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On the Space VLBI Mathematical Model with Nutation Parameters

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Abstract. Space Very Long Baseline Interferometry (SVLBI) is an important space technology for geodesy and geodynamics to a very high accuracy level. At present, the SVLBI mathematical model of geodetic parameters does not consider nutation parameters. In this paper the nutation parameters will be added to the rotation matrix, and the mathematical model of SVLBI observations with the nutation parameters will be derived. Finally, the estimability of parameters in the mathematical model will be analyzed.

1. Introduction

The most important and complicated mission of modern geodesy and geodynamics is definition, realization and interconnection of different reference systems, which include the Conventional Inertial System (CIS) fixed in space and defined by radio sources, the Conventional Terrestrial System (CTS) fixed to the earth and defined by a series of observation stations on the ground, and the Dynamic Reference System defined by the movement of the satellite. At present, Space VLBI (SVLBI) is the unique space technology that can directly connect the three reference frames with high accuracy. SVLBI is often used for other applications in geodesy and geodynamics like determination of geocentric positions of VLBI stations, estimation of the gravity field of the earth, satellite orbit determination.

There have been some studies [1, 4] on the mathematical model to estimate the parameters that can interconnect the reference frames by using SVLBI observations, but in this mathematical model nutation correction parameters ($\Delta\psi$ and $\Delta\varepsilon$) are not included. In the model there is a rotation matrix which includes Greenwich apparent time correction SG and polar motion corrections ξ,η, which are called earth rotation parameter corrections, ERP's. By using the rotation matrix, the radio source's CIS coordinates can be transformed to CTS. Anyway there are no radio source's true celestial coordinates in actual observations. There are just $J2000.0$ CIS coordinates of the radio source. Nutation parameters are needed to transform CIS to true celestial coordinates.

Along with the development of technology, the long life of deep space exploration satellite makes it possible to observe nutation parameters. The aim of this work is to add nutation parameters to the mathematical model of SVLBI observations. In particular the rotation matrix will be written with nutation correction parameters and the SVLBI observations mathematical model with the nutation correction parameter will be derived. Then, the estimability of parameters in the mathematical model will be analyzed.

2. Mathematical Model

2.1. Rotation Matrix

Actually observation stations are fixed in CTS while the observing radio source is fixed in $J2000.0$ CIS. The satellite can be seen to have true celestial coordinates. To establish the right time delay observation mathematical model it is necessary to transform these different reference systems to the same one. This transformation includes ERP's, nutation and precession parameters [5].

The rotation matrix with ERP corrections has been derived, the precession parameters can be calculated and in this research are considered as constant because the precession cycle is too long to be observed by current SVLBI technology. So the final model just needs to add nutation corrections.

The $J2000.0$ CIS can be transformed to average celestial coordinates at epoch t by precession correction, which can be obtained by astrophysical methods. Then the true celestial coordinates at epoch t can be obtained by nutation correction.

The rigid nutation matrix is expressed as

$$N = R_X(-\varepsilon - \Delta\varepsilon) \cdot R_Z(-\Delta\psi) \cdot R_X(\varepsilon).$$

The elements of the nutation matrix are functions of ε, the mean obliquity of the ecliptic, that can be obtained by astronomic methods and will be considered constant during the derivation, $\Delta\psi$ and $\Delta\varepsilon$ that are respectively the nutation in longitude and nutation in obliquity corrections. The transformation of the $J2000.0$ average position to true position at any epoch can be performed using the nutation rotation matrix. The objective of this paper is to obtain the SVLBI observation mathematical model with nutation correction in longitude and nutation correction in obliquity.

