

Interpretation of VLBI Results in Geodesy,
Astrometry and Geophysics

The Phase Variations of Retrograde Free Core Nutation

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Abstract. The application of an envelope method to the analysis of celestial pole offsets in the interval 1984-2007, published by IERS and NEOS services, has shown that the mean value of the RFCN period is close to -444 solar days. However there are three consecutive intervals of time when this period sharply changed and had values -421.4 ± 0.1 , -614.7 ± 1.1 , and -447.1 ± 0.2 days.

1. Introduction

The basic feature of the Earth's interior is the presence of a hot fluid outer core capable to free near-daily spin in a cavity limited by lower mantle. In a Celestial Reference System (CRS) this spin causes circular offsets of the Celestial Intermediate Pole (CIP) with a period of about -430.2 solar days which refers to the Free Core Nutation (FCN). The modern theory of the Earth's rotation [1] asserts that the FCN period is induced by the ellipticity of the fluid outer core and should be constant over a long time.

However, because of active physical processes in the Earth's core, its dynamics hardly can be named "free". It is considered that the outer fluid core consists basically of iron and its central part named the inner core consists of iron and nickel in a rigid crystal condition. Both parts of the core can contain impurities of silicates and other light elements. Being hot but cooling down, the fluid core has convective and gravitational movements, which due to high electric conductivity of iron and nickel create a magnetic field penetrating all of Earth's body and going out into space. This phenomenon is referred to as an electromagnetic dynamo.

It is established also that the external layer of the outer core rotates more slowly than the mantle of the Earth, which cannot be explained by the viscosity of the core but by its electromagnetic connection with the mantle [2, 3]. It results in the western drift, no-dipole part of the magnetic field.

There are numerous publications devoted to the influence of the electromag-

netic connections of the core and mantle on angular velocity and polar motion of the Earth. The problem related to CIP offsets was discussed only in one work [4], where it is shown that in 1992 and 1998 changes of the FCN phase coincided with geomagnetic jerks. Below it will be shown that modern VLBI observations give new important information for studying the fluid core of the Earth and its influences on geodynamics.

2. Stochastic Model of RFCN

After 1984 the accuracy of VLBI observations has increased so that it is practically possible to study FCN as variation of CIP offsets with the amplitude limited by 0.5 ms. It appears that this near-harmonic variation has a negative frequency; therefore it started to be named Retrograde Free Core Nutation (RFCN). To the naked eye, the amplitude of RFCN varies over a wide range. Therefore, in the numerical models constructed by T. Herring in 1998 [5] and S. Lambert in 2007 [6], it is represented as piecewise linear functions under the condition that the RFCN period is constant and equal to -430.21 solar days. Simultaneously an analysis of amplitude and phase variations was undertaken by Z. Malkin and D. Terentev in 2003 [7].

In addition to S. Lambert's most exact model (LAM), we constructed another numerical model of RFCN by means of the least squares collocation technique (LSC) [8]. The basic difference of this method to all other linear procedures is that it uses the covariance function of the desired signal as a priori information. Generally, for the construction of such a model it is necessary to remove a low-frequency trend and then a white noise from the observational series [9, 10]. The last procedure is defined by the formula $\mathbf{t} = \mathbf{Q}_{tt}(\mathbf{Q}_{tt} + \sigma^2 \mathbf{I})^{-1} \mathbf{l}$ where \mathbf{l} is the vector of one component of the observed CIP offsets (dX or dY), \mathbf{t} is a vector of corresponding model, σ^2 is a variance of white noise of observation errors, \mathbf{Q}_{tt} is a Toeplitz covariance matrix. This matrix is constructed from the positive definite function $q_{tt}(\tau)$ as model of empirical auto-covariance $q_{ll}(\tau)$. The variance σ^2 is defined as the difference between these functions at the zero point.

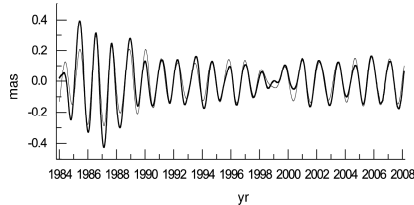


Figure 1. X-component of RFCN as follows from LSC and LAM models (thick and thin lines, respectively)

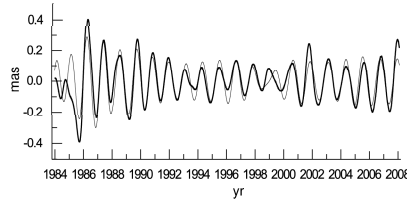


Figure 2. Y-component of RFCN as follows from LSC and LAM models (thick and thin lines, respectively)

The comparison of the LAM and LSC models are shown in Fig. 1-2. The RMS of IERS(dX) and IERS(dY) offsets and their residuals with respect to LAM and LSC models are given in Tabl. 1. The values in columns 3 and 4 correspond to a filtering of the series, whereas columns 2 and 5 show the accuracy of the predictions. For all available IERS and NEOS series the LSC model appreciably better corresponds to observations than the LAM model.

