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Simulation of Local Tie Accuracy on VLBI Antennas

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Abstract

We introduce a new mathematical model to compute the centering parameters of a VLBI antenna. These include the coordinates of the reference point, axis offset, orientation, and non-perpendicularity of the axes. Using the model we simulated how precisely parameters can be computed in different cases. Based on the simulation we can give some recommendations and practices to control the accuracy and reliability of the local ties at the VLBI sites.

1. Model

Most of the models used in the calculation of the VLBI antenna reference point have been based on a 3D circle fitting [1], [2]. The main restriction of those models is that observations must be planned so that the tracks of the points form circles in the three dimensional space. If we use the VLBI antenna angle readings as additional observations we can calculate the reference point and axis offset also from scattered points. This kind of model was first presented by Lösler [4]. We parameterized our model differently so that the telescope axes are presented in the same three dimensional Cartesian system as the observed coordinates [3]. The basic assumptions are that points on the antenna structure rotate about the secondary axis and that the secondary axis rotates about the primary axis, but no pre-defined geometry or order of observations are assumed. The model is suitable for different types of telescope mounting and can be used with the data collected during the normal use of a telescope.

The position vector of a target (or a GPS antenna attached to a radio telescope) X is the sum of three vectors: the position vector of the reference point X_0, the axis offset vector $(E - X_0)$ rotated by angle α about the azimuth axis a, and a vector from the eccentric point E to the antenna point p rotated about the elevation axis e by angle β and about the azimuth axis by angle α (Fig. 1). Unknown parameters are X_0, E, a, e, and p. Observations are coordinates X for each antenna point and epoch and VLBI antenna angle readings α and β for every epoch. The estimated values of E, e, and p are those of an antenna initial position which may be zero for both angles.

The basic equation and the rotation matrix of our model are

$$X_0 + R_{\alpha,a}(E - X_0) + R_{\alpha,a} R_{\beta,e} p - X = 0 \tag{1}$$

$$R(\alpha, a) = \cos\alpha \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + (1 - \cos\alpha) \begin{pmatrix} xx & xy & xz \\ xy & yy & yz \\ xz & yz & zz \end{pmatrix} + \sin\alpha \begin{pmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{pmatrix} \tag{2}$$

The axis $a = (x\ y\ z)^T$ is in a three-dimensional Cartesian coordinate system. Because the rotation axes are unit vectors and the reference point is the intersection of the primary axis with the

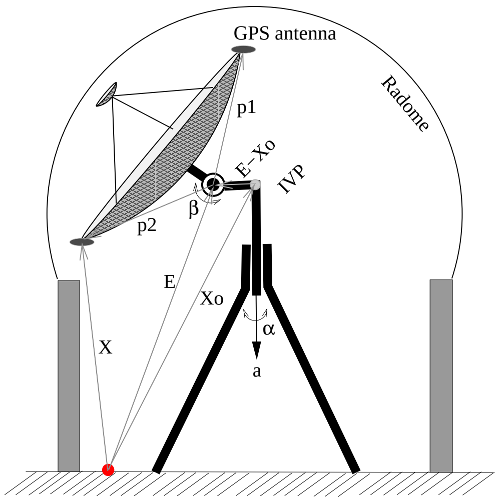

Figure 1. Geometry of the model.

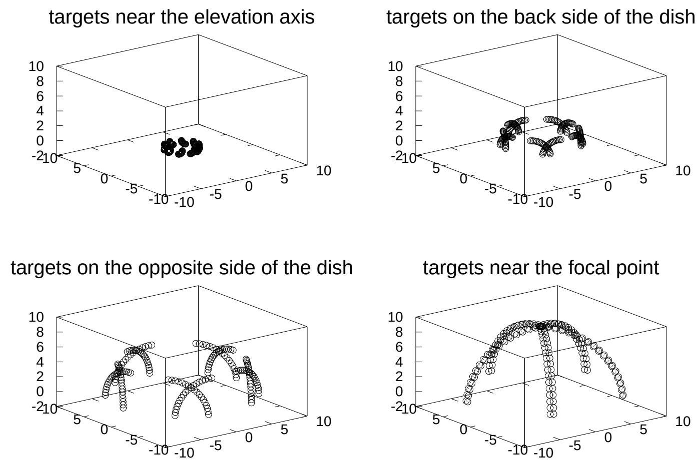

Figure 2. The geometry of observations in the four case studies.

shortest vector between the primary and secondary axes [1], four conditions between parameters are necessary:

$$1)\ \ a^T a - 1 = 0; \quad 2)\ \ e^T e - 1 = 0; \quad 3)\ \ (E - X_0)^T a = 0; \quad 4)\ \ (E - X_0)^T e = 0 \tag{3}$$

The solution of the above inverse problem can be reached by iteration in a linearized least squares mixed model with conditions between the parameters. The weight matrix of the observations is the inverse of their covariance matrix. The covariance matrix of parameters is therefore the inverse of the normal equation matrix in the least squares adjustment. The precision of the model parameters is a function of the number and geometry of the observations.

