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A Comparison of General Relativity Theory Evaluations using VLBI and SLR: Will GGOS Improve These Results?

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Abstract

Constant instrumental and data analysis upgrades throughout the development of the VLBI technique have delivered a continuous improvement in the accuracy of the evaluations of Parameterized Post Newtonian (PPN) parameter γ. Lunar Laser Ranging can be used to estimate PPN parameter β, as well as test possible temporal variation of the gravitational constant. Satellite Laser Ranging can be used to evaluate frame dragging. New applications using SLR data recently have attempted to estimate γ and β directly. These different space geodesy techniques and their potential to evaluate GRT will be affected positively by the development of the Global Geodetic Observing System. It was found that if improvements in VLBI estimates are extrapolated to 2020, results will equal that of γ estimates obtained by radiometric tracking data of the Cassini spacecraft on its approach to Saturn (the best estimate to date) but with a more robust error budget.

1. Introduction

The observational techniques of space geodesy, VLBI (Very Long Baseline Interferometry), GNSS (Global Navigation Satellite Systems), SLR/LLR (Satellite or Lunar Laser Ranging), and DORIS (Doppler Orbitography and Radiopositioning Integrated by Satellite) have reached measurement accuracies of such a level that general relativistic effects (GRT), are considered on a routine basis in data processing and parameter estimations. Data must be analyzed within the framework of a post-Newtonian formalism [5] and the complete context within which the modeling is performed, i.e. reference and time frames, solar body ephemerides, signal propagation, and observables (such as VLBI delay, laser pulse travel time, and satellite clock frequency) must consider GRT [6]. This paper briefly considers the expected impact of the Global Geodetic Observing System (GGOS) on the ability of VLBI to evaluate the space curvature parameter γ.

2. Expected GGOS Contributions

It is expected that GGOS will bring an improvement of a factor of ten in global accuracies in the observational as well as the theoretical components of space geodesy, implying a ten-fold improvement in measuring accuracies, reference frames and their maintenance, modeling, and station network geometry. This should translate into the accuracies involved when doing relativity tests using space geodesy. Additional geophysical instrument co-location (gravimeter, accelerometer, and seismometer) could lead to ‘tuned’ site-specific Love numbers to improve geophysical modeling (for instance the elastic coefficient of Earth). Improvements are expected in the ITRF, ICRF, EOPs, and models (e.g., gravity fields, earth and pole tides, ocean and atmospheric loading, and precise orbit determination). This *should* lead to an improvement in the evaluation of GRT using space geodesy techniques, in particular VLBI, SLR, and LLR. Also required are improved GRT delay models, from 1 ps to 0.3 ps or less for both VLBI and SLR/LLR (i.e. sub-mm accuracy), e.g. by developing post post-Newtonian models. Alternative (scalar-tensor) theories predict small deviations from GRT values at a level of: $\gamma - 1 = 1 \times 10^{-6}$ to 1×10^{-7}; VLBI and SLR (or any other technique) cannot reliably obtain these values yet.

Why test GRT? The strength of gravity is given by Newton's (scalar theory) gravitational constant G; it is important to evaluate possible changes in G with time (which would cause an evolving scale of the solar system). In addition one would want to test GRT to evaluate measurement of space-time curvature and gravitational delay, verify Einstein's equivalence principle, measure frame dragging and relativistic precession of orbits (geodetic (de Sitter) precession) as well as evaluate PPN parameters β (nonlinearity in superposition of gravity) and γ (amount of space curvature produced by unit test mass).

3. Technique-Dependent Sensitivity to Model and Observational Errors

Estimates of GRT using VLBI are sensitive to intrinsic source structure, the contribution to the delay of the wet neutral atmosphere, uneven North-South VLBI network distribution, and solar coronal plasma for smaller Sun elongations, which produces large path-length changes. Satellite laser ranging GRT estimates are sensitive to gravity field model errors in even zonal coefficients (J2, J4, J6,...), orbit perturbation (model) errors, and the contribution to the delay of the atmosphere (the need to incorporate azimuth-dependent components). GRT can be embedded in gravity field models and weak network geometry, especially in the Southern Hemisphere, and only some satellites are suitable for GRT tests (LAGEOS 1/2 and LARES (recently launched)). Lunar laser ranging GRT estimates are affected by the sparsity of the network (nothing in the Southern Hemisphere), very limited data quantity (due to extreme difficulties in ranging to the Moon and a very low return rate), and the depth signature effect in lunar reflector arrays for single photon returns.

Figure 1. 1-m Cassegrain (ex OCA) which will be used as part of an SLR/LLR system being developed at HartRAO, with the 26-m VLBI antenna (mid-background) and the 15-m SKA prototype (far background). The 15-m antenna is being equipped for S/X VLBI and will be used for IVS EOP measurements.

HartRAO has commenced with the development of a Satellite and Lunar laser ranger in collaboration with NASA and the Observatoire de la Côte d'Azur (OCA). A Southern Hemisphere LLR (see Figure 1) will strengthen the geometry of the LLR network and should improve the determination of the orientation of the Moon as well as improve GRT tests using LLR. A dual system