The 5^{th} IVS General Meeting Proceedings, 2008, p. 157–162

Improved Estimation in VLBI through Better Modeling and Analysis

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Abstract. We report on the effect of several modifications to the standard VLBI data analysis using the CONT05 dataset. These include: 1) varying re-weight constants; 2) introducing elevation dependent noise; 3) introducing correlation between observations; 4) use of VMF instead of NMF.

1. Introduction

The VLBI observable, colloquially called the “delay”, is the difference in arrival time of a signal at two different antennas. Roughly speaking, the delay is measured by cross-correlating the signals received at the two stations and searching for a peak. This process has an uncertainty associated with it which, for clarity, we call measurement noise and denote by σ_{meas}. This noise is inherent to the correlation process and depends only on the signal strength, sensitivity of the antennas, frequency sequence, and number of bits recorded. One can show, assuming SNRs commonly used in VLBI, that the measurement noise on different baselines is uncorrelated. The normal assumption in VLBI is that the observational noise σ_{obs} equals the measurement noise: $\sigma_{meas} = \sigma_{obs}$.

There are several indications that this assumption is false. First, the reduced χ^2 from individual session solutions is much larger than it should be based on $\sigma_{meas} = \sigma_{obs}$. This implies that there are other noise sources, or that we are not correctly modeling the system. The standard approach is to re-weight the observations by adding additional noise until $\chi^2 \simeq 1$. We demonstrate below that this reduces baseline scatter. Second, even after re-weighting, baseline length scatter is larger than it should be based on the formal errors of individual sessions. Third, comparison of EOP measurements from simultaneous VLBI sessions are inconsistent with the formal errors. Fourth, comparison of EOP measurements from VLBI and GPS indicate that the formal errors from one or both techniques are two optimistic.

There are many other error sources besides measurement noise: 1) phase cal errors; 2) RF interference in the signals; 3) other correlator related errors;

4) source structure; 5) source position errors; 6) errors in geophysical models; 7) mis-modeling clocks and/or atmospheres; 8) underparametrizing the time variation of clocks and/or atmospheres; etc. All of these increase the noise of individual observations. Many also introduce correlation between observations.

In the following sections we present several modifications to the standard *solve* data-analysis, and study the effect on the CONT05 sessions observed during Sep. 12-26, 2005. This dataset has several advantages for this kind of work. The number of sessions (15) allows us to meaningfully compute statistics. The short time span reduces seasonal effects, and allows us to determine the intrinsic accuracy of the technique. Moreover, σ_{meas} is much smaller than for most VLBI sessions. Because of this, you may be able to see improvements in analysis which would not be apparent if σ_{meas} were larger.

In this note we use baseline scatter as a measure of goodness. A reduction in the scatter is strong internal evidence that the solution is better. In our studies we also looked at the agreement between VLBI and IGS estimates of EOP, which is an independent external test. Space constraints preclude us from describing these results in detail. In all cases where the baseline scatter was reduced, the agreement with IGS improved.

In Sec. 2 we examine the effect of re-weighting the data. Re-weighting has been standard practice at Goddard since at least the early 1990s.

In Sec. 3 we look at the effect of adding noise proportional to the mapping function. Such a term arises from some kinds of atmosphere mis-modeling. Including these terms reduces baseline scatter. Interestingly enough, we get better results if we turn-off re-weighting. This implies that much of the unmodeled noise is due to atmosphere-modeling error.

In Sec. 4 we look at the effect of introducing elevation dependent correlations between observations ("correlated noise"). This is a natural extension of the work of Sec. 3. In fact, elevation dependent noise is an approximation to elevation dependent correlation. We demonstrate that this further reduces the scatter.

At Goddard the standard mapping function is NMF (Niell Mapping Function) [4]. In Sec. 5 we study the effect of using the more recent mapping function VMF (Vienna Mapping Function) of J. Boehm and H. Schuh [1]. We show that this reduces scatter compared to NMF. We obtain the best results when we combine VMF with correlated noise and turn off re-weighting.