2.2. Mathematical Model of Time Delay Observation

Starting from the mathematical models as found in literature [1, 4], a simple ground-space time delay observation equation with nutation correction, can be expressed as:

$$d^I_{ijkl} = -\left\{ \begin{bmatrix} X_j \\ Y_j \\ Z_j \end{bmatrix}^T R_2(-\xi) R_2(-\eta) R_2(\theta_k) - \begin{bmatrix} X^I_K \\ Y^I_K \\ Z^I_K \end{bmatrix}^T \right\} \times \\ \times \left\{ N \begin{bmatrix} \cos\delta_l \cos\alpha_l \\ \cos\alpha_l \sin\alpha_l \\ \sin\alpha_l \end{bmatrix} \right\} + c[\Delta C^I_{0rj} - \Delta C^I_{1rj}(t_k - t_0)]\,, \tag{1}$$

where X_j, Y_j and Z_j are the CTS coordinates of observation station; X^I_K, Y^I_K and Z^I_K are true celestial coordinates of satellite S^I at epoch t_K; α_l and δ_l are $J2000.0$ CIS right ascension and declination of the radio source, which include precession correction, at epoch t_K; N is the nutation matrix; R is the rotation matrix with ERP's parameters; c is the velocity of light; t_0 is the beginning observing epoch; ΔC^I_{0rj} and ΔC^I_{1rj} are the clock error and clock drift of the reference clock in monitor station and clock P_j at observation station.

The observation station's CTS coordinates and radio source's average celestial coordinates can be transformed to true celestial coordinates by using the equation (1). It is the basic of the derivation of mathematical models.

The expression can be rewritten as the following:

$$d^I_{ijkl} = -X_j A1 + Y_j A2 + Z_j A3 + X^I_K A4 + Y^I_K A5 + Z^I_K A6 + \\ + c[\Delta C^I_{0rj} - \Delta C^I_{1rj}(t_k - t_0)]\,, \tag{2}$$

where $A1$ to $A6$ are the expanding coefficients of ground observations station's coordinates and satellite's coordinates. The following mathematical approximations will be used in the derivation:

$$\cos(\varepsilon + \Delta\varepsilon) \cong \cos\varepsilon \cos\Delta\varepsilon - \sin\Delta\varepsilon \sin\varepsilon,$$
$$\sin(\varepsilon + \Delta\varepsilon) \cong \sin\varepsilon \cos\Delta\varepsilon + \sin\Delta\varepsilon \cos\varepsilon.$$

And the following approximate transformations will also be used because ξ, η, $\Delta\psi$ and $\Delta\varepsilon$ are small:

$$\begin{matrix} \sin\xi \cong \xi, \sin\eta \cong \eta \\ \cos\xi \cong \cos\eta \cong 1 \end{matrix}\,,\ \begin{matrix} \cos(\Delta\psi) \cong 1 \\ \sin(\Delta\psi) \cong \Delta\psi \end{matrix}\,,\ \begin{matrix} \cos(\Delta\varepsilon) \cong 1 \\ \sin(\Delta\varepsilon) \cong \Delta\varepsilon \end{matrix}\,,\ \begin{matrix} \Delta\psi\Delta\psi \cong 0 \\ \Delta\psi\Delta\varepsilon \cong 0 \\ \Delta\varepsilon\Delta\varepsilon \cong 0 \end{matrix}\,,\ \begin{matrix} \xi\eta \cong 0 \\ \xi\xi \cong 0 \\ \eta\eta \cong 0 \end{matrix}\,.$$

So the forms of Ai are:

$$A1 = -\Delta\psi \cos\varepsilon \cos\delta_l \sin(\alpha_l - \theta_k) + \cos\delta_l \cos(\alpha_l - \theta_k) \\ - \Delta\psi \sin\varepsilon \sin\delta_l \cos\theta_k - \Delta\varepsilon \sin\delta_l \sin\theta_k + \xi \sin\delta_l\,,$$
$$A2 = \Delta\psi \cos\varepsilon \cos\delta_l \cos(\alpha_l - \theta_k) + \cos\delta_l \sin(\alpha_l - \theta_k) \\ + \Delta\psi \sin\varepsilon \sin\delta_l \sin\theta_k - \Delta\varepsilon \sin\delta_l \cos\theta_k - \eta \sin\delta_l\,,$$

$$
\begin{aligned}
A3 = & -\xi \cos\delta_l \cos(\alpha_l - \theta_k) + \eta \cos\delta_l \sin(\alpha_l - \theta_k) + \\
& \cos\delta_l (\Delta\psi \cos\alpha_l \sin\varepsilon + \Delta\varepsilon \sin\alpha_l) + \sin\delta_l \,, \\
A4 = & \cos\delta_l (\cos\alpha_l - \Delta\psi \cos\varepsilon \sin\alpha_l) - \Delta\psi \sin\varepsilon \sin\delta_l \,, \\
A5 = & \Delta\psi \cos\varepsilon \cos\alpha_l \cos\delta_l - \Delta\varepsilon \sin\alpha_l + \sin\alpha_l \cos\delta_l \,, \\
A6 = & \Delta\psi \sin\varepsilon \cos\alpha_l \cos\delta_l + \Delta\varepsilon \sin\alpha_l \cos\delta_l + \sin\alpha_l \,.
\end{aligned}
$$