2. Simulation

By simulation we can study e.g. the following questions:

- the limits of the model (minimum number of observations, geometry),
- robustness of the model,
- how good the initial values of the parameters must be for linearization,
- how many target points to choose,
- what is the best place for targets,
- how to choose the antenna positions,
- what is the accuracy of the reference point, the antenna offset and the axes orientation,
- how accurate must the coordinate observations be,
- what is the minimum number of observations to get a reliable solution,
- how the random errors propagate to the solution,
- how the systematic errors propagate to the solution, and
- how the blunders can be detected.

In our simulations we varied the placements and the number of targets, the number of VLBI antenna positions, and the precision of the target points. We calculated the variances of the reference point coordinates, the axis offset, and the angle between the axes using two different strategies.

In the first case we computed the inverse of the normal equation matrix in least squares adjustment which is the covariance matrix of the unknown parameters of the model. Because the axis offset and the angle between axes are not the model parameters they must be calculated. The angle between the axes is

$$\varphi = \arccos(a^T e) \tag{4}$$

and the axis offset

$$o = \pm\sqrt{(E - X_0)^T (E - X_0)} \tag{5}$$

The sign of the offset o is negative if the direction of the offset vector is opposite to the antenna opening direction. The variances of the angle and the offset are reached by using the law of variance propagation.

In the second case we added the normal distributed random errors to observations and repeated the adjustment thousands of times with different observations. Then we calculated the standard deviations from the adjusted parameters and derived axis offsets and angles. Besides the azimuth-elevation type antenna orientation, we tested the model with X/Y- and polar-mount types of antennas. We have also applied the model to the Metsähovi VLBI telescope using real data. In our simulations we used local topocentric coordinate systems. When using real data, the system is chosen according to the target point coordinate system.

2.1. Geometry of Observations and Precision

We simulated four different observation geometries (Fig. 2) to find the best places for the targets:

1. The targets or prisms were near the elevation axis and only a few meters from the azimuth axis (used, e.g., in Onsala local tie measurements [4], [5]).
2. The targets were on the back side of the antenna dish (used, e.g., in the Metsähovi local tie measurements with tacheometers)
3. The target points or GPS antennas were on the opposite side of the dish (used e.g. in the Metsähovi tie measurements with GPS)
4. The targets were near the focal point on the apex of the quadripod (here the target point coordinates were approximated from the Metsähovi telescope dimensions)

In order to get realistic target or GPS point coordinates we used the actual antenna dimensions in the simulations [7]. For comparable results we used in all cases two targets at opposite sides of the telescope, although for example in Onsala they had eight targets in the real measurements [5]. In our simulations we derived the weight matrix of observations from the diagonal covariance matrix of coordinates X for each antenna point and epoch, and VLBI antenna angle readings α and β for every epoch. We varied the precision of the target coordinates between 0.0001–0.05 m for the horizontal components and 0.0002–0.10 m for the vertical components. The precision of the telescope angle readings were 0.0001° in all cases. In each case there were 90 antenna positions. The results of the simulations are presented in Figures 3 and 4.

2.2. Number of Targets, Number of VLBI Antenna Positions and the Precision

For this experiment we chose the first type of observation geometry from our list above: targets were near the elevation axis. We varied the number of targets from two to ten. The precision of the target points was 0.01 m for horizontal and 0.02 m for vertical components. The 18 elevations

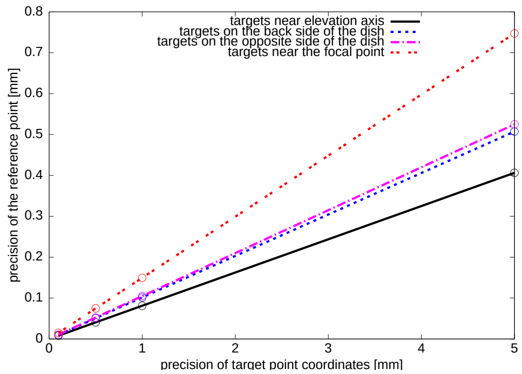

Figure 3. Precision of the reference point in different target geometries.

