Volker Tesmer et al.: An Advanced Stochastic Model for VLBI Observations and its Application to VLBI Data Analysis, IVS 2004 General Meeting Proceedings, p.296–300
http://ivscc.gsfc.nasa.gov/publications/gm2004/tesmer1

An Advanced Stochastic Model for VLBI Observations and its Application to VLBI Data Analysis

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Abstract

Further refinements of the functional representation of the geometric-physical properties of the VLBI observations mostly need big efforts and are not possible with any precision. Although the stochastic model is an important part of the VLBI observation equations, the stochastic properties of VLBI observations have not been studied in detail so far. The idea is to interpret discrepancies between the functional model and the observations as variances of the observations. In particular, the characterisation of station and elevation dependent influences is of limited precision. Remaining influences can be modelled by specific stochastic properties of the observations. This paper focusses on the application of a refined stochastic model of the observations to the estimation of VLBI parameters.

1. Refinement of the Traditional Stochastic Model of VLBI Observations

The traditional stochastic VLBI model $\mathbf{\Sigma}_{yy}$ consists of a factor σ_0^2 to describe the common variance level and the cofactor matrix $\mathbf{Q}$ of all observations. $\mathbf{Q}$ is composed of the variances σ_i^2 of the n observations (i=1,2,...,n), derived from the correlation process (approximated by the SNR, see [3]) and a constant σ_{const}^2 to consider quasi-random deficiencies in the functional model:

$$\mathbf{\Sigma}_{yy} = \sigma_0^2\mathbf{Q} = \sigma_0^2 \ (diag(\sigma_1^2, \sigma_2^2, ..., \sigma_n^2) + \sigma_{const}^2) \ . \tag{1}$$

Deficits of the traditional stochastic VLBI model are to be expected in station, elevation and source dependent parts of the observations' variances. Additionally, the value of the additive constant is usually only roughly known. VLBI observations are also correlated with each other (caused by the correlation process, due to deficits of the tropospheric modelling and deficits of the modelling of the station coordinates). This has been neglected so far. Until now, their values were not determined by means of rigorous methods (for details see [4] or [5]).

The refined stochastic model (eq. 2) is in contrast to the traditional stochastic model (eq. 1) more differentiating, because it allows us to describe more than just one stochastic quantity (σ_0^2 to describe the common variance level of all observations):

$$\mathbf{\Sigma}_{yy} = \theta_1\mathbf{V}_1 + \theta_2\mathbf{V}_2 + ... + \theta_k\mathbf{V}_k \ = \ \textstyle\sum_{m=1}^{k} \theta_m\mathbf{V}_m \ . \tag{2}$$

If, e.g., θ_1 shall describe the common variance level only for the observations i=1 and i=2, the corresponding matrix must simply be chosen to be $\mathbf{V}_1 = diag(\sigma_1^2, \sigma_2^2, 0, ..., 0)$. An additive constant for all observations (e.g. called θ_2) can be characterised with $\mathbf{V}_2$ being an identity matrix. θ_3 can represent a correlation coefficient of the observations i=1 and i=2 if $\mathbf{V}_3$ is an empty matrix except the elements $\mathbf{V}_3(1,2) = \mathbf{V}_3(2,1) = \sqrt{(\sigma_1^2 \cdot \sigma_2^2)}$.

2. Estimated Variance Components of the Refined of the Stochastic Model

The components of the type according to section 1 were determined by means of a Minimum Norm Quadratic Unbiased Estimation (MINQUE) as described in [1, p. 246f] or [2, p. 303f]. For all assumed deficits of the traditional stochastic model, corresponding components could be estimated. Concerning the correlations between observations it was found that in present solutions VLBI observations can be considered as almost uncorrelated (a largest value of 0.2 was detected due to the correlation process).

