev, 2003a, 2003b). This result is also confirmed by means of another method, Short-time Periodogram with Gabor Function, proposed by T. Shirai (Shirai *et al.*, 2004). Of course, found variability of the FCN period may be fully or partially a transformation of the variations of the FCN phase, which maybe has more geophysical meaning. However, geophysical considerations of the FCN variability lie beyond this study.

In this paper we develop a practical model for computation of the FCN contribution to the celestial pole offset taken into account variability both amplitude and period (phase) of the FCN oscillation.

2. Computation of the FCN Parameters

Three most long and dense VLBI nutation series obtained at the Goddard Space Flight Center (GSF), Institute of Applied Astronomy (IAA), and U.S. Naval Observatory (USN), each containing more than 3000 estimates of the nutation angles for the period from 1979 up to now were used for investigation of the FCN parameters.

Firstly, estimates of $d\psi$ and $d\varepsilon$ w.r.t. the IAU1976/1980 nutation model computed at the GSF and USN were transformed to the dX_c and dY_c w.r.t. IAU2000A model (IAA series already contains this data).

Then combined series was computed. Since preliminary analysis showed that the three input series are of very similar quality, no weighting was applied, however formal errors reported in the input series were scaled for uniformity. After that, input series were averaged, saving only the epochs present in all the series. Band-pass Gaussian filtering was applied to the combined series. Transfer function of the filter at the FCN frequency is 0.988.

At this step we also computed the smoothed values at equally sampled epochs with 20-day step. However, a smoothed series given at the original epochs also can be used for analysis with a similar final result (Malkin and Terentev, 2003a, 2003b).

The FCN amplitude time series was computed using the simple formula

$$A(t) = \sqrt{(dX_c(t))^2 + (dY_c(t))^2}\,, \tag{1}$$

Indeed, using such an approach we suppose that all the differences can be attributed to the FCN, and this seems to be a good approximation to reality. However, a resonance impact on the nutation terms at the frequencies close to the FCN evidently should be accounted for in future developments.

Finally, the FCN period variation was computed using a wavelet analysis technique as described in Malkin and Terentev (2003a, 2003b). For this analysis we used program WWZ developed by the American Association of Variable Star Observers.

3. Computation of the FCN Contribution

Let us consider how a model with variable amplitude and period (phase) can be used in practice. One can describe the FCN term as

$$\begin{aligned} dX_c &= A(t)\sin(\Phi(t))\,, \\ dY_c &= A(t)\cos(\Phi(t))\,. \end{aligned} \tag{2}$$

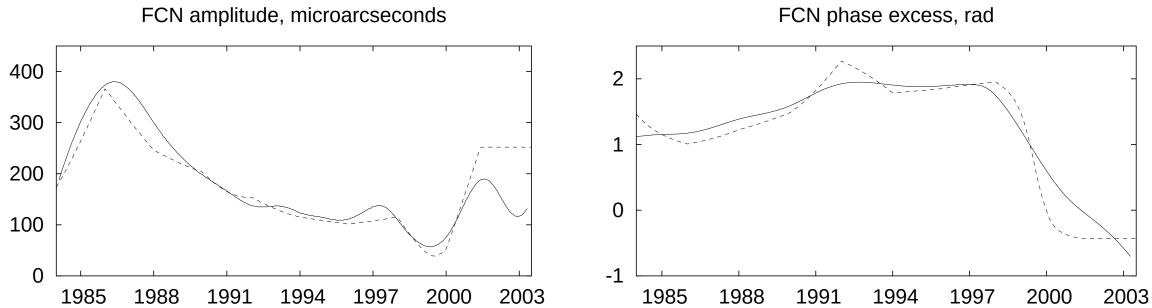

Figure 1. The FCN amplitude and phase (after removing linear trend corresponding to the constant FCN period 430.2^d) variations found in this study (solid line), and the MHB2000 model (dashed line).

Mathematically (not geophysically, indeed!), we can suppose three equivalent models for the FCN phase $\Phi(t)$

$$\Phi(t) = \begin{cases} \dfrac{2\pi}{P(t)}\, t + \Phi_0 \,, \\[2ex] \dfrac{2\pi}{P_0}\, t + \Phi(t) \,, \\[2ex] \dfrac{2\pi}{P(t)}\, t + \Phi(t) \,, \end{cases} \tag{3}$$

where P is the FCN period, and zero subscripts mean constant values. In other words, we can consider three models: with variable period and constant phase, variable phase and constant period, or variable both period and phase. Of course, this is a subject of geophysical consideration, but does not matter for an empirical FCN model using time variations of the FCN parameters found from analysis of the observed data.

