John M. Gipson: Correlation Due to Station Dependent Noise in VLBI, IVS 2006 General Meeting Proceedings, p.286–290
http://ivscc.gsfc.nasa.gov/publications/gm2006/gipson

Correlation Due to Station Dependent Noise in VLBI

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Abstract

I study the effects of including station dependent delay noise in the analysis of geodetic VLBI. Such terms increase the observational noise, as well as introducing correlation between observations. I demonstrate by looking at CONT05 and the R1 & R4 sessions from 2005 that introducing such noise terms reduces baseline scatter and gives more realistic formal errors.

1. Introduction

The VLBI observable, colloquially called the "delay", is the difference in arrival time of a signal at two different stations. The delay is measured by correlating the signals received at the stations. This process has a uncertainty associated with it which, for clarity, I call *measurement noise* and denote by σ_{meas}. One can show, assuming SNRs commonly used in VLBI, that the measurement noise on different baselines is uncorrelated. The standard assumption in VLBI processing is that the *observational noise* in VLBI is just the measurement noise. This has the corrolary that the VLBI observations are independent.

There are several indications that this assumption is false. First, χ^2 from individual session solutions is much larger than it should be. Second, the scatter of baseline lengths is larger than it should be based on the formal errors. Third, comparison of EOP measurements from simultaneous VLBI sessions are inconsistent with the formal errors. All of these are indicative of incorrect modeling and/or unmodeled error sources.

There are many other error sources besides measurement noise. For example: 1) phase cal errors; 2) RF interference in the signals; 3) other correlator related errors; 4) source structure; 5) source position errors; 6) errors in geophysical models; 7) mismodeling clocks and/or atmospheres; 8) underparametrizing the time variation of clocks and/or atmospheres; etc. All of these increase the noise of individual observations. Many also introduce correlation between observations.

In this paper I look at the special case of error sources which manifest themselves as station dependent delay, e.g., mismodeling atmosphere delay. These cause changes in the measured delay. This will show up as excess delay noise. Since this delay error is common for all baselines involving a station, these observations are no longer independent, and the covariance matrix is no longer diagonal.

Other scientists have studied the stochastic model used in VLBI. Schuh and Wilkin [1] derived empirical correlation coefficients from 19 VLBI sessions, but did not take the next step of modifying the normal equations. Schuh and Tesmer [2] derived empirical correlation coefficients, and, together with the a priori variance, σ^2_{meas}, constructed the covariance matrix. They demonstrated that this improved repeatability on 36 IRIS-S sessions from December 1994 through December 1998. Tesmer [3] and Tesmer and Kutterer [5] modified the covariance matrix by inflating the diagonal terms with additional contributions due to sources, stations, and elevations. They found a reduction in the scatter of station position of a few percent.

In the next section, I present the least squares equations for VLBI. In Section 3, I discuss how station dependent delay modifies the covariance matrix. In Section 4, I study the effect of including "clock-like" errors and errors due to mismodeling the atmosphere. Using two data sets, CONT05, and the R1s & R4s during 2005, I demonstrate that including these terms increase the formal error of our baseline estimates, making them more realistic, and decreases baseline scatter, indicating that the estimates are actually better. I conclude with a discussion of future work.

2. Least Squares Equations in VLBI

The VLBI observable is the time delay $\tau_{ij}(t)$ between two stations i, j at some epoch t. The delay is a function of various parameters A_a. In the linear approximation the observed delay is:

$$\tau_{ij}(t) = \tau_{0,ij}(t) + \sum_a A_a \frac{\partial \tau_{ij}(t)}{\partial A_a} + \varepsilon_{ij,obs}(t) = \tau_{0,ij}(t) + \sum_a A_a F_{a,ij}(t) + \varepsilon_{ij,obs}(t) \tag{1}$$

$\tau_{0,ij}(t)$ is the a priori delay and $\varepsilon_{ij,obs}(t)$ is the noise associated with the observation, and $F_{a,ij}(t)$ the partial derivative of the delay. Let Ω be the covariance matrix of the observations:

$$\Omega_{ijt,klt'} =< \varepsilon_{ij,obs}(t)\varepsilon_{kl,obs}(t') > \tag{2}$$

The least squares equations are given by:

$$\sum_a \left(\sum_{ijt} \sum_{klt'} F_{b,ij}(t) F_{a,kl}(t') \Omega^{-1}_{ijt,klt'} \right) A_a = \sum_{ij} \sum_t F_{b,ij}(t) \Omega^{-1}_{ijt,klt'} \left(\tau_{kl}(t') - \tau_{0,kl}(t') \right) \tag{3}$$

These equations can be formally inverted to solve for the A_a:

$$A = \left(F^T \Omega^{-1} F \right)^{-1} F^T \Omega^{-1} \tau_{o-c} \tag{4}$$

3. Effect of Station Dependent Delay Noise on Covariance Matrix

Let the delay $\tau_{i,obs}$ at a station i be given by:

$$\tau_{i,obs} = \tau_{i,geom} + \tau_{i,mod} + \varepsilon_{i,A} + \varepsilon_{i,B} + \varepsilon_{i,C} + ... \tag{5}$$