In contrast, some of the deficits of the variances of the traditional stochastic model (especially the station and elevation dependent parts) were found to be very significant and clear. Therefore the refined stochastic model was chosen to consist of the following parts (values and their formal errors see table 1, graphically displayed in figure 1):

$$\begin{aligned}\mathbf{\Sigma}_{yy} &= \theta_{var\ level}\mathbf{V}_{var\ level} \\ &+ \theta_{additive}\mathbf{V}_{additive} \\ &+ \textstyle\sum_{j=1}^{47}\theta_{station^A\ j}\mathbf{V}_{station^A\ j} + \sum_{j=1}^{47}\theta_{station^B\ j}\mathbf{V}_{station^B\ j} \\ &+ \textstyle\sum_{m=1}^{8}\theta_{elev^A\ m}\mathbf{V}_{elev^A\ m} + \sum_{m=1}^{8}\theta_{elev^B\ m}\mathbf{V}_{elev^B\ m} \quad .\end{aligned} \tag{3}$$

Table 1. 57 components for the refined stochastic model of VLBI observations.

type of θ	est.	value θ	type of θ	est.	value θ	type of θ	est.	value θ
var level	0.395	± 0.0011	PIETOWN	0.145	± 0.0093	FORTLEZA	0.145	± 0.0049
additive	0.206	± 0.0012	NRAO 140	0.281	± 0.0463	MK-VLBA	0.309	± 0.0132
WESTFORD	0.168	± 0.0034	DSS45	0.120	± 0.0168	OV-VLBA	0.193	± 0.0126
HRAS 085	0.459	± 0.0084	NRAO85 3	0.238	± 0.0058	CRIMEA	0.351	± 0.0140
MOJAVE12	0.190	± 0.0051	NOTO	0.209	± 0.0085	NYALES20	0.174	± 0.0046
RICHMOND	0.312	± 0.0056	HOBART26	0.270	± 0.0135	NRAO20	0.103	± 0.0047
WETTZELL	0.062	± 0.0032	KASHIM34	0.352	± 0.0222	YEBES	0.265	± 0.0150
ONSALA60	0.149	± 0.0048	MATERA	0.157	± 0.0052	URUMQI	0.023	± 0.0167
KASHIMA	0.321	± 0.0091	LA-VLBA	0.075	± 0.0064	TSUKUB32	0.081	± 0.0108
HATCREEK	0.432	± 0.0226	EFLSBERG	-0.079	± 0.0159	$\varepsilon = 5° - 8°$	1.213	± 0.0111
OVRO130	0.235	± 0.0252	FD-VLBA	-0.183	± 0.0051	$\varepsilon = 8° - 11°$	0.769	± 0.0074
HAYSTACK	0.350	± 0.0181	SANTIA12	0.304	± 0.0194	$\varepsilon = 11° - 15°$	0.570	± 0.0056
KAUAI	0.322	± 0.0067	KP-VLBA	0.399	± 0.0136	$\varepsilon = 15° - 20°$	0.396	± 0.0045
GILCREEK	0.072	± 0.0032	NL-VLBA	0.306	± 0.0112	$\varepsilon = 20° - 30°$	0.249	± 0.0031
KWAJAL26	0.338	± 0.0252	HN-VLBA	0.569	± 0.0187	$\varepsilon = 30° - 45°$	0.103	± 0.0026
ALGOPARK	0.067	± 0.0051	OHIGGINS	0.401	± 0.0432	$\varepsilon = 45° - 65°$	0.034	± 0.0024
HARTRAO	0.205	± 0.0084	BR-VLBA	-0.071	± 0.0073	$\varepsilon = 65° - 90°$	(0)	(± 0)
MEDICINA	0.076	± 0.0059	DSS15	0.023	± 0.0154			
SESHAN25	0.319	± 0.0140	KOKEE	0.160	± 0.0040			
DSS65	0.097	± 0.0090	SC-VLBA	0.948	± 0.0178			

In earlier investigations (e.g. [4] or [5]), some of the estimated variance components were doubtful or not representative, respectively. Most of them were not determined from a sufficiently large number of observations, like the components for most of the VLBA telescopes, YEBES and EFLSBERG, as well as the components for the observations below 5° elevation. Here, the observations of additional sessions were added to the analysis (now 2211 sessions instead of 2124) and the observations below 5° were strictly removed from the analysed data sets. As a consequence, all of the 57 components can be considered as stable and reliable estimates (see also section 4).

