The 5th IVS General Meeting Proceedings, 2008, p. 427–432

Vienna VLBI2010 PPP Simulations

Andrea Pany [1,2], *Jörg Wresnik* [1], *Johannes Böhm* [1], *Harald Schuh* [1]

[1] *Institute of Geodesy and Geophysics, Vienna University of Technology, Austria*

[2] *Recipient of a DOC-fFORTE fellowship of the Austrian Academy of Sciences, Austria*

Abstract. Within the frame of Vienna's contribution to VLBI2010, a precise point positioning (PPP) simulation software has been developed. Similar to the simulation studies carried out with OCCAM and Calc/Solve, an artificial group delay time series is generated, consisting of wet slant delays provided by a turbulence model, stochastic variations of station clock and thermal noise of the receiving system. In contrast to the simulations that are based on VLBI analysis software packages, the delay time series are generated for single stations. The group delays are used as observations to perform a PPP - with the classical Gauss-Markov approach as well as with Kalman filtering. Different parameterizations of zenith wet delay, i.e. gradients and spherical harmonics, are tested using different time intervals and constraints. Performing a PPP for all stations of a VLBI schedule allows comparisons to the results of OCCAM and Calc/Solve simulations in order to validate how close the results of PPP are to what can be expected from real VLBI data.

1. Introduction

With the establishment of the Global Geodetic Observation System (GGOS) in 2006, it has become clear that VLBI, being the only space geodetic technique that is linking terrestrial and celestial reference frame, should be able to provide sub mm accuracy on global baselines in near future. Based on the recommendations and suggestions of Working Group 3 (WG3) [5], which was entrusted with the development of strategies to reach this high level of accuracy, Monte Carlo simulations are carried out within the IVS. Main goal of these simulation studies is to investigate different scheduling strategies, antenna slew speeds and the impact of network size, as well as to improve analysis strategies. Currently, the results of three Monte Carlo simulators are contributing to VLBI2010. In addition to Monte Carlo simulators based on the VLBI analysis software packages Calc/Solve and OCCAM [8], a VLBI2010 Precise Point Positioning (PPP) Simulator has been developed. Heart of the Monte Carlo PPP Simulator is a fake delay generator that produces artificial group delay observables

$$delay_{group} = mf_w(el) \cdot zwd + clk + wn. \tag{1}$$

In (1), zwd denotes tropospheric zenith wet delay, mf_w wet mapping function, el elevation, clk clock, and wn white noise. The elevation angles are taken from artificial VLBI2010 schedules which were generated either by John Gipson (NVI/Inc., NASA GSFC, USA) or by Anthony Searle (Natural Research Council, NRC, Canada). The generation of artificial zenith wet delays is based on turbulence theory [7], following the approach proposed by Nilsson et al. [6]. The parameters driving the turbulence, i.e. wind velocity and wind direction, the refractive index structure constant C_n, and the effective height H of wet troposphere, are derived from numerical weather models or from empirical fits to radiosonde data, respectively, and are provided by Tobias Nilsson from Onsala Space Observatory, Sweden. Variations of station clock are simulated as sum of random walk and integrated random walk stochastic processes [8], typically with power spectral densities corresponding to Allan standard deviations (ASD) of 10^{-14} at 50 min, 10^{-14} at 15 min, or $2 \cdot 10^{-15}$ at 15 min. The white noise accounts for the thermal noise of the receiving system. For the PPP simulations, a white noise of $4/\sqrt{2}$ ps per station is added. The artificial group delay observables are used as observations to perform a PPP, either with the classical Gauss-Markov approach or a Kalman filter. The parameters to be estimated are troposphere parameters, clock parameters and station coordinate residuals. Goal of the PPP simulations is to re-gain the single contributions of troposphere and clock to the cumulative group delay. Troposphere, clock and the vertical component of station position are highly correlated. The separation of these parameters is possible because of their distinct dependencies on elevation angle or zenith distance, respectively. The whole procedure (simulation of group delay observables, estimation of parameters) is repeated 25 times to obtain a sample of output parameters that can be analyzed statistically.

2. The PPP Simulator

2.1. PPP with Kalman Filtering

The PPP Simulator is based on a covariance propagating Kalman filter, as described thoroughly by e.g. Gelb [2]. Tropospheric wet delays are modeled as zenith wet delay (mapped with a mapping function) with superimposed gradients, all of which are treated as random walk stochastic processes, using variance rates of 0.7 ps^2/s for zenith wet delays and 0.5 ps^2/s for gradients. This corresponds to the parametrization that is used in the OCCAM Kalman Filter for VLBI2010 simulation studies [8]. Clocks are modeled with a rate (deterministic parameter), and a random walk offset (variance rate 1 ps^2/s). Station coordinates are estimated as deterministic parameters.

When using a Kalman filter, only those observations between the first time epoch and a specific time epoch t_k are included in the filter estimate at t_k.

