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Considering a Priori Correlations in the IVS Combined EOP Series

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Abstract

The current combination of EOP time series actually implies a violation of the basic rule that the same data cannot be used twice in an adjustment process. This fact is presently neglected by treating the input data of the IVS Analysis Centers as "new" or independent data. However, this deficiency can be mitigated by introducing proper correlation coefficients between the Analysis Centers. Different approaches for deriving such correlations as well as results are presented in this paper.

1. Introduction

The generation of the IVS combined EOP series actually implies a violation of the basic rule that an observation can only be used once in an adjustment process. However, due to different modelling of the observations or different data editing and outlier elimination procedures utilized by the Analysis Centers it can be assumed that the resulting EOP series can be treated as (approximately) independent data[1]. Nevertheless, it is possible to process estimates from the same set of observations in a rigorous way by using the correlations between the estimates. In the case of the IVS combined series two types of correlations have to be considered:

- correlations between EOP components ρ_{x_p,y_p}, $\rho_{x_p,dUT1}$, $\rho_{y_p,dUT1}$ and $\rho_{d\psi,d\varepsilon}$, reported by the respective Analysis Center with
$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$$
- correlations between the Analysis Centers

Both types of correlations are necessary to form a rigorous combination. As an example eq. 1 shows the covariance matrix for two Analysis Centers and two components as it should be used in the combination.

$$\mathbf{\Sigma_{combi}} = \begin{vmatrix} \sigma^2_{x_1} & \sigma_{12} & \sigma_{x_1 y_1} & \sigma_{x_1 y_2} \\ & \sigma^2_{x_2} & \sigma_{x_2 y_1} & \sigma_{x_2 y_2} \\ & & \sigma^2_{y_1} & \sigma_{12} \\ & & & \sigma^2_{y_2} \end{vmatrix} \tag{1}$$

The covariance matrix contains three types of covariances:

σ_{12} – covariances between different Analysis Centers

$\sigma_{x_1 y_1}$ – covariances between EOP components of the same Analysis Center

$\sigma_{x_1 y_2}$ – "mixed" covariances (between Analysis Centers and EOP components)

[1] The IGS uses this approach (Kouba, pers. communication).

In this paper we develop the correlations between the Analysis Centers. For clarity reasons the correlations between the components ρ_{x_p,y_p}, $\rho_{x_p,dUT1}$, $\rho_{y_p,dUT1}$ and $\rho_{d\psi,d\varepsilon}$ and the "mixed" covariances are neglected here.

EOP series from five IVS Analysis Centers are currently being combined to form the official IVS product (two OCCAM solutions from AUS – Geoscience Australia and IAA – Institute of Applied Astronomy, St. Petersburg and three Calc/Solve solutions from BKG – Bundesamt für Kartographie und Geodäsie, GSF – Goddard Space Flight Center, and USN – U.S. Naval Observatory). Embarking on a rigorous approach we first develop a correlation matrix purely on the basis of a thought experiment taking into account the general facts of the different solutions (table 1). Small correlations are entered between Analysis Centers using different software packages, high correlations between the same software package and even a little higher between GSF and USN, using the same software and the same procedures.

Table 1. A priori correlation matrix from a thought experiment

	AUS	BKG	GSF	IAA	USN
AUS	–	0.2	0.2	0.7	0.2
BKG		–	0.7	0.2	0.7
GSF			–	0.2	0.8
IAA				–	0.2
USN					–

2. Covariances and Correlations

This section summarizes briefly the formulas for the derivation of a theoretical covariance matrix. These formulas were mainly taken from [4]. The basis is a two-dimensional random vector as a realization of a two-dimensional random variable. The two observation vectors $\mathbf{l}_1$ and $\mathbf{l}_2$ contain the EOP time series determined by two different Analysis Centers. The observations can be grouped pairwise according to the epoch of observation.

