Mamoru Sekido et al.: VLBI Delay Model for a Radio Source at Finite Distance, IVS 2006 General Meeting Proceedings, p.59–63
http://ivscc.gsfc.nasa.gov/publications/gm2006/sekido

VLBI Delay Model for a Radio Source at Finite Distance

Mamoru Sekido [1], *Toshio Fukushima* [2]

[1] *Kashima Space Research Center/National Institute of Information and Communications Technology*
[2] *National Astronomical Observatory Japan*
Contact author: Mamoru Sekido, e-mail: sekido@nict.go.jp

Abstract

A new VLBI Delay model for radio source at finite distance is presented. The geometrical effect of curved wavefront is fully considered with pseudo source vector defined by $\vec{K} = (\vec{R}_{01} + \vec{R}_{02})/(R_{01} + R_{02})$. Our VLBI delay model gives delay in the scale of terrestrial time (TT) with the baseline vector on the TT-scale. Since the new delay model is in a similar form as the current standard VLBI delay model (consensus model), implementation into a current VLBI analysis software, such as CALC/SOLVE and OCCAM, will be relatively easy. Our VLBI delay model is regarded as expansion of the consensus model from infinite to finite region in terms of distance to radio source. Accuracy of the new delay model is better than 1 ps in ground based VLBI observations of radio sources. The applicable range is from radio sources at altitude of 100 km or more. Analytical correction terms to adapt the consensus model to finite-distance radio sources is also given under the condition that the distance to the radio source is farther than 10 pc. This may be useful for analysis of parallax for galactic radio sources.

1. Introduction

The standard VLBI delay model, the so-called consensus model [1, 2], is designed to compute accurately the time differences of signal arrival between two stations on the earth for a radio source at infinite distance. However the effect of curved wave fronts has to be taken into account for radio sources at finite distance. For radio sources closer than 200 kpc, the effect of a curved wavefront will exceed 1 ps on VLBI observations with 12000 km baseline [7]. Pulsars, maser sources, and all radio sources in our galaxy are included. Radio sources in the solar system ,such as planets and space probes, are important targets from VLBI observation; however intolerable errors will be caused if the effect of curved wavefront is not considered. Moyer [3] provides a delay model by computing the difference of light time for two legs from a spacecraft to observation stations. This is a straight forward approach and it has been used in deep space missions by JPL/NASA in practice. As this model is fairly different from the current VLBI delay model, an implementation into VLBI analysis software is not so simple. VLBI-like approaches, in which the difference of light time is computed analytically, were proposed by several authors [4, 5, 6]. Fukushima proposed an iterative scheme to determine VLBI delay in the lunar project[5]. Klioner gives analytical formulas for radio sources in the solar system [4]. These models, however, do not give a relation between the baseline and the time delay observable in the TT-frame, in which the practical delay measurements are performed, but in the TDB-frame or Barycentric Celestial Reference System (BCRS), which are not directly accessible. BCRS is defined in Resolution B1.3 of IAU General Assembly in 2000 (see appendix of reference [2]). TDB-frame is a kind of the barycentric celestial reference system that differs from BCRS only by a scaling factor L_B. Its time coordinates is regarded as Barycentric Dynamical Time (TDB). TT-frame is a non-rotation celestial reference system differing from Geocentric Celestial Reference System (GCRS) by a constant scale L_G. The GCRS is also defined in IAU Resolution

B1.3 as a non-rotation local flat coordinate system near the earth, in which the spatial scale is defined as consistent with Geocentric Coordinate Time (TCG). The spatial scale of the TT-frame is consistent with Terrestrial time (TT). Current major VLBI analysis software packages, such as CALC/SOLVE and OCCAM, deal with delay data and station coordinates in the TT-frame. The International Terrestrial Reference System (ITRS) is defined in the GCRS, although realizations of the ITRS, such as ITRF2000, are actually given in the TT-frame. Due to these reasons, our new VLBI delay model is intended to give the relation between baseline vector and delay observable in the TT-frame. This paper presents formulas from our new VLBI delay model. See paper [7] for detail of the derivation of the formulas and comparison with other models.

2. The New VLBI Delay Model

The basic formula of the new VLBI delay model is given as

$$(TT_2 - TT_1)_{\text{Finite}} = \left\{ -\left[1 - 2\frac{W_{\mathrm{E}}}{c^2} - \frac{\vec{V}_{\mathrm{E}}^2 + 2\vec{V}_{\mathrm{E}}\cdot\vec{v}_2}{2c^2}\right]\frac{\vec{K}\cdot\vec{b}}{c} - \frac{\vec{V}_{\mathrm{E}}\cdot\vec{b}}{c^2}\left[1 + \hat{\vec{R}}_2\cdot\frac{\vec{V}_2}{c} - \frac{(\vec{V}_{\mathrm{E}} + 2\vec{V}_2)\cdot\vec{K}}{2c}\right] + \Delta T_{g,21}\right\} \Big/ \left[(1 + \hat{\vec{R}}_2\cdot\frac{\vec{V}_2}{c})(1+H)\right], \quad (1)$$

