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Secular Decrease of the Earth's Ellipticity from the Analysis of VLBI Data of 1984–2006, and the Long-term Systematic Errors of the Precession-Nutation Models IAU 2000 and IAU 2006

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Abstract. At present, the series of the Celestial Pole offsets, provided by VLBI geodetic programs, become sensitive to the secular decrease $\dot{e}$ of the Earth's ellipticity e (the effect of the so-called post-glacial rebound). Indeed, the SLR-based value $\dot{e} = -7.9 \times 10^{-9}$/cy should manifest itself in the VLBI residuals of the precession angle ϕ as a quadratic trend -8 mas/cy^2 (the trend is negative for the adopted left-hand coordinate frame). For the 25-year time-span of the VLBI data, the effect reaches 0.5 mas, a value which is quite detectable. Unfortunately, this effect can not be studied making use of any of two standard IAU 2000/2006 models of the precession-nutation motion, as the present analysis has revealed significant positive quadratic trends of unknown origin in the residuals of the angle ϕ of 23 ± 4 mas/cy^2 for IAU 2000 and an even bigger 30 ± 4 mas/cy^2 for IAU 2006. As a result, processing the VLBI data with these theories leads to the large positive rate $\dot{e} = (27 \pm 4) \times 10^{-9}$/cy which is incompatible with the SLR value and physically meaningless. The IAU 2006 estimate is corrected for $\dot{e}$ value that has been already embedded into this model to account for the SLR results. On the other hand, applying the numerical theory ERA of the Earth's rotation [3, 4], the estimate $\dot{e} = -(14 \pm 4) \times 10^{-9}$/cy has been obtained, statistically in accordance with the SLR-based value of $\dot{e}$. For the increasing time-span T of observations, the error of the VLBI-derived rate $\dot{e}$ falls as $1/T^2$ while that of the SLR-based rate diminishes only as $1/T$; thus, the VLBI technology in the near future will provide the most accurate estimates for this important geodynamical parameter. For that, however, usage of an Earth's rotation theory more adequate than the adopted IAU 2006 is indispensable.

1. Introduction

In our papers [3] and [4], the experience of constructing the numerical theory ERA of the Earth's rotation, adequate in accuracy to the best VLBI-based series of the Earth's rotation parameters, is described. The version ERA-2005 was shown to provide a better fit to the VLBI data than the analytical theory IAU 2000 adopted as the international standard up to the year 2006, when it was replaced by the new IAU 2006 standard of the same level of accuracy. In [4], by comparing ERA-2005 with IAU 2000, it was predicted that since the year 2007 the IAU 2000 residuals of the precessional angle ϕ will not oscillate around zero level but will keep their sign, increasing the absolute values with T^2 for the time interval T. The sign of this trend is negative, but one should bear in mind that these results have been given for the right-hand reference frame, in accordance with the standard mathematical practice, while in astronomical work, a left-hand reference frame is commonly used to make the precession motion positive. That is the reason why the residuals $d\phi$ in [4], as well as in the present paper, are of the opposite sign to those calculated in most other astronomical studies. In the Abstract of the present paper, the numerical results are reformulated for the left-hand reference frame to avoid possible confusion.

The detected significant positive quadratic trend in the precession angle of IAU 2000 is rather surprising, because the adopted value of the secular decrease $\dot{e}$ of the ellipticity (given in IERS standard in the form of the secular rate of the coefficient of C_{20} of the geopotential) should manifest itself as a negative trend of about -8 mas/cy^2. As the model of the new IAU 2006 standard includes this correction, one should expect that the discrepancies of IAU 2006 with VLBI data will become even bigger.

In the present paper, the long-term systematic errors of the IAU 2000/2006 theories are studied by direct analysis of their residuals against VLBI data and making use of the numerical model ERA as a reference only as an additional control. The main difficulty of this approach is the unmodelled Free Core Nutation (FCN) oscillations of the period $T_{FCN} \approx 435$ days which conceal the systematic errors of the forced precession-nutation motion. As the 22-year time-span of the available VLBI data is long enough, the FCN oscillations may be filtered out by various methods making it possible to study the long-term behavior of the residuals. The striking feature of the filtered IAU 2000 and IAU 2006 residuals proves to be large quadratic trends in the precession angle ϕ. In the right-hand coordinate system used, they are -23 mas/cy^2 and -30 mas/cy^2 for IAU 2000 and IAU 2006, respectively.