In practice, one can compute $\Phi(t)$ as

$$\Phi(t) = \int\limits_{t_0}^{t} \frac{2\pi}{P(t)}\, dt + \varphi_0 \,, \tag{4}$$

where φ_0 is the parameter to be adjusted.

Variations of the FCN amplitude $P(t)$ and phase $\Phi(t)$ are shown in Figure 1 along with the corresponding FCN parameters included in the MHB2000 model which is, in fact, also a model with variable phase and amplitude, though this is not stated explicitly (we used the text of the FCN_NUT routine included in the MHB_2000 code to extract the FCN(MHB) amplitude and phase variations). One can see that both models show similar behavior of the FCN parameters, however new approach allow us to get continues, non-inflecting and predictable functions $A(t)$ and $\Phi(t)$. Comparing these two models one should keep in mind that MHB2000 model is developed only for the period till 2001.4, and after this epoch the difference between the models grows rapidly.

Figure 2 shows spectra of the differences between observed nutation series and the IAU2000A model computed for raw differences and after removing the FCN contribution. One can see that the FCN signal is completely eliminated.

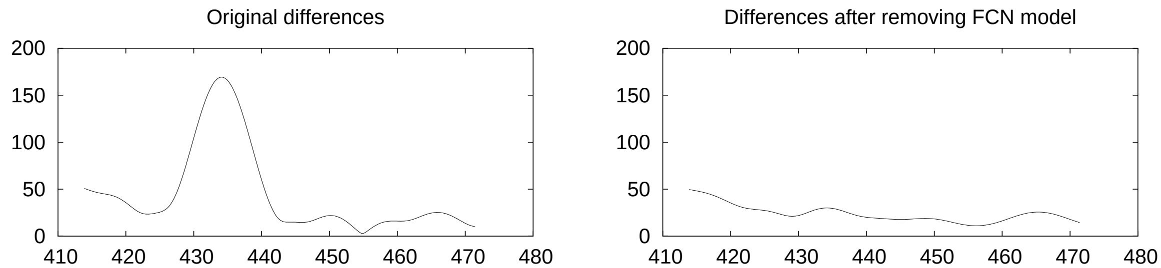

Figure 2. Spectrum of the differences between observed nutation series and the IAU2000A model, period in days, amplitude in μas.

Table 1. WRMS of differences with two FCN models, μas (No – no FCN model, MHB – MHB2000 FCN model, New – model proposed in this paper applied; NEOS – NEOS-A VLBI sessions observed in 1993–2001, R1R4 – IVS R1 and R4 VLBI sessions only observed since 2002).

Series	All sessions			NEOS			R1R4		
	FCN model			FCN model			FCN model		
	No	MHB	New	No	MHB	New	No	MHB	New
GSF	166	146	138	138	122	120	134	150	102
IAA	170	152	144	140	123	123	138	154	111
USN	161	144	136	138	122	122	136	156	107
Mean	156	136	126	131	113	112	129	146	97

However, the differences between observed nutation series and model have a noise of various origins with the rms compatible with the FCN contribution. To estimate the actual contribution of the FCN model to this noise we have computed rms of differences between the observations and the IAU2000A model after applying three different FCN models: no FCN (raw differences), extracting the MHB2000 FCN term, and extracting the FCN term according to the model described here. The results are shown in Table 1.

One can see that accounting for the FCN contribution leads to decreasing of the differences. Especially interesting is the last part of the table corresponding to the period of observations 2002–2003. Using the MHB2000 FCN model for this period leads to degradation of differences between observations and the IAU2000A model, which is natural for this model is developed only for the epochs until 2001.4.

A FCN model with variable period and phase allow us to try a new approach to FCN prediction. One can consider two possibilities. The first one is a prediction of actual FCN contribution, which is developed *e.g.* in Brzeziński and Kosek (2004). Another possibility is to predict functions $A(t)$ and $\Phi(t)$ separately, and then use predictions to construct the FCN contribution using the formulas given above.

4. Conclusions

We have developed a new FCN model with variable amplitude and period (phase) which provides a computation of a continuous FCN contribution to the celestial pole offset with good accuracy for whole interval of the VLBI observations, and convenient prediction of the FCN contribution. Using this model allows us to reduce the differences between VLBI observations and model to the level 100μas.

It is clear that the proposed model is a purely empirical one. Considerable efforts should be made to understand the physical origin of the variability of the FCN period and/or phase, and its consequences in a theory of nutation.

The proposed model is routinely used in the VLBI processing at the Institute of Applied Astronomy since September 2003.

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