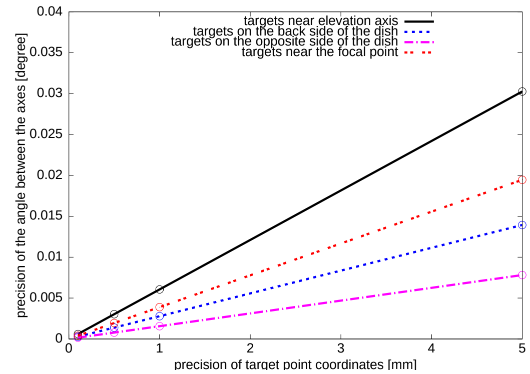

Figure 4. Precision of the angle between the axes in different target geometries.

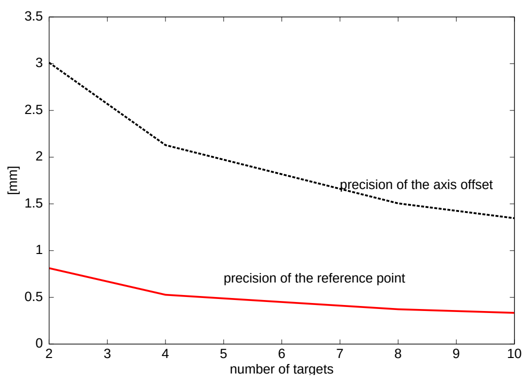

Figure 5. Increasing the number of targets.

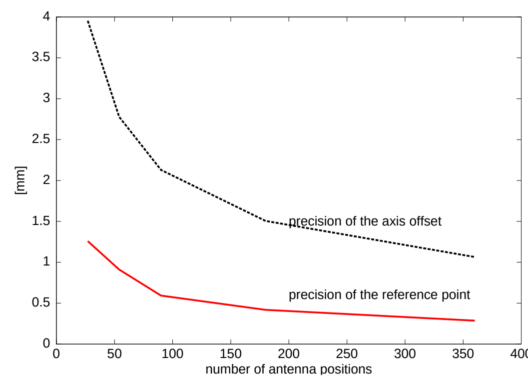

Figure 6. Increasing the antenna positions.

in five azimuths give 90 antenna positions together. In Figure 5 we can see that the precision of the reference point is at sub-mm level with only two targets and 90 antenna positions, but if a mm level precision is required for the axis offset, then the precision of the observations must also be at the mm level.

What happens if we increase the number of antenna positions? We chose the second type of the observation geometry from our list: targets on the back side of the dish. We varied the number of azimuths from three to ten but kept the number of elevations fixed to 18. The number of targets was four and the precision of target points 0.01 m for horizontal components and 0.02 m for vertical components. In Figure 6 we see that the precision of the reference point is below the mm level after 50 antenna positions. Also the axis offset is near the mm level if we increase the number of antenna positions to 360. In our simulation, azimuth values were chosen equally around the azimuth axis.

The model is suitable also for X/Y or polar type mounts. As a demonstration we adopted the dimensions of the Hobart 26 m VLBI antenna with 8.1913 m axis offset [2], [6], and [7]. We used 42 antenna positions (three positions [-20,0,20] for X and 14 positions [-65,65] with steps of 5° for Y) and generated 1000 observation sets with normal-distributed random errors and adjusted the model parameters. In the simulation we assumed that the two GPS antennas or targets were on the opposite side of the dish. With these assumptions we computed the variation of axis offset and the angle between the axes. 95% of the solutions of the axis offset were between 8.1834 m and 8.1996 m, if the precision of the observed coordinates was 1 cm. When we used mm-level target point precision and the same antenna positions, we got 8.1905 m and 8.1922 m. To achieve better accuracy we need to increase the number of antenna positions where the observations are made.

3. Summary of the Results and Conclusion

The precision of the antenna axis offset depends mostly on the achievable precision of the target coordinates, the number of targets, and the number of VLBI antenna positions. If we use the same number of targets and the same number of antenna positions, all target geometries which we tested gave almost the same precision for the axis offset. The most precise reference point coordinates will be achieved if the targets are placed near the elevation axis. It is possible to achieve a sub-millimeter precision for the coordinates of the reference point and the antenna offset by increasing the number of VLBI antenna positions, even if the precision of target points is on the cm level.

In real measurements it is not possible to choose the places of the targets arbitrarily, but they must instead be installed where they are visible from the observing points. It may not be possible to give unambiguous answers on how many targets or how many antenna positions one must have. It is still recommendable to simulate the accuracy before the measurement work. Our model is powerful and very easy to use for this purpose.

Acknowledgement

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