$$\mathbf{L} = \begin{vmatrix} L_1 \\ L_2 \end{vmatrix}; \quad \mathbf{l}^T = \begin{vmatrix} \mathbf{l}_1^T \\ \mathbf{l}_2^T \end{vmatrix} = \begin{vmatrix} l_{11} & l_{12} & \cdots & l_{1n} \\ l_{21} & l_{22} & \cdots & l_{2n} \end{vmatrix} = \begin{vmatrix} \begin{pmatrix} l_{11} \\ l_{12} \end{pmatrix} \begin{pmatrix} l_{21} \\ l_{22} \end{pmatrix} \cdots \begin{pmatrix} l_{n1} \\ l_{n2} \end{pmatrix} \end{vmatrix} \tag{2}$$

Eq. 3 gives the definition of the expectation values of the random variables.

$$E(\mathbf{L}) = \begin{vmatrix} E(L_1) \\ E(L_2) \end{vmatrix} = \begin{vmatrix} \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} l_{1j}; & \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} l_{2j} \end{vmatrix}^T = \begin{vmatrix} \mu_1 \\ \mu_2 \end{vmatrix} = \boldsymbol{\mu} \tag{3}$$

In the next step true errors can be calculated according to eq. 4.

$$\boldsymbol{\varepsilon}^T = \begin{vmatrix} \boldsymbol{\varepsilon}_1^T \\ \boldsymbol{\varepsilon}_2^T \end{vmatrix} = \begin{vmatrix} l_{11}-\mu_1 & l_{12}-\mu_1 & \cdots & l_{1n}-\mu_1 \\ l_{21}-\mu_2 & l_{22}-\mu_2 & \cdots & l_{2n}-\mu_2 \end{vmatrix} = \boldsymbol{l}^T - \boldsymbol{\mu}\boldsymbol{e}^T \tag{4}$$

The covariance matrix follows according to

$$\boldsymbol{\Sigma}_{\mathbf{ll}} = E\left\{ \begin{vmatrix} \boldsymbol{\varepsilon}_1^T \\ \boldsymbol{\varepsilon}_2^T \end{vmatrix} \begin{vmatrix} \boldsymbol{\varepsilon}_1^T & \boldsymbol{\varepsilon}_2^T \end{vmatrix} \right\} = E\left\{ \begin{vmatrix} \boldsymbol{\varepsilon}_1^T\boldsymbol{\varepsilon}_1 & \boldsymbol{\varepsilon}_1^T\boldsymbol{\varepsilon}_2 \\ \boldsymbol{\varepsilon}_2^T\boldsymbol{\varepsilon}_1 & \boldsymbol{\varepsilon}_2^T\boldsymbol{\varepsilon}_2 \end{vmatrix} \right\} = \begin{vmatrix} \sigma_{01}^2 & \sigma_{12} \\ \sigma_{21} & \sigma_{02}^2 \end{vmatrix} \tag{5}$$

with $E\left(\boldsymbol{\varepsilon}_1^T\boldsymbol{\varepsilon}_1\right)=\sigma_{01}^2$, $E\left(\boldsymbol{\varepsilon}_2^T\boldsymbol{\varepsilon}_2\right)=\sigma_{02}^2$ and $E\left(\boldsymbol{\varepsilon}_1^T\boldsymbol{\varepsilon}_2\right)=E\left(\boldsymbol{\varepsilon}_2^T\boldsymbol{\varepsilon}_1\right)=\sigma_{12}=\sigma_{21}$.
In the last step the correlation matrix can be deducted by simple matrix algebra (cf. [3])

$$\mathbf{R}=\begin{vmatrix} 1 & \rho_{12} \\ \rho_{21} & 1 \end{vmatrix} \tag{6}$$

with $\mathbf{R}=\mathbf{F}\,\mathbf{\Sigma_{ll}}\,\mathbf{F}$ and $\mathbf{F}=diag\,(1/\sigma_{01},1/\sigma_{02})$.