For the larger time interval 1984–2019, a similar analysis of the differences between the current version ERA-2006 of the numerical theory ERA and IAU 2000/2006 has been carried out. Note that for constructing the version ERA-2005, the effect of the secular decrease of the ellipticity e was ignored. Taking into account this effect, the estimates of the quadratic trends in the precessional angle of IAU 2000/2006 theories derived directly from the VLBI data in the interval 1985–2006 and those derived from the differences between ERA-2006 and IAU 2000/2006 in the interval 1985–2019 became to be in a good accordance, while for the analogous differences of ERA-2005 with IAU 2000/2006 the deviations were two times bigger than the observed ones.

Table 1. Statistics of residuals $\sigma_{d\theta}$, $\sigma_{d\phi}$, mas

series	σ_θ	N_{used}	N_{del}	$\sin\theta\sigma_{d\phi}$	N_{used}	N_{del}
GSFC	0.130	3775	140	0.122	3790	125
USNO	0.130	3474	108	0.125	3470	112
BKG	0.135	3091	162	0.125	3086	167
IAA	0.136	3183	172	0.126	3189	166

2. Numerical Theory ERA-2006 of the Earth Rotation and Estimation of the Secular Decrease of the Earth's Ellipticity

Slightly improving the dynamical model of [3] and correcting minor programming deficiencies, somewhat better fitting to the VLBI data in comparison with ERA-2005 and significant improvement in comparing with the IAU 2000/2006 theories have been reached. In the ERA-2006 model, only three empiric parameters have been introduced: two corrections to the amplitudes of annual harmonics (formally, they are seasonal variations of the ellipticity e) and the parameter E_2 which has the meaning of a scale-factor for the harmonics brought about by dissipation in the fluid core (Sec. 3.4 of [4]). In contrast to ERA-2006, six empirical parameters have been estimated for ERA-2005, including four coefficients of the annual harmonics. The coefficients of four annual harmonics of ERA-2005 were strongly correlated, and reducing their total number only to two coefficients made the analysis of the VLBI data more robust, practically without any degradation of accuracy. Unfortunately, the estimated amplitudes of the seasonal oscillations of the ellipticity e exceed by an order of magnitude the analogous amplitudes reliably derived from LAGEOS data. Thus, they still should be related to as some empiric parameters of unclear physical meaning.

We have analyzed to what degree the choice of the series for the Celestial Pole offsets may affect the resulting numerical theory. For that, in addition to the series GSFC of the Goddard Space Flight Center, the following three alternative series were used: USNO (US Naval Observatory), BKG (Bundesamt für Kartographie und Geodäsie) and IAA (Institute of Applied Astronomy). For each constructed numerical theory, Tabl. 1 gives the total number of observations being used, the number of observations deleted (the standard 3σ criterion for outliers being used), and the weighted root mean square (wrms) errors for the interval January 1984 – March 2007. Tabl. 1 shows that the GSFC series, firstly, is most complete and, secondly, demonstrates the best accuracy. The wrms errors of the residuals in Tabl. 1 should be compared with those for IAU 2000: $\sigma_{d\theta} = 0.203$ mas and $\sin\theta\sigma_{d\phi} = 0.158$ mas (the same outliers being deleted).

For each of these series, the value $\dot{e}$ was estimated and compared (after

Table 2. Estimates of the Love number k_2 and the secular trend $\dot{c}_{20}$

Series	k_2	$\dot{c}_{20} \times 10^{-10}$
GSFC	0.27204(28)	21(6)
USNO	0.27197(30)	27(6)
BKG	0.27215(32)	27(7)
IAA	0.27288(32)	40(7)

conversion) with the adopted satellite-based secular rate $\dot{c}_{20} = 12 \times 10^{-10}$/cy of the normalized coefficient c_{20} of the geopotential [6]. The actually estimated parameter $\dot{e}/e$ was transformed into the corresponding value $\dot{c}_{20}$ making use of the relation $\dot{c}_{20}/c_{20} = -\dot{e}/e$. The derived estimates of $\dot{c}_{20}$ are given in the last column of Tabl. 2. All the values $\dot{c}_{20}$ in Tabl. 2 agree at the 3σ level with the adopted value $\dot{c}_{20}$ (for the IAA series, on the 4σ level).