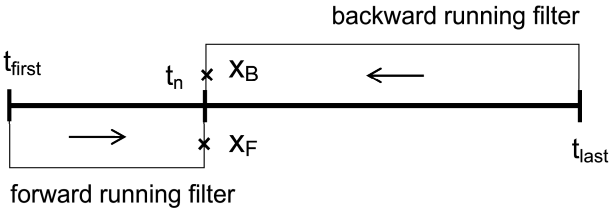

Figure 1. Principle of smoothing [2]

For the estimation of stochastic parameters, all available information should be included. Herring et al. [3] thus proposed to use a smoothing algorithm. A smoother is a combination of two Kalman filters that run in opposite direction (Fig. 1). The forward running filter (FRF) starts at the first time epoch, the backward running filter (BRF) at the last time epoch. At time epoch t_k, both filters together include all available observations. The smoothed estimate x_S is computed as weighted average of the FRF and BRF estimates (x_F and x_B) at each time epoch:

$$x_S = x_F + P_F(P_F + P_B)^{-1}(x_B - x_F)^{-1}, \tag{2}$$

where P_F and P_B are the covariance estimates of FRF and BRF, respectively. The final solution for deterministic parameters, i.e. clock rate and station position, is taken from the final FRF estimate, whereas stochastic parameters, i.e. zenith wet delays, gradients, and clock offset are taken from the smoothed solution.

2.2. PPP with the Classical Gauss-Markov Model

The stochastic model of the classical Gauss-Markov method is widely known and shall not be discussed in detail here. As for the Kalman filter solution, tropospheric wet delays are modeled as zenith wet delay (mapped with a mapping function) with superimposed gradients, which are all parameterized as piecewise linear functions with rather short estimation intervals (e.g. 6 min). The rates of the piecewise linear functions are constrained to zero by introducing pseudo-observations with standard deviations of 15 mm/h for zenith wet delays and 48 mm/h for gradients. Clocks are modeled as quadratic polynomial, estimated once per 24 hours, with superimposed piecewise linear functions (1 hour intervals, constrained with 54 mm/h). Station coordinates are estimated once per 24 hour session.

2.3. Validation of the PPP Simulator

For validation, a comparison to OCCAM Kalman Filter results was carried out. Since OCCAM is evaluating baselines, it has not only to separate the contributions of troposphere and clock, but also the contributions of the two stations forming the baseline. The PPP results might thus be too optimistic.

Another difference is that the stochastic model of the OCCAM Kalman Filter is more sophisticated than the stochastic model of the PPP Simulator. E.g., the OCCAM Kalman Filter performs a reweighting of observations when the difference between prediction and observation is too large. This is not the case for the PPP software, since it is only used for simulation studies, where no outliers will occur unless simulated. In order to carry out a valid comparison, the simulations of group delay observables were performed with the same turbulent zenith wet delay and clock time series. For each 24 hour session and station, the 3D position error was computed. The rms of 3D position errors over 25 sessions were computed for each station and are plotted in Fig. 2, which shows OCCAM results in black and PPP results in grey. As expected, the PPP solution is a little too optimistic, but, regardless of that, the same conclusions can be drawn from the results.

3. Investigations and Results

The PPP is, in case of VLBI, a purely theoretical approach. Nevertheless, due to the simplicity and compactness of the simulator, it is a very powerful tool that helps understanding the highly correlated main stochastic error sources — troposphere and clock. Since the input to the PPP is exactly known, a direct comparison of input and output is possible as well as investigations on how errors in troposphere and clock parameter estimation impact on the determination of geodetic parameters such as station position. The PPP Simulator was used to carry out studies on the length of estimation intervals for the piecewise linear functions, constraints and variance rates, different troposphere models (i.e. gradients and spherical harmonics), the impact of different clocks and white noises, elevation dependent downweighting and the influence of the parameters driving the turbulence on position repeatabilities. As an example, the investigation on modeling tropospheric wet delay with spherical harmonics will be discussed in more detail below.

The functional form used for modeling tropospheric wet delays is based on the approach by Böhm and Schuh [1]:

$$\Delta L(el, az) = mf(el) \cdot L^z + mf(el) \cdot \mathrm{ctg}(el) \cdot SH_{nm}(zd, az). \tag{3}$$

In (3), ΔL denotes tropospheric wet delay, el elevation angle, az azimuth, mf mapping function, L^z zenith wet delay, SH_{nm} spherical harmonics of degree n and order m and zd zenith distance. The spherical harmonics are scaled with $ctg(el)$ to account for the increasing horizontal change in refractive index along the ray path when the zenith distance increases [1]. Several spherical harmonics expansions of different degrees and orders were tested on their ability to model the fast variations of turbulent zenith wet delay on short time scales. For the PPP Kalman Filter, spherical harmonics of degree and order 1 have been found to yield best results. In this case,

$$SH_{nm}(zd, az) = (-\sin(zd)) \cdot [a_{11} \cdot \cos(az) + b_{11} \cdot \sin(az)], \tag{4}$$

where a_{11} and b_{11} are the coefficients of SH_{11}. The classical gradient model as recommended by the IERS Conventions [4] is given as

$$\Delta L(el, az) = mf(el) \cdot L^z + mf(el) \cdot \text{ctg}(el) \cdot [G_N \cdot \cos(az) + G_E \cdot \sin(az)], \quad (5)$$

where G_N and G_E denote north and east gradient, respectively. The troposphere parameters to be estimated are L^z, a_{11} and b_{11} for the spherical harmonic model and L^z, G_N and G_E for the classical gradient model, the number of troposphere parameters thus being the same in both cases. Substituting (4) in (3) and comparing with (5) it can be seen that the only difference between these two approaches is their different dependence on zenith distance.