Table 1. RMS of CIP offsets and residuals with LAM and LSC models in mas

Span of observations in MJD	45700-46274	46275-50370	50371-54446	54467-54518
Offsets number	575	4096	4096	52
IERS(dX)	.534	.353	.198	.143
IERS(dX)-LAM	.506	.322	.179	.135
IERS(dX)-LSC	.485	.315	.176	.130
IERS(dY)	.430	.366	.222	.296
IERS(dY)-LAM	.453	.338	.202	.191
IERS(dY)-LSC	.400	.329	.193	.142

3. Phase Analysis

The theory outlined below is referred to as an envelope method [11]. This method allows to evaluate the amplitude and phase of stochastic processes for each moment of observation. We shall consider CIP offsets as near-harmonic oscillations on a plane (X, Y), which is tangent to the celestial sphere in a point set by the IAU 2000 precession-nutation theory:

$$x(t) = E(t) \cos F(t), \quad y(t) = E(t) \sin F(t),$$

$$E(t) = \sqrt{x(t)^2 + y(t)^2}, \quad F(t) = \arctan(y(t)/x(t)).$$

Let $F(t) = qt + p(t)$, where q is a negative frequency and $p(t)$ is a phase variation not containing a linear trend. Designating $u = |q| > 0$ we obtain $F(t) = -ut + p(t)$ and the following relations:

$$x(t) = A(t) \cos ut + B(t) \sin ut, \quad y(t) = -A(t) \sin ut + B(t) \cos ut,$$

$$A(t) = x(t) \cos ut - y(t) \sin ut, \quad B(t) = x(t) \sin ut + y(t) \cos ut,$$

$$E(t) = \sqrt{A(t)^2 + B(t)^2}, \quad p(t) = \arctan(B(t)/A(t)).$$

LSC models of the same name CIP offsets for IERS and NEOS series appeared very close, therefore they have been averaged. The amplitudes and phases of RFCN for both models were calculated with an accepted period of $P_0 = -1/u = -444$ day. This period was chosen because the global phase trend

over the entire span of used observations was found to be equal to zero. The functions $E(t)$ and $p(t)$ obtained after removing phase jumps equal to $\pm\pi$ are shown in Fig. 3-4. As seen from Fig. 4 the phase linear trend was calculated for three sequential intervals of time bounded by MJD dates 46275, 51080, 51520, 54518 for the LSC model and 46275, 50844, 51402, 54518 for LAM. Two inner dates were obtained by the least squares method assuming that the total trend was uninterrupted.

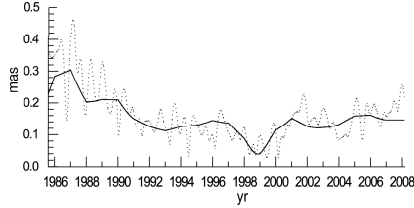


Figure 3. RFCN amplitudes derived from the LAM (solid line) and LSC (dotted line) models

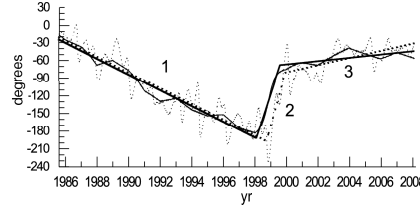


Figure 4. RFCN phase variations and their trends derived from the LAM (solid line) and LSC (dotted line) models

Let the observed phase be $p(t) = (du)t + dp(t)$ in degrees, where du is the factor for a linear trend [degree/year], and $dp(t)$ is a residual variation not containing a linear trend. Because the frequency of the envelope function is equal to $F(t) = -ut + p(t) = -ut + (du)t + dp(t) = -(u - du)t + dp(t)$, the new absolute value of the RFCN frequency will be $w = u - du$. The coefficients of the linear phase trend for the three mentioned above intervals of observation du_1, du_2, du_3 are obtained by the least squares method. Taking into account the relations $P = -1/w$, $\sigma_P = \sigma_w/w^2$, we receive the corresponding periods (Tabl. 2).

Table 2. Linear phase trend and periods of RFCN

Model	Par.	1	2	3	Dim.
LAM	du	-14.2 ± 0.1	$+80.5 \pm 0.2$	$+2.0 \pm 0.1$	degree
LSC		-14.2 ± 0.1	$+96.9 \pm 0.7$	$+5.2 \pm 0.1$	per year
LAM	P	-423.7 ± 0.1	-609.8 ± 0.3	-447.0 ± 0.1	solar
LSC		-421.4 ± 0.1	-614.7 ± 1.1	-447.1 ± 0.2	days

4. Conclusions

- The mean value of the RFCN period in the time span 1985.6–2008.2 is approximately equal to -444 solar days and is not constant.
- In the initial interval from 1985.6 until 1998.0, the period amounted to

about -422.5 days. Then, during 1998, it suddenly changed up to near -612 days, and after another year it again changed to -447 days.

- As is apparent from Fig. 1-3, the continuous process of RFCN practically ceased in 1999 and a sharp change in phase and period occurred subsequently.
- The first numerical RFCN model was constructed by T. Herring based on observational data up to 1998. Therefore the period that he derived (-430.2 days) is close to the one mentioned above for the same interval. S. Lambert did not reconsider T. Herring's RFCN period.
- If changes of the RFCN frequency are real, the resonance effects in terrestrial tides and lunar-solar nutation must be unstable, which has great significance for geodynamics. To check this effect it is necessary to construct a new transfer function for resonances, obtain a new decomposition for the lunar-solar nutation for the three mentioned above intervals of time and reprocess all VLBI observations. The new series of CIP coordinates should be more precise than the previous.

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