Among other findings obtained with ERA-2006, it seems to be noteworthy that the new estimate of the Love number $k_2 = 0.27204(28)$ is compatible with the estimate $k_2 = 0.27272(36)$ of [4] based on the VLBI observations of 1984–2005. Shirai and Fukushima [7] obtained the estimate $k_2 = 0.2788(11)$ from earlier VLBI data of 1980–2000 making use of the original analytical theory of the precession-nutation motion. All the VLBI-based estimates of k_2 given above are in reasonably good agreement. Shirai and Fukushima [7] also presented a brief review of k_2 values obtained by other (not VLBI) methods since 1968. They are quite inaccurate, varying in the range from 0.252 to 0.343. Note that the value $k_2 = 0.29525$, adopted in the last version of the IERS standards, was obtained from purely theoretical considerations. It seems improbable that such calculations could ensure the correct five digits given by the IERS standards. Thus, one can see that the available VLBI-based Celestial Pole offsets provide an excellent opportunity of estimating k_2 with an accuracy unattainable by other methods. In [5] which presents the theoretical basis of the IAU 2000 nutation, VLBI data also have been analyzed but no explicit estimate of k_2 is given. However, if one derives k_2 from the published value of the so-called compliance κ making use of the standard relation $\kappa = ek_2/k_s$ ($k_s = 0.93831$ is the secular Love number), the derived value $k_2 = 0.2953(22)$ strongly disagrees with other VLBI-based results. It practically coincides with the a priori value $k_2 = 0.29525$ confirming the guess of [3] that the IAU 2000 model is of semi-empirical nature being based on the Fourier analysis of the residuals.

In the next section, four methods of studying the long-term systematic errors of the IAU 2000/2006 and ERA-2006 theories from the analysis of the VLBI data are described. In good accordance, the results demonstrate significant quadratic trends in the VLBI residuals ($d\phi \approx -23$ mas/cy^2 for IAU 2000 and $d\phi \approx -30$ mas/cy^2 for IAU 2006) and the absence of such a trend in ERA-2006.

3. Long-Term Systematic Errors of IAU 2000 and IAU 2006 from the Analysis of VLBI Data

3.1. Determination of the Rate $\dot{e}$ Making Use of the IAU 2000 and IAU 2006 Theories

Firstly, the rate $\dot{e}$ was estimated for the IAU 2000/2006 theories using the partials that had been calculated for ERA-2006 in the process of the numerical integration by varying the parameters under estimation. The physical meaning of these parameters is explained in [4]. It is important that they include the initial values of the angular velocity of the fluid core as a whole and those of its inner part. As it is shown in [4], these parameters make it possible to model satisfactorily the complex two-modal structure of the FCN oscillations in the time interval of the available VLBI data. Such approach proved to be workable also for the analytical theories IAU 2000/2006 significantly reducing the amplitudes of the FCN oscillations in the post-fit residuals. As a result, the wrms errors of the post-fit residuals are drastically improved: $\sigma_{d\theta} = 0.136$ mas and $\sin\theta\sigma_{d\phi} = 0.134$ mas while for the original pre-fit residuals of IAU 2006 they were: $\sigma_{d\theta} = 0.203$ mas and $\sin\theta\sigma_{d\phi} = 0.158$ mas. (The estimated correction $\dot{e}$ for IAU 2006 has been added to the theoretical value $\dot{e} = -7.9 \times 10^{-9}$/cy used in this model; however, when deriving the apparent secular trend in IAU 2006, this reduction should not be applied). The corresponding quadratic trends $d\phi$ in IAU 2000/2006 are as follows:

$$d\phi = \begin{cases} -23 \pm 3 \text{ mas/cy}^2 \text{ for IAU 2000,} \\ -30 \pm 3 \text{ mas/cy}^2 \text{ for IAU 2006.} \end{cases} \qquad (1)$$

3.2. Analysis of the Averaged VLBI Residuals of IAU 2000 and IAU 2006 Theories

In a second approach, the averaged values of the IAU 2000/2006 and ERA-2006 residuals were obtained by filtering out the FCN oscillations, as well as the oscillations of higher frequencies, by linearly fitting the raw residuals for each of the adjoining 435-day intervals of the FCN oscillations. In this fit, the weights for the residuals were assigned in accordance with their a priori errors. For more reliability, this procedure was repeated twice shifting the initial date of the averaging by half a period of the FCN oscillations. For a better filtering of these oscillations, we restricted the time interval by the maximally possible number of the complete FCN periods. As a result, 38 points were obtained for each of the two Euler angles of the three theories under consideration (IAU 2000, IAU 2006 and ERA-2006; see Fig. 1 where the top and bottom plots present the residuals $d\phi$ and $d\theta$, respectively). The most salient feature of the $d\phi$ plot is the pronounced parabolic trend in both the IAU 2000 and IAU 2006 theories, and the absence of a similar trend in the ERA-2006 residuals. To fit these data, we use the model