3. Determination of Correlation Coefficients

The target of this investigation is to determine empirical correlation coefficients between the Analysis Centers. Two cases have to be distinguished (cf. [4]):

1. True values are unknown. In this case mean values have to be calculated in a first step. The mean values are then used to calculate a vector of corrections $\mathbf{v}$.

$$\bar{\mathbf{x}}=\begin{vmatrix} \bar{x}_1 \\ \\ \bar{x}_2 \end{vmatrix}=\frac{1}{n}\begin{vmatrix} \sum_{i=1}^{n} l_{1i} & \sum_{i=1}^{n} l_{2i} \end{vmatrix}^T;\ \mathbf{v}^T=\begin{vmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{vmatrix}=\begin{vmatrix} \bar{x}_1-l_{11} & \bar{x}_1-l_{12} & \cdots & \bar{x}_1-l_{1n} \\ \bar{x}_2-l_{21} & \bar{x}_2-l_{22} & \cdots & \bar{x}_1-l_{2n} \end{vmatrix} \tag{7}$$

The empirical covariance matrix can be obtained according to eq. 8.

$$\bar{\mathbf{\Sigma}}_{\mathbf{ll}}=\frac{1}{n-1}\,\mathbf{v}^T\mathbf{v}=\frac{1}{n-1}\begin{vmatrix} \boldsymbol{v}_1^T\boldsymbol{v}_1 & \boldsymbol{v}_1^T\boldsymbol{v}_2 \\ \\ \boldsymbol{v}_2^T\boldsymbol{v}_1 & \boldsymbol{v}_2^T\boldsymbol{v}_2 \end{vmatrix}=\begin{vmatrix} s_1^2 & s_{12} \\ \\ s_{21} & s_2^2 \end{vmatrix} \tag{8}$$

2. True values $\tilde{L}_i$ or expectation values μ_i are known. In this case the empirical covariance matrix follows directly:

$$\bar{\mathbf{\Sigma}}_{\mathbf{ll}}=\begin{vmatrix} \frac{1}{n}\boldsymbol{\varepsilon}_1^T\boldsymbol{\varepsilon}_1 & \frac{1}{n}\boldsymbol{\varepsilon}_1^T\boldsymbol{\varepsilon}_2 \\ \\ \frac{1}{n}\boldsymbol{\varepsilon}_2^T\boldsymbol{\varepsilon}_1 & \frac{1}{n}\boldsymbol{\varepsilon}_2^T\boldsymbol{\varepsilon}_2 \end{vmatrix}=\begin{vmatrix} s_1^2 & s_{12} \\ \\ s_{21} & s_2^2 \end{vmatrix} \tag{9}$$

In both cases the result is the empirical correlation coefficient:

$$\boxed{\rho=\frac{s_{12}}{s_1\,s_2}}$$

4. Significance Tests

Empirical correlation coefficients r are normally distributed:

$$r\sim N\left(\rho,\frac{1-\rho^2}{\sqrt{n-1}}\right) \tag{10}$$

for $n\longrightarrow\infty$. In case of small samples a transformation is needed (eq. 11) This transformation permits the use of the normal distribution even for small samples (e.g. [2]).

$$z=\frac{1}{2}\log\frac{1+r}{1-r},\ z\sim N\left(\zeta,\sigma_z\right)\quad\text{and}\quad\frac{z-\zeta}{\sigma_z}\sim N\left(0,1\right) \tag{11}$$

with

$$\zeta = \frac{1}{2}\log\frac{1+\rho}{1-\rho} + \frac{\rho}{2\,(n-1)} \tag{12}$$

and

$$\sigma_z = \frac{1}{\sqrt{n-3}} \tag{13}$$

In addition it is possible to give confidence regions for the calculated correlation coefficients. Another possibility to express the significance test for correlation coefficients using the t-distribution is given in [5].

5. Results

To investigate the correlation coefficients a subset of sessions common to all Analysis Centers series currently incorporated in the combination was chosen (NEOS-A, R1 and R4). The number of 634 sessions was sufficient to calculate significant correlation coefficients. As already pointed out in section 3 two cases have to be distinguished:

1. True values are unknown. This test failed because this method cannot remove the deterministic signal properly. It turned out that all correlation coefficients are 1, hence the resulting covariance matrix is singular.
2. "True" values can be simulated by using a series of higher accuracy level or an independent series ([1]). A suitable series to remove the deterministic signal from the input EOP series of the Analysis Centers is the IERS C04 series.