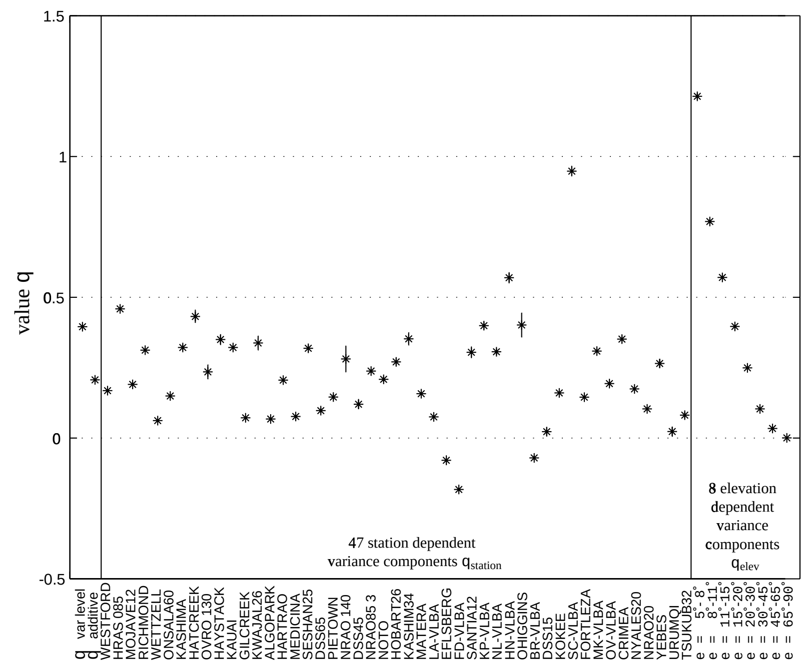

Figure 1. 57 components for the refined stochastic model of VLBI observations.

3. Indirect Influence of the Refined Stochastic Model on Parameter Estimations

When applying the refined stochastic model to parameter estimations, care has to be taken regarding indirect effects which are mainly connected with (for details see [4]):

- the weights and the respective impact of the pseudo observations for the constraints of auxiliary clock and tropospheric parameters,
- the power of outlier tests which compare observations' residuals with their formal errors,
- influence of observations under very low elevations, which can decisively affect the variances of the tropospheric parameters as well as their covariances with station positions, the EOP and the parameters of the station clocks.

The following readjustments yielded optimum efficiency of the refined stochastic model with respect to improved parameter estimations:

- change the weights of the constraints for:
  - 1 h rates of the PWLF for the tropospheric ZD from 15 mm/h to 10 mm/h,
  - 1 h rates of the PWLF for the clocks from 40 mm/h to 30 mm/h,
  - 24 h gradient offsets from 0.3 mm to 0.5 mm,
- change the criterion for outlier rejection from 3 σ to 3.5 σ,
- change the cut off angle from 8° to 5°.

4. Important Aspects Concerning the Applicability to VLBI Sessions

The estimated components were tested in several ways concerning their stability (w.r.t. time dependencies, dependencies due to network configurations, etc.). It could very clearly be shown, that they represent all VLBI data used at DGFI (more than 2000 sessions between 1984 and 2001).

As the traditional stochastic model of the observations is suspected to have deficiencies which are at least partially due to functionally not ascertainable differences between the functional model and the observations, it is assumed that the estimated components are dependent on the functional model used. However, investigations showed clearly that the results of a MINQUE are quite insensitive to slight changes in the parameterisation like fixing or estimating station positions. Significant changes could only be created artificially like e.g. by upweighting pseudo-observations in the solutions by the factor 100 (pseudo-observations are by default used to stabilise auxiliary troposphere and clock parameters by constraining them to the value zero with a certain empirically derived variance). This indicates clearly that the estimated components can be applied to all standard VLBI solutions.