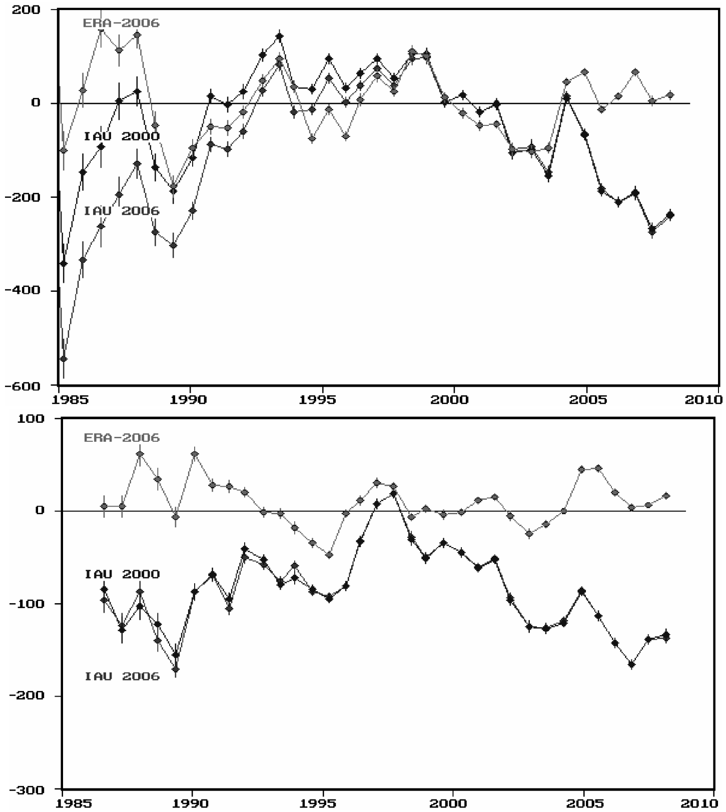

Figure 1. Averaged residuals of IAU 2000/2006 and ERA-2006 (in μas) $d\phi$ (top) and $d\theta$ (bottom)

$$d\phi = A_0 + A_1 T + A_2 T^2 + A_{\cos}\cos f + A_{\sin}\sin f, \qquad (2)$$

$$d\theta = B_0 + B_1 T + B_2 T^2 + B_{\cos}\cos f + B_{\sin}\sin f, \qquad (3)$$

where T is counted from the epoch JD2000 in centuries, f is the fundamental argument of the main 18.6 year harmonics of the nutational theory.

As the coefficients $A_{\sin}$ and $A_{\cos}$ appeared to be within their statistical errors, they were excluded from the final solution for which the quadratic errors of IAU 2000/2006 models have been obtained as follows:

$$d\phi = \begin{cases} -25 \pm 3 \text{ mas/cy}^2 \text{ for IAU 2000,} \\ -33 \pm 3 \text{ mas/cy}^2 \text{ for IAU 2006.} \end{cases} \qquad (4)$$

For ERA-2006, the coefficient A_2 is within its statistical error and thus negligible. Note that estimates (1) and (4) are in a good agreement notwithstanding that they have been obtained in quite different ways.

Viewing the averaged residuals of $d\theta$ of IAU 2000/2006 in Fig. 1 (the bottom plot), one might suggest that they also demonstrate significant quadratic trends. And indeed, if the terms $B_{\cos}$, $B_{\sin}$ are ignored, the coefficient B_2 of the quadratic approximation becomes rather large. However, the general five-parametric solution gives a negligible quadratic trend B_2 but rather large values for the shift B_0 and the cosine amplitudes $B_{\cos}$. The most reliable estimates correspond to the solution in which the quadratic term B_2 is set equal to zero. The coefficients B_0 and $B_{\cos}$ for this solution are consistent with those for the five-parametric solution 2 and have twofold lesser error (due to the absence of any strong correlations).

For a control, we also tried the version in which the constant shift and the amplitudes of the sine and cosine harmonics of the FCN oscillations were estimated for each of the adjoining 435-day intervals (repeating the process by shifting these intervals on the half of the FCN period). After that, the obtained estimates of the shifts were fitted by the models (2), (3). The results appeared to be in accordance with the estimates given above. In particular, for the quadratic term in precession angle, the following estimates were obtained:

$$d\phi = \begin{cases} -22 \pm 3 \text{ mas/cy}^2 \text{ for IAU 2000,} \\ -30 \pm 3 \text{ mas/cy}^2 \text{ for IAU 2006,} \end{cases} \tag{5}$$

which are practically coincide with estimates (1).