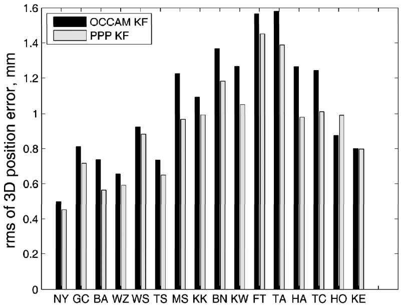

Figure 2. Comparison of rms of 3D position error of OCCAM (black) and PPP (grey) Kalman Filter for the VLBI2010 schedule st16uni_30_6_230. The clock was simulated with an ASD of 10^{-14} at 50 min. Variance rates: 0.7 ps^2/s for zenith wet delay, 0.5 ps^2/s for gradients, 1 ps^2/s for clock offset

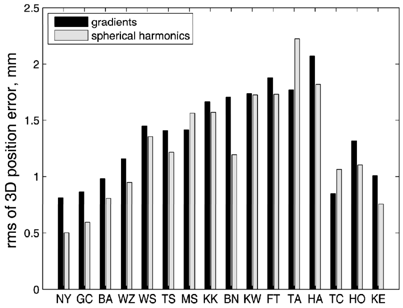

Figure 3. Comparison of rms of 3D position error for the PPP Kalman Filter for the VLBI2010 schedule st16uni_60_12_230. The clock was simulated with an ASD of 10^{-14} at 50 min. Variance rates: 0.7 ps^2/s for zenith wet delay, 0.5 ps^2/s for gradients, 0.01 ps^2/s for spherical harmonics and 1 ps^2/s for clock offset

Fig. 3 presents rms of 3D position error for a VLBI2010 16 station test schedule and shows a clear improvement for almost all stations if using spherical harmonics instead of the gradient model. The significant degradation for station Tahiti (TA) was shown to be due to the specific sequence of random noise used for the generation of the turbulent zenith wet delay time series.

4. Summary and Conclusions

Though a theoretical approach, the PPP Simulator was found to be very helpful for VLBI2010 simulation studies. Of course one has to bear in mind that results obtained with PPP might be too optimistic and have thus to be interpreted with care. Findings concerning improvement of analysis strategies have to be confirmed with VLBI analysis software packages. Nevertheless, the PPP Simulator has numerous advantages that make it a powerful tool: Based on MATLAB script files, it is compact and clear, making it easy to implement modifications. Analyzing data for a couple of single stations instead of station networks, makes it possible to test many different strategies in very short time. Promising strategies can then be applied to the analysis of all stations in a VLBI2010 network. Furthermore, since the contributions of zenith wet delay and station clock to the artificial group delay observables are known, Monte Carlo simulations of this kind, and — due to their simplicity — especially the PPP simulations, enable a basic understanding of how troposphere and clock parameters are correlated and how errors in their determination affect the estimation of geodetic parameters.

References

[1] Böhm, J., H. Schuh. Spherical Harmonics as a Supplement to Global Tropospheric Mapping Functions and Horizontal Gradients. Proceedings of the 15-th European VLBI for Geodesy and Astrometry Working Meeting. D. Behrend and A. Rius (eds.), 2001;
http://www.ieec.fcr.es/hosted/15wmevga/proceedings/boehm/boehm.html.

[2] Gelb, A. Applied Optimal Estimation. MIT Press, Cambridge Mass., 1974.

[3] Herring, T., I. Shapiro. Geodesy by Radio Interferometry: The Application of Kalman Filtering to the Analysis of Very Long Baseline Interferometry Data. J. Geophys. Res., v. 95, No B8, 12561–12581, 1990.

[4] McCarthy, D., G. Petit. IERS Conventions 2003. Verlag des Bundesamtes für Kartographie und Geodäsie, Frankfurt am Main, 2004.

[5] Niell, A., A. Whitney, B. Petrachenko, et al. VLBI2010: Current and Future Requirements for Geodetic VLBI Systems. IVS Memorandum 2006-008v01, 2004.

[6] Nilsson, T., R. Haas, G. Elgered. Simulations of atmospheric path delays using turbulence models. Proceedings of the 18-th European VLBI for Geodesy and Astrometry Working Meeting. J. Böhm, A. Pany, H. Schuh (eds.), 2007;
http://mars.hg.tuwien.ac.at/evga/proceedings/S64_Nilsson.pdf.

[7] Treuhaft, R., G. Lanyi. The effect of the dynamic wet troposphere on radio interferometric measurements. Radio Science, v. 22, No 2, 251–265, 1987.

[8] Wresnik, J., J. Böhm, A. Pany, et al. Towards a new VLBI system for geodesy and astrometry. Adv. Geosci., AOGS 2007, 2007 (accepted by the editor).