   Fig. 1 shows a comparison of the "thought experiment" matrix and the correlation matrix for $d\psi$ after removal of the deterministic signal from the EOP series subtracting the IERS C04 series.

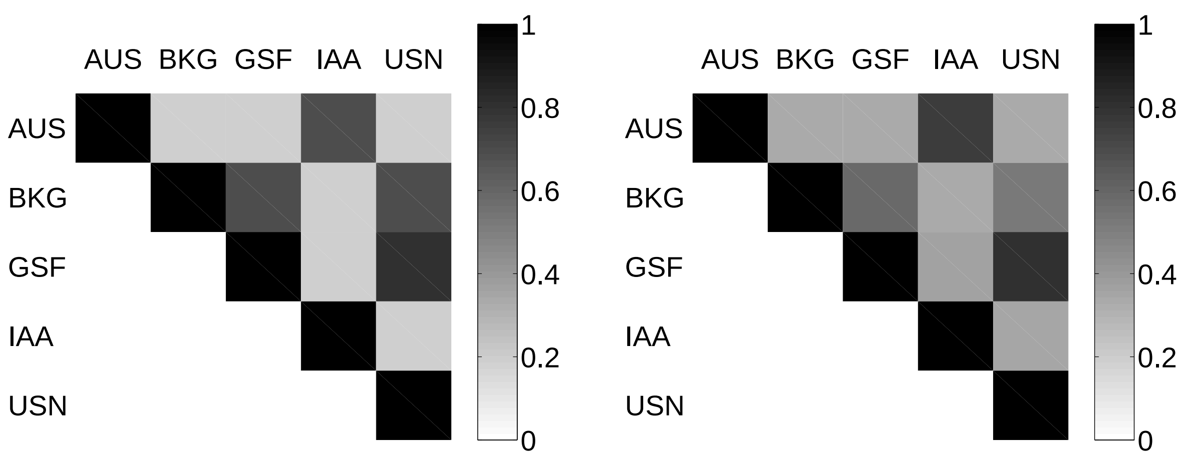

Figure 1. Correlation matrix from thought experiment (cf. tab. 1) (left) and correlation matrix for parameter $d\psi$ after removing the deterministic signals subtracting IERS C04.

To test this approach a combined series was calculated using the correlation coefficients shown on the right hand side of fig. 1. Figure 2 shows a comparison of the uncorrelated and the correlated approach for x_p. The differences are mainly within the range of ± 100 μas.

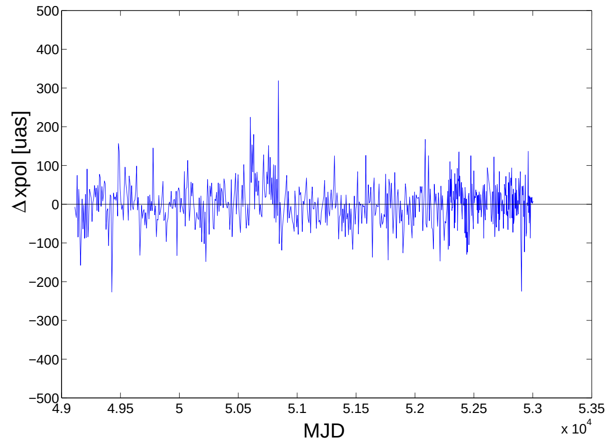

Figure 2. Differences between EOP combinations using a covariance matrix which contains covariances between the Analysis Centers and a diagonal covariance matrix respectively for parameter x_p.

6. Conclusions and Outlook

The use of correlation coefficients between Analysis Centers is another step forward to a rigorous combination of EOP time series. It has been shown that the correlation matrix computed using real data agrees surprisingly well with the one developed by purely taking into account known facts. As it had to be expected the average formal errors of the combined series increased by a factor of ~ 1.4 as compared to the combination where the results of the Analysis Centers are treated as uncorrelated. In the next step, the correlations between the individual parameters and between the Analysis Centers have to be consolidated to form a single covariance matrix containing both types of correlations. Only then the formal errors of the combined earth orientation parameters will become really meaningful.

References

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