3.3. Analysis of the Differences Between ERA-2006 and the Theories IAU 2000 and IAU 2006 in the Interval 1985–2020

In this method, the differences of the theories ERA-2006 and IAU 2000/2006 were calculated in the larger time interval 1985–2019 and studied after averaging them in the same way as described in Section 3.2. For this interval, we have obtained 60 averaged points for each of two Euler's angles. In Fig. 2, the top plot presents the differences of the angles ϕ, the bottom one those for the angle θ (note the different scales in the plots). One can see that the main peculiarity of the ϕ-differences is again the quadratic trend. After fitting the model (2) to the ϕ-differences, the following estimates were obtained for the quadratic terms:

$$d\phi = \begin{cases} -25.5 \pm 0.7 \text{ mas/cy}^2 \text{ for IAU 2000,} \\ -31.9 \pm 0.7 \text{ mas/cy}^2 \text{ for IAU 2006.} \end{cases} \tag{6}$$

These quadratic trends should be attributed to the errors of IAU 2000/2006 but not of ERA-2006.

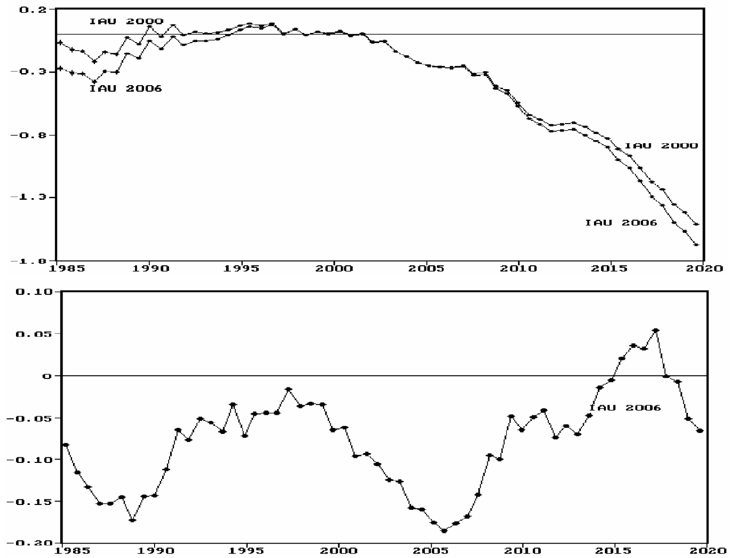

Figure 2. Averaged differences ERA-2006 minus IAU 2000, and ERA-2006 minus IAU 2006 (in mas) $d\phi$ (top) and $d\theta$ (bottom)

The main feature of the θ-differences between ERA-2006 and IAU 2000/2006 is the constant shift $B_0 = -0.096(4)$ mas and the cosine amplitude $B_{\cos} = -0.066(5)$ of the main nutation term of the period 18.6 years. These values are consistent with the estimates $B_0 = -0.090(5)$ mas, $B_{\cos} = -0.048(7)$ derived directly from the analysis of the VLBI residuals and thus it is clear that they are the errors of the IAU 2000/2006 theories.

In Fig. 3, the original (non-averaged) differences in the Euler's angles $d\theta$, $d\phi$ (in the sense ERA-2006 minus IAU 2000) are presented for the time span 1985–2019 (the solid lines). The FCN oscillations seen in Fig. 3 have arisen because the model of these oscillations is embedded automatically into the numerical theory ERA-2006 while no such model is available in this time interval for the IAU 2000/2006 theories. On these plots, the residuals of IAU 2000 against the VLBI data (supplied with the bars of the a priori errors) are also presented. The black solid line should be considered as a prognosis of the deviations of the IAU 2000 model from the future VLBI data up to the year 2020. It is predicted that the deviation of the precession angle of IAU 2000/2006 in 2020 will exceed 2 mas.