2. Re-weighting VLBI Data

If the VLBI observations were correctly modeled, and $\sigma_{obs} = \sigma_{meas}$, we should find $\chi^2 \simeq 1$ for individual sessions. Typically $\chi^2 \simeq 3 - 4$, indicating that: 1) we have underestimated σ_{obs}; 2) are mis-modeling the system; or 3) both. The standard fix is to add extra noise ϵ^2_{rewt} to the observations:

$$\sigma^2_{t,ij,obs} = \sigma^2_{t,ij,meas} + \epsilon^2_{t,ij,rewt}\,, \tag{1}$$

until $\chi^2 \simeq 1$. The subscripts t and ij label the time and the stations involved in the the observation. Although in principle the re-weight constants $\epsilon^2_{t,ij}$ can vary arbitrarily from observation to observation, in practice analysts usually make one of the following simplifying assumptions:

1. “global re-weights” are the same for all observations;
2. “station re-weights” depend only on the stations in an observation;
3. “baseline re-weights” depend only on the baselines in an observation.

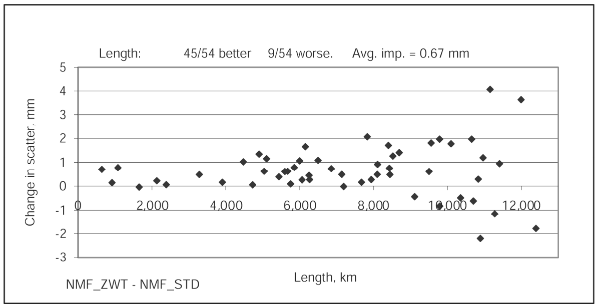

Figure 1. Reweighting reduces scatter

At Goddard we generally use either station or baseline re-weight constants. Fig. 1 plots the difference in baseline scatter between a solution with no re-weighting and one using standard baseline re-weighting. Each point represents a single baseline. If the point is above the X-axis, re-weighting reduces the scatter of 45/54 baselines, and the average reduction is 0.67 mm. Generally speaking, the longer the baseline, the greater the reduction in scatter. It is remarkable that re-weighting–which depends only on what is happening within a session–has such a large effect on intra-session scatter.

3. Elevation Dependent Noise

Delay residuals are larger at lower elevations, which is evidence of atmosphere mis-modeling. There are many plausible functional forms for this. We assume a noise term proportional to the mapping function $M_i(el_i)$. The particular mapping function is not important, and using a simple $1/sin(el)$ gives similar results. Analogous to (1) we assume that

$$\sigma^2_{ij,obs} = \sigma^2_{ij,meas} + \epsilon^2_{ij,rewt} + (\epsilon_i M_i(el_i))^2 + (\epsilon_j M_j(el_j))^2 \, .$$

The noise coefficients ϵ_i could be site and session dependent. For simplicity we assume the same values for all stations (we have investigated the more general case). We ran solutions varying ϵ_i from 0 to 15 ps. All of these values reduced the scatter, with the greatest reduction at $\epsilon_i \simeq 6\ ps$.