The unknown geodetic parameters will be:
$\{X_j, Y_j, Z_j, X_K^I, Y_K^I, Z_K^I, \Delta\psi, \Delta\varepsilon, \xi, \eta, \theta_k, \alpha_l, \delta_l, \Delta C_{0ij}, \Delta C_{1ij}\}$

2.3. Analysis of Relation Between Parameters

To estimate the unknown parameters of equation (2), this has to be linearized:

$$
d(d_{ijkl}^I) = \sum_P C_P dP \,,
$$

where C_P are the required partial derivatives from of the time delay with respect to the parameters of interest, indexed by P. They are as follows:

$$
C_{X_j} = A1, \quad C_{Y_j} = A2, \quad C_{Z_j} = A3, \quad C_{X_K^I} = A4, \quad C_{Y_K^I} = A5, \quad C_{Z_K^I} = A6,
$$

$$
\begin{aligned}
C_{\Delta\psi} = & X_j(-\cos\varepsilon \cos\delta_l \sin(\alpha_l - \theta_k) - \sin\varepsilon \sin\delta_l \cos\theta_k) + \\
& + Y_j(\cos\varepsilon \cos\delta_l \cos(\alpha_l - \theta_k) + \sin\varepsilon \sin\delta_l \sin\theta_k) + Z_j \cos\delta_l \cos\alpha_l \sin\varepsilon + \\
& + X_K^I(-\cos\varepsilon \sin\alpha_l \cos\delta_l - \sin\varepsilon \sin\delta_l) + Y_K^I \cos\varepsilon \cos\alpha_l \cos\delta_l + \\
& + Z_K^I \sin\varepsilon \cos\alpha_l \cos\delta_l \,,
\end{aligned}
$$

$$
\begin{aligned}
C_{\Delta\varepsilon} = & X_j \sin\delta_l \sin\theta_k - Y_j \sin\delta_l \cos\theta_k + Z_j \cos\delta_l \sin\alpha_l \\
& + Y_K^I \sin\alpha_l + Z_K^I \sin\alpha_l \cos\delta_l \,,
\end{aligned}
$$

$$
C_\xi = X_j \sin\delta_l - Z_j \cos\delta_l \cos(\alpha_l - \theta_k) \,,
$$

$$
C_\eta = -Y_j \sin\delta_l + Z_j \cos\delta_l \sin(\alpha_l - \theta_k) \,,
$$

$$
\begin{aligned}
C_{\theta_k} = & X_j(\Delta\psi \cos\varepsilon \cos\delta_l \cos(\alpha_l - \theta_k) + \cos\delta_l \sin(\alpha_l - \theta_k) + \\
& + \Delta\psi \sin\varepsilon \sin\delta_l \sin\theta_k - \Delta\varepsilon \sin\delta_l \cos\theta_k) + \\
& + Y_j(\Delta\psi \cos\varepsilon \cos\delta_l \sin(\alpha_l - \theta_k) - \cos\delta_l \cos(\alpha_l - \theta_k) + \\
& + \Delta\psi \sin\varepsilon \sin\delta_l \cos\theta_k + \Delta\varepsilon \sin\delta_l \sin\theta_k) + \\
& + Z_j(- \xi \cos\delta_l \sin(\alpha_l - \theta_k) - \eta \cos\delta_l \cos(\alpha_l - \theta_k)) \,,
\end{aligned}
$$