4. Concluding Notes

From the above considerations, the following conclusions may be derived:

1. The available VLBI data are accurate enough for detecting the secular decrease $\dot{e}$ of the ellipticity e. Applying the numerical theory ERA-2006, the

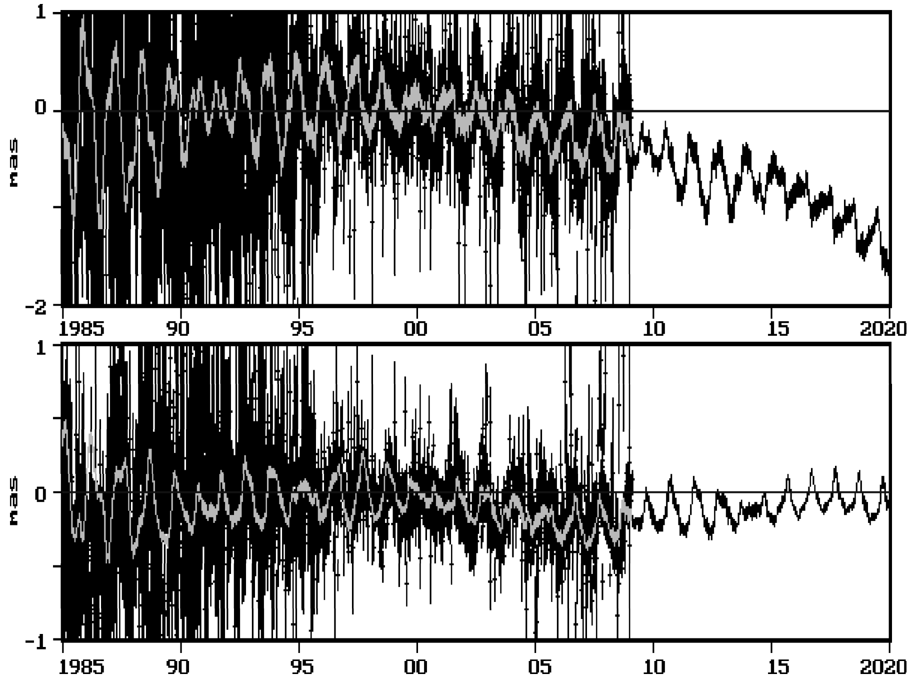

Figure 3. Prognosis of IAU 2000 residuals (in mas): $d\phi$ (top) and $d\theta$ (bottom)

estimated $\dot{e}$ proves to be in accordance with the results of satellite tracking. As the error of the VLBI-based $\dot{e}$ decreases with T^{-2} with increasing time interval T, while the error of the SLR estimates diminishes only with T^{-1}, the VLBI technology will provide the most accurate estimates for this important geodynamic characteristic in the near future.

2. The conclusion of [4] about the fast increasing systematic error of the IAU 2000 precession angle is confirmed by the direct analysis of the VLBI residuals of IAU 2000, independently of the validity of the ERA numerical theories. It is clear now why the larger value $d\phi \approx -40$ mas/cy^2 (instead of $d\phi \approx -22$ mas/cy^2 of the present work) has been obtained for the quadratic trend $d\phi$ from the analysis of the differences ERA-2005 minus IAU 2000 in [4]. The reason is the disregard of $\dot{e}$ in the ERA-2005 model.

3. The large and steadily increasing systematic error $d\phi = -22 \pm 3$ mas/cy^2 of IAU 2000 and the even bigger $d\phi = -30 \pm 3$ mas/cy^2 of IAU 2006 may be easily corrected introducing these quadratic terms into the precession formalism of IAU 2000 or IAU 2006. That is an acceptable approach for practical applications but not for the scientific study of geophysical effects in the Earth's rotation. Unfortunately, this empiric term would not be unique in the IAU 2000/2006 theories; see Introduction to [3] for more details. In particular, these

theories ignore the torques caused by the tidal friction, as well as by the tides in the fluid core, instead making use of the semi-empiric approach and including the large empirical secular trend -25.24 mas/cy into the obliquity motion. In [8] it is shown that this trend has arisen mainly (but not completely) due to ignoring some indirect lunar perturbations in the rigid body part of nutation. An introduction of this empirical trend makes IAU 2006 unsuitable for the study of the geophysical nature of dissipation which brings about the secular obliquity rate.

The IAU 2000/2006 errors in precession motion are too small to be detected by the classic optical observations of the previous two centuries. Because the errors increase with T^2, the earlier VLBI observations also were insensitive to this effect, and only at present, after a 25-year time-span for the geodetic VLBI observations, the effect became quite noticeable.

4. Our analysis of the systematic errors of IAU 2006 has demonstrated that the adoption of the P03 precession as the international standard IAU 2006 at the IAU General Assembly (Prague, 2006) was a false step, because the detected P03 errors substantially exceed those of the previous standard IAU 2000.

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