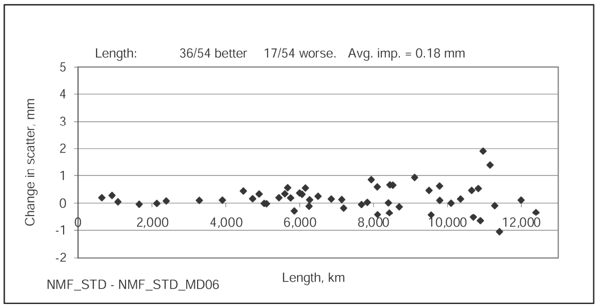

Figure 2. Elevation dependent noise

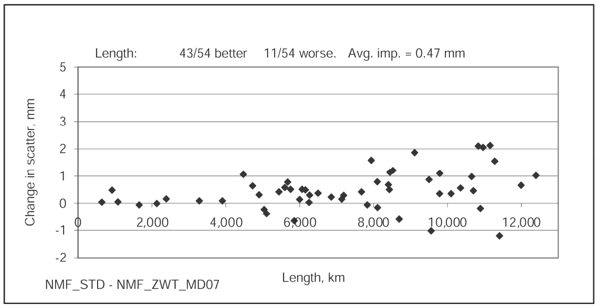

Figure 3. Elevation dependent noise with no re-weighting

Fig. 2 plots the difference in scatter between the standard solution and one with 6 ps of noise. Including this effect reduces scatter on 36/54 baselines, and the average improvement is 0.18 mm. Both solutions used our standard reweights. Fig. 3 plots results of a similar solution where re-weighting is turned off. Surprisingly, this further improves the solution: 43/54 baselines are improved, and the average improvement is 0.47 mm.

4. Correlation Between Observations

Since atmosphere mis-modeling is station dependent, it effects all observations involving a given station in a given scan in the same way. In addition to adding noise to the observation, it introduces correlations between observations. The normal equations must be modified to correctly handle this. The details of this are beyond the scope of the present paper, but are given in [2, 3]. Other scientists [5–10] have also studied correlation between observations. Space constraints preclude us from adequately discussing their work, and we refer the reader to their papers. Fig. 4 plots the difference between the standard solution and one where we have introduced 6 ps of correlated elevation dependent noise.

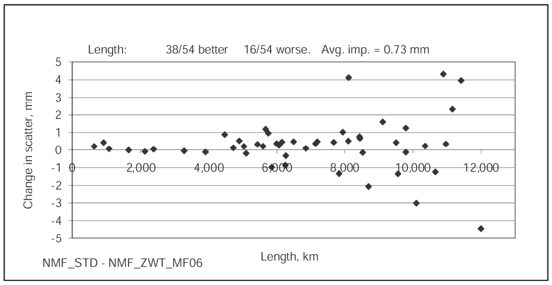

Figure 4. Incorporating correlated noise improves the solution

Average scatter is reduced by 0.73 mm. Most (38/54) baselines are improved. A few baselines involving HARTRAO are degraded.

5. Comparison of NMF and VMF Mapping Functions

The standard mapping function used at Goddard is NMF. This has the advantage that it is easily calculable. It has the disadvantage that it completely ignores the weather. In contrast, VMF uses ECMF data to derive station and session specific mapping functions. Fig. 5 plots the difference between

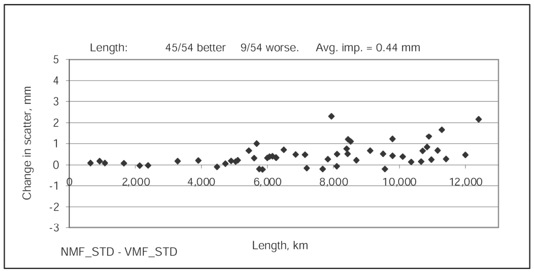

Figure 5. VMF

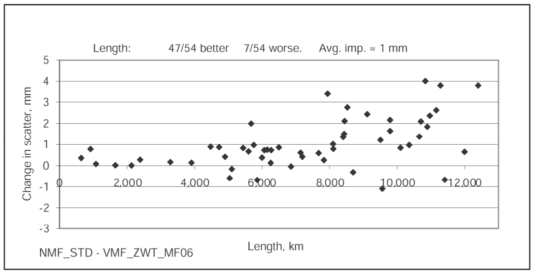

Figure 6. VMF with correlation and no re-weighting

a solution using NMF and one using VMF. On average baseline scatter is reduced by 0.44 mm, and 45/54 baselines improve. In Fig. 6 we have turned off re-weighting and included correlation effects. In this case, 47/54 baselines are better, and the average improvement is 1.0 mm.

6. Conclusions

In this note we presented several alternatives to the standard VLBI analysis. We demonstrated that re-weighting reduces scatter. Including elevation dependent noise further reduces scatter. Curiously enough, the results are better in this case if re-weighting is turned off. Including the effect of correlation between observations results in a significant reduction in scatter. We showed that using VMF reduces scatter compared to NMF. Our best results use VMF, correlated elevation dependent noise, and no re-weighting.

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