$\tau_{i,geom}$ is the geometric delay. $\tau_{i,mod}$ incorporates both calibration and modeling terms. The $\varepsilon_{i,A}$ are station dependent delay error terms. The observational noise $\varepsilon_{ij,obs}(t)$ for baseline ij is:

$$\varepsilon_{ij,obs}(t) = \varepsilon_{ij,meas}(t) + \varepsilon_{ij,A}(t) + \varepsilon_{ij,B}(t) + \varepsilon_{ij,C}(t) + ... \tag{6}$$

where $\varepsilon_{ij,meas}(t)$ is the measurement noise due to the correlation process, and the remaining terms are due to different kinds of station dependent delay error: $\varepsilon_{ij,A}(t) = \varepsilon_{i,A}(t) - \varepsilon_{j,A}(t)$.

The following assumptions simplify the evaluation of the covariance matrix:

1. Different kinds of delay error are uncorrelated: $< \varepsilon_A \varepsilon_B >= 0$ for $A \neq B$.
2. Delay errors at different times are uncorrelated: $< \varepsilon_{ij,A}(t)\varepsilon_{kl,A}(t') >= 0$ for $t \neq t'$.

3. Delay errors at different stations are uncorrelated.

The covariance matrix is $\Omega =< \varepsilon_{obs}^2 >$. By assumption 1, the cross terms vanish, and the covariance matrix is just a sum of terms:

$$\begin{aligned} \Omega &= < \varepsilon_{meas}^2 > + < \varepsilon_A^2 > + < \varepsilon_B^2 > + < \varepsilon_C^2 > ... \\ &= \Omega_{meas} + \Omega_A + \Omega_B + \Omega_C .... \end{aligned}$$

The first term Ω_{meas} is the (diagonal) covariance matrix associated with the measurement process, and the remaining terms are the covariance matrices associated with each type of noise.

Assumption 2 implies that the covariance matrix is block diagonal, with each block being the covariance matrix for a single scan. By assumption 3, cross-terms involving different stations vanish. The diagonal elements for baseline ij are:

$$\Omega_{A,ij,ij}(scan) =< (\varepsilon_{i,A} - \varepsilon_{j,A})^2 >=< \varepsilon_{i,A}^2 + \varepsilon_{j,A}^2 >= \sigma_{i,A}^2 + \sigma_{j,A}^2 \tag{7}$$

i.e., just the sum of the noise terms for each station. The off-diagonal terms of the covariance matrix are non-zero *if, and only if* the baselines have a station in common. In this case we have:

$$\Omega_{A,ij,il}(scan) = -\Omega_{A,ij,li} =< (\varepsilon_{i,A} - \varepsilon_{j,A})(\varepsilon_{i,A} - \varepsilon_{l,A}) >=< \varepsilon_{i,A}^2 >= \sigma_{i,A}^2 \tag{8}$$

Note that both the diagonal and off-diagonal terms depend *only* on the variance of the noise. Hence station dependent delay noise has two effects: 1) The noise level of the observations is increased; and 2) Observations involving a common station at a given time are correlated.

Since the covariance matrix is block diagonal, building up the normal equations given in Eq. (3) is straightforward. This is done on a scan by scan basis: 1) compute the covariance matrix for a given scan; 2) invert it; and 3) compute the contribution of this scan to the normal matrix.

4. Clock Noise and Azimuthal Asymmetry Mismodeling in VLBI

In this section I look at the efffect of incorporating station dependent delay error using two VLBI data sets: 1) CONT05 is close to the current state of the art in geodetic VLBI, and all of the sessions are contiguous in time; 2) The R1 & R4 sessions during 2005 are a relatively good operational network with sessions over a prolonged period of time.

I modified the GSFC *solve* analysis software to take into account the effects of clock-like error and atmospheric azimuthal mismodeling. Clock like delay error can be caused by underparametrizing the clock variation, by errors in the cable calibration, and other sources. I assume that the variance is uniform and time independent:

$$\sigma_{clk}^2 = a_{clk}^2 \tag{9}$$

One source of atmosphere mismodeling is to neglect the effect of turbulence. Assuming a 2/3 power law for spatial fluctuations, the unmodeled azimuthal variance is:

$$\sigma_{az}^2 = \left(a_{az} \times \cot^{2/3}(el)\right)^2 \tag{10}$$

Figure 1 plots the difference in baseline repeatability, as a function of baseline length, between the standard solution and a solution assuming that $a_{az} = 15$ ps at each station. Points above (below) the x-axis are baselines where the scatter in the standard solution is larger (smaller) than in the new solution. For 46 out of 54 baselines, the new solution reduces the scatter and *improves* the VLBI solution. The average improvement is 0.81 mm, or 11.1%. The average χ^2 of the baseline scatter from the standard solution is 2.16, indicating that the formal errors are too optimistic by $\sqrt{2.16} \simeq 1.47$. The average χ^2 from the new solution is 1.08, indicating that the formal errors are too optimistic by a factor of 1.04.