where suffices 0,1,2, and E indicate radio source, observation station 1, 2, and geocenter, respectively. W_E is the gravitational potential given by $W_E = \sum_{J\neq E} GM_J/|\vec{X}_E - \vec{X}_J|$. $\vec{X}_i$ is the position vector of $i = (0,1,2,E)$ in the TDB-frame. The TDB-frame is chosen as barycentric celestial reference system in our derivation, because the position and velocity of objects in the solar system are given by planetary ephemeris such as JPL ephemeris DE405. $\vec{V}_i$, $(i = 2, E)$ is the coordinate velocity of i with respect to the solar system barycenter (SSB). $\vec{v}_2$ is the geocentric station vector of station 2. $\vec{b}$ is a baseline vector given by the transformation of the coordinates in ITRF2000 into the celestial reference frame (see chapter 5 of [2]).$\vec{K}$ is a pseudo source vector introduced by Fukushima [5], by which the geometrical effect of the curved wavefront is expressed. It is given by

$$\vec{K} \stackrel{\text{def}}{=} \frac{\vec{R}_1 + \vec{R}_2}{R_1 + R_2}, \text{ where } \vec{R}_i = \vec{X}_i(T_1) - \vec{X}_0(T_0), \text{ and } R_i = |\vec{R}_i|, \quad (2)$$

where i=1,2. The position vectors $\vec{X}_0(T)$ should be given by a predicted orbit as function of time in the TDB-frame. The position vector in TDB-frame $\vec{X}_i$ (i=1,2) are given with geocentric station coordinates in the TT-frame $(\vec{\xi}_i, TT_1)$ as

$$\vec{X}_i(T_1) = \vec{X}_{\mathrm{E}}(T_1) + \left(1 - \frac{W_{\mathrm{E}}}{c^2} - L_C\right)\vec{\xi}_i(TT_1) - \left(\frac{\vec{V}_{\mathrm{E}}\cdot\vec{\xi}_i(TT_1)}{2c^2}\right)\vec{V}_{\mathrm{E}}, \quad (3)$$

where $\vec{X}_E, \vec{X}_J, \vec{V}_2$, and W_E are given by planetary ephemeris. Scaling factor $L_C = 1.48082686741\times 10^{-8} \pm 2\times 10^{-17}$ [8]. $\hat{\vec{R}}_2$ is a unit vector given by $\hat{\vec{R}}_2 = \vec{R}_2/R_2$. The epoch, when the signal departed from radio source 0, is denoted T_0 in TDB and the arrival time at station 1 is T_1. Observed delay data are time tagged with UTC. Here the arrival time at station 1 is denoted as UTC_1. Then the corresponding TT_1 is computed from UTC_1 by

$$TT_1 = (\mathrm{TT} - \mathrm{TAI}) + (\mathrm{TAI} - \mathrm{UTC}) + UTC_1, \tag{4}$$

where (TT-TAI) is 32.184 for historical reason. (TAI-UTC) is 32 sec in 2005 and is 33 sec from 0h UTC on 1st January 2006. Then T_1 in TDB is computed by using time ephemeris $\Delta T_{\oplus}(T_{eph})$ [8] as

$$T_1 = TT_1 + \Delta T_{\oplus}(TT_1) - \Delta T_{\oplus}(TT_0) + \frac{V_E \cdot \vec{\xi}_1}{c^2}, \tag{5}$$

where TT_0 corresponds to 0h UT on January 1st, 1977. $\vec{\xi}_1$ is the geocentric vector of station 1 in TT-frame. The epoch of the signal emission from the radio source in TDB (T_0) is obtained by solving the light time equation

$$T_0 = T_1 - \frac{\left|\vec{X}_0(T_0) - \vec{X}_1(T_1)\right|}{c} - 2\sum_J \frac{GM_J}{c^2} \ln \frac{R_{1J} + R_{0J} + R_{01}}{R_{1J} + R_{0J} - R_{01}}, \Delta T_{g,01}. \tag{6}$$

The last term is the gravitational effect in the ray path from radio source 0 to observation station 1. In this term, the position of the gravitating body J must be evaluated at the epoch of closest approach of the photon to the gravitating body. The light time equation (6) may be solved by numerical iteration such as the Newton-Raphson method and the solution converges very rapidly. The term H in the denominator of equation (1) is the correction term with Halley's method [7].

$$H = \left|\frac{\vec{V}_2}{c} \times \widehat{\vec{R}}_2\right|^2 \frac{\vec{K} \cdot \vec{b}}{2R_2}. \tag{7}$$

The gravitational effect $\Delta T_{g,21}$ is composed from several terms as discussed by Klioner [4]: Post-Newtonian ΔT_{pN}, effect in the field of moving body ΔT_M, influence of quadruple field ΔT_Q, rotation of the bodies ΔT_R, and the post-post-Newtonian effects ΔT_{ppN}.

$$\Delta T_{g,21} = \Delta T_{pN} + \Delta T_M + \Delta T_R + \Delta T_Q + \Delta T_{ppN}. \tag{8}$$