$$
\begin{aligned}
C_{\alpha_l} = & X_j(-\Delta\psi \cos\varepsilon \cos\delta_l \cos(\alpha_l - \theta_k) - \cos\delta_l \sin(\alpha_l - \theta_k)) + \\
& + Y_j(-\Delta\psi \cos\varepsilon \cos\delta_l \sin(\alpha_l - \theta_k) + \cos\delta_l \cos(\alpha_l - \theta_k)) + \\
& + Z_j(\xi \cos\delta_l \sin(\alpha_l - \theta_k) + \eta \cos\delta_l \cos(\alpha_l - \theta_k) + \\
& + \cos\delta_l(-\Delta\psi \sin\alpha_l \sin\varepsilon + \Delta\varepsilon \cos\alpha_l)) + \\
& + X_K^I(\cos\delta_l(-\sin\alpha_l - \Delta\psi \cos\varepsilon \cos\alpha_l)) \\
& + Y_K^I(-\Delta\psi \cos\varepsilon \sin\alpha_l \cos\delta_l - \cos\alpha_l(\Delta\varepsilon - \cos\delta_l)) \\
& + \mathrm{Z}_K^I(-\Delta\psi \sin\varepsilon \sin\alpha_l \cos\delta_l + \cos\alpha_l(\Delta\varepsilon \cos\delta_l + 1)) \,,
\end{aligned}
$$

$$
\begin{aligned}
C_{\delta_l} = & X_j(-\sin\delta_l(-\Delta\psi\cos\varepsilon\sin(\alpha_l-\theta_k)+\cos(\alpha_l-\theta_k))+ \\
& -\cos\delta_l(\Delta\psi\sin\varepsilon\cos\theta_k-\Delta\varepsilon\sin\theta_k+\xi))+ \\
& +Y_j(-\sin\delta_l(\Delta\psi\cos\varepsilon\cos(\alpha_l-\theta_k)+\sin(\alpha_l-\theta_k))+ \\
& +\cos\delta_l(\Delta\psi\sin\varepsilon\sin\theta_k-\Delta\varepsilon\cos\theta_k-\eta))+ \\
& +Z_j(\sin\delta_l(\xi\cos(\alpha_l-\theta_k)+\eta\sin(\alpha_l-\theta_k)+\Delta\psi\cos\alpha_l\sin\varepsilon+\Delta\varepsilon\sin\alpha_l)+ \\
& +\cos\delta_l)+ \\
& +X_K^I(-\sin\delta_l(\cos\alpha_l-\Delta\psi\cos\varepsilon\sin\alpha_l)-\Delta\psi\sin\varepsilon\cos\delta_l)+ \\
& +Y_K^I(-\sin\delta_l(\Delta\psi\cos\varepsilon\cos\alpha_l-\sin\alpha_l))+ \\
& +\mathrm{Z}_K^I(-\sin\delta_l(\Delta\psi\sin\varepsilon\cos\alpha_l+\Delta\varepsilon\sin\alpha_l))\,,
\end{aligned}
$$

$$
C_{\Delta C_{0ij}} = c, \quad C_{\Delta C_{1ij}} = c(t_k - t_0)\,.
$$

So the linear dependencies are as follows:

$$
C_\xi = X_j C_{Z_j} - Z_j C_{X_j} + B_1, \quad C_\eta = Z_j C_{Y_j} - Y_j C_{Z_j} + B_2, \quad C_{\alpha_l} = C_{\theta_k} + B_3,
$$

where B_1 to B_3 are periodic variables. From the derivation above, it is clear that linear dependencies between some parameters are present. These linear dependencies will make the normal equation matrix rank deficient, therefore the normal equation will be singular. It means that not all the parameters can be estimated correctly. The ground station parameters and the earth rotation parameters can not be separated. Further more, an obvious linear dependency exists between α_l and θ_k.

3. Conclusion

Nutation is an important parameter to connect CTS and CIS and it is significant to geodesy and geodynamics. VLBI, especially SVLBI, is a main technology for observing nutation by now. It is important to add nutation correction to SVLBI observation mathematical model.

In this paper, the expansion of the observation equation has been made and the partial derivatives of SVLBI parameters are discussed. From the result of the model derivation, it is clear that linear dependencies between some partial derivative of the SVLBI observation with nutation correction exist. It means that all the parameters except α_l and θ_k can be estimated by the SVLBI ground-space time delay observations. In the future, some mathematical method will be investigated to calculate these parameters.

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