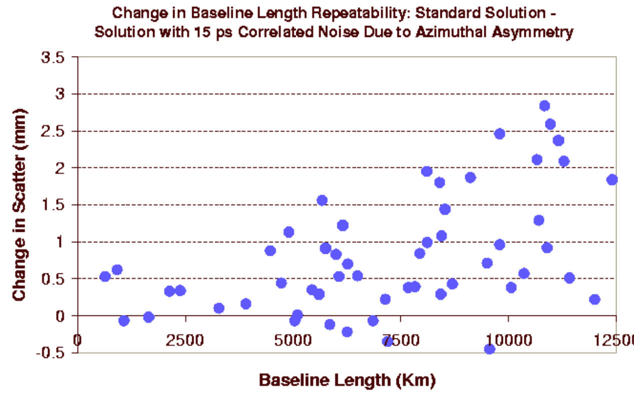

Figure 1. Difference in baseline repeatability for the CONT05 data set between the standard solution and one incorporating 15 ps of noise due to azimuthal asymmetry. Points above (below) the x-axis are baselines where the scatter is reduced for the new (standard) solution.

I ran a series of solutions using different values for a_{clk} and a_{az}. For each solution I considered all baselines with more than 10 observations, found the best fit line through the baseline lengths, and calculated χ^2 per degree of freedom and the WRMS about the best fit line. Table 1 summarizes these results. For each solution, Table 1 displays the WRMS and χ^2 averaged over all baselines. Also displayed is the average change in scatter compared to the standard solution, expressed in millimeters and in per cent, and the number of baselines where the scatter is reduced. Incorporating the effect of clock-like noise makes χ^2 more realistic, i.e., closer to 1, but does not reduce baseline scatter very much. In contrast, including the effect of azimuthal assymetry reduces χ^2 and reduces the baseline scatter: The formal errors are more realistic, and the solution is better. The optimal value for a_{az} is about 15 ps.

Table 1. Effect of clock and atmosphere station dependent delay.

		CONT05					R1s & R4s for 2005				
a_{clk}	a_{az}	Avg	WRMS	Avg. Imp.		#BL	Avg.	WRMS	Avg. Imp.		#BL
ps	ps	χ^2	mm	mm	%	Imp.		mm	mm	%	Imp.
0	0	2.16	7.56	-	-	-	1.94	12.27	-	-	
5	0	1.96	7.52	.04	0.6%	34/54	1.83	12.25	0.02	0.1%	24/51
10	0	1.4	7.51	.05	1.0%	31/54	1.64	12.24	0.02	0.0%	24/51
15	0	1.21	7.54	.02	0.6%	28/54	1.46	12.30	-0.03	-0.6%	23/51
0	5	1.75	7.17	0.39	5.2%	49/54	1.73	12.11	0.16	1.7%	37/51
0	10	1.34	6.87	0.69	9.8%	48/54	1.46	11.99	0.28	2.9%	35/51
0	15	1.08	6.75	0.81	11.1%	46/54	1.25	12.00	0.27	2.8%	32/51
0	20	0.91	6.73	0.72	11.0%	45/54	1.10	12.11	0.17	2.0%	29/51
10	10	1.20	6.99	0.57	8.4%	49/54	1.35	12.06	0.21	2.0%	34/51

One possibility is that the improvement in baseline scatter is due entirely to inflating of the diagonal components of the covariance matrix, i.e., the only important effect of station-dependent noise is to increase the noise of the observations, and the correlation between observations can be ignored. To test this theory I reran some of the above solutions, setting the off-diagonal terms of the covariance matrix to 0. The results are displayed in Table 2. Although the baseline scatter is reduced, the amount of reduction is only half that of using the full covariance matrix, indicating that the correlations are important, and including them improves the solution.

Table 2. Effect of clock and atmosphere station dependent delay (diagonal components only).

		CONT05					R1s&R4s for 2005				
a_{clk}	a_{az}	Avg.	WRMS	Avg. Imp.		#BL	Avg.	WRMS	Avg. Imp.		#BL
ps	ps	χ^2	mm	mm	%	Imp.	χ^2	mm	mm	%	Imp.
0	0	2.16	7.56	-	-	-	1.94	12.27	-	-	
0	5	1.94	7.42	0.14	2.5%	44/54	1.79	12.20	0.08	0.9%	34/51
0	10	1.61	7.28	0.28	5.0%	42/54	1.57	12.15	0.15	1.6%	34/51
0	15	1.36	7.23	0..33	6.2%	40/54	1.38	12.18	0.13	1.5%	32/51

5. Conclusion and Future Work

Including station dependent delay noise has the potential to reduce baseline scatter estimates and result in more realistic formal errors. The effect of including clock-like errors is relatively small. In contrast, including the effect of atmospheric asymmetry results in a dramatic decrease in baseline scatter. This improvement is not due simply to inflating the observational errors, but depends as well on the correlations introduced in the measurement.

Under current investigation are extensions to this work such as: 1) Include other sources of station dependent delay, such as mapping function error. 2) The present work assumed that the variance was station independent. It seems plausible that this would vary from station-to-station. 3) The tests done in the present paper showed impovement based on internal consistency. Another test is to compare VLBI results with those from other techniques, e.g., GPS.

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