Because of (partially large) dependencies between estimated components, it is not recommended to restrict the model to parts of the set of components. This means that always all six components (one for the variance level, one for the additive part, two for the telescopes and two for the elevations) have to be applied according to eq. 3. If not, the derived variance-covariance matrix $\mathbf{\Sigma}_{yy}$ might be distorted or even meaningless.

5. Application of the Refined Stochastic Model

One of the major motivations for the investigations concerning the stochastic model of the observations was to improve VLBI solutions. In the following sections 5.1 and 5.2, two different solution setups are described, which are mostly suitable for such tests.

5.1. Repeatability of Estimated Station Positions of the Refinements

For 2211 sessions between 1984 and 2001, station positions were computed session by session. The datum for each single session was NNR (no-net-rotation) and NNT (no-net-translation) w.r.t. a solution for station positions and velocities, computed from all these 2211 sessions in order to avoid systematic discrepancies which could distort the results. As all knowledge concerning the time dependent physical behaviour of the station positions was modelled a priori, it is assumed that the smaller (or less significant, respectively) the residual position estimates are, the better is the modelling of the corresponding observations.

Table 2. Repeatability of estimated station positions, determined from 2211 sessions.

	$\frac{\mathbf{RMS_{new}}}{\mathbf{RMS_{old}}}$	$\frac{\mathbf{WRMS_{new}}}{\mathbf{WRMS_{old}}}$
latitute	95.9 %	97.4 %
longitude	95.8 %	96.6 %
radial	96.8 %	99.9 %

The results indicate very clearly that the estimated parameters improve in the meaning as described before. In contrast to the preliminary results presented in [4], most of the results also became more realistic concerning their formal errors (except the radial components).

5.2. Similarity of EOP from Simultaneous NEOS-A and CORE-A Sessions

134 of the VLBI sessions stored at DGFI were carried out simultaneously by the two independently observing NEOS-A and CORE-A networks (start and end time of the respective schedules differed less than 15 minutes). Daily EOP were computed for each single session, fixing the TRF

to a solution for station positions and velocities (as mentioned in the last section) to avoid systematic differences due to inhomogenities of the TRF. For this test it is assumed that the better the modelling of the observations is, the smaller (or less significant, respectively) are the differences between the estimated corrections to the EOP determined from the two networks.

Table 3. Similarity of EOP from 67 pairs of simultaneous NEOS-A and CORE-A sessions.

	$\frac{\mathbf{RMS_{new}}}{\mathbf{RMS_{old}}}$	$\frac{\mathbf{WRMS_{new}}}{\mathbf{WRMS_{old}}}$
$\mathbf{X_P}$	99.2 %	98.6 %
$\mathbf{Y_P}$	88.0 %	87.2 %
$\mathbf{\Delta UT1}$	95.4 %	94.0 %
PSI	83.6 %	88.7 %
EPS	99.8 %	98.8 %

Similar to the results presented in the last section, the estimated parameters improve. Nevertheless, the refinements of the stochastic model do not influence all EOP the same way, which has not yet been fully understood. Maybe this is due to the small number of data points (67 only), whereby these estimates represent a certain network and/or observing geometry only.

6. Conclusions and Outlook

The presented results demonstrate very clearly that the largest deficiencies of the traditional stochastic model are found in its station and elevation dependent attributes. Another result was that in present solutions, VLBI observations can be considered as almost uncorrelated. Refinements of the stochastic model can be applied to almost all standard VLBI solutions, no matter if the primary target parameters are the EOP or station positions. Furthermore, it became very clear that standard VLBI solutions can be improved using the refined stochastic model. Even further improvements of this approach could be achieved by a more sophisticated description of stochastical properties of VLBI observations such as a function which models a station-wise elevation dependent weighting. But, one has to consider that further progress in the functional modelling of the VLBI observations (like, e.g., an improved description of the tropospheric influences) may affect the corresponding stochastic attributes significantly.

This research has made use of the IVS VLBI observation data. It was partially supported by the German research association “Deutsche Forschungsgemeinschaft (DFG)”, contract (DR 143/11-1).

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