The Post-Newtonian term (ΔT_{pN}) is the most significant and it must be included at least for the sun, moon and major planets (Jupiter, Saturn, Venus, Mars, and the Earth). This term is given by

$$\Delta T_{pN} = 2\sum_J \frac{GM_J}{c^3} \ln \left(\frac{R_{2J} + R_{0J} + R_{20}}{R_{2J} + R_{0J} - R_{20}}\right)\left(\frac{R_{1J} + R_{0J} - R_{10}}{R_{1J} + R_{0J} + R_{10}}\right). \tag{9}$$

According to Klioner [4], the post-post-Newtonian term of the Sun becomes a few hundreds of a ps in case of grazing rays and several ps when the source direction is 1 deg. away from the Sun. The term ΔT_Q reaches a few tens of a ps when the ray passes through the rim of Jupiter or Saturn. The term ΔT_M of Jupiter and ΔT_R of the sun reaches 0.5 ps when the ray passes the rim of these gravitating bodies. Refer to literature [9, 4] for formulas of each gravitational effect.

3. Adapting the Consensus Model for Finite-Distance Radio Source

For source distances up to 200 kpc and an Earth-diameter baseline the curved wavefront causes delay effects of larger than 1 ps. A full consideration of this effect is indispensable for observing radio source in the solar system. However, when the distance to the radio source from the earth is larger than 10 pc, the effect of the curved wavefront can be approximated with a few correction terms with parallax vectors $\vec{p}_M$ and $\vec{p}_2$ as

$$\Delta\tau_{\rm Finite} - \Delta\tau_{\rm IERS} = (\Delta T_{g,12} - \Delta t_g) + \frac{\vec{b}\cdot\vec{p}_M}{c}\left(1 - \vec{k}\cdot\frac{\vec{V}_2}{c}\right) - \frac{\vec{k}\cdot\vec{b}}{c}\left(\vec{p}_2\cdot\frac{\vec{V}_2}{c} - H\right) + O(b\varepsilon^2), \tag{10}$$

where $\vec{p}_M = \vec{\varepsilon}_M - (\vec{\varepsilon}_M \cdot \vec{k})\vec{k}$, $\vec{p}_2 = \vec{\varepsilon}_2 - (\vec{\varepsilon}_2 \cdot \vec{k})\vec{k}$. $\vec{\varepsilon}_i = \vec{X}_i/R$ (i=1,2), and $\vec{\varepsilon}_M = (\vec{\varepsilon}_1 + \vec{\varepsilon}_2)/2$. See paper [7] for details of the derivation of the approximation. The right hand side of equation (10) can be used as correction terms to the consensus model adapting it to finite distance radio sources. Galactic radio sources such as pulsars and maser sources are candidates for such targets. Parallax measurements have been traditionally made by mapping source positions on the celestial sphere with data observed in multiple seasons. Then, the parallax parameter is estimated by fitting their apparent motion with a model. This technique has been used used since the era of classical optical measurement. And the same technique is still used in modern VLBI, in which the delay observable is directly available. Equation (10) suggests the possibility of parallax parameter estimation by the least squares technique, directly using the delay data-set from multiple seasons. See paper [7] for the partial derivative of the delay with respect to the parallax parameter. Equation (10) gives the daily variation with 40 ps amplitude in the case of PSR1937+21 (3.6kpc), for example. This approach may enhance the precision of parallax measurement. Possible difficulty in this approach may be delay resolution to detect the variation and clock discontinuity among VLBI sessions at different epochs.

References

[1] Eubanks, T. M. A Consensus Model for Relativistic Effects in Geodetic VLBI. Proc. of the USNO workshop on Relativistic Models for Use in Space Geodesy: 60–82, 1991

[2] McCarthy, D. D. and Petit, G. IERS Conventions, IERS Technical Report No.32, 2003.

[3] Moyer, T. D. Formulation for Observed and Computed Values of Deep Space Network Data Types for Navigation, JPL Monograph 2 (JPL Publication 00-7) 2000. JPL Deep Space Communications and Navigation Series, John Wiley & Sons. Inc., Hoboken New Jersey ISBN 0-471-44535-5

[4] Klioner, S, "General relativistic model of VLBI observables", in Proc. AGU Chapman Conf. on Geodetic VLBI: Monitoring of Global Change, Cartger, W. E.(ed.) NOAA Technical Report NOS 137 NGS 49, American Geophysical Union, Washington D.C, 188–202, 1991.

[5] Fukushima, T. Lunar VLBI observation model. Astron. Astrophys. 291: 320–323, 1994.

[6] Sovers, O. J., J. L. Fanselow, and C. S. Jacobs, Astrometry and Geodesy with radio interferometry: experiments, models, results, Reviews of Modern Physics, 70: No.4: 1393–1454, 1998.

[7] Sekido, M and T. Fukushima, A VLBI delay model for a finite distance radio source. J. Geodesy, 2006, in print.

[8] Irwin A. W. and Fukushima T. A numerical time ephemeris of the Earth. Astron. Astrophys. 348: 642–652,1999.

[9] Brumberg V. A., Post-post-Newtonian propagation of light in the Schwarzschild field. Kinematica Fizika Neb. 3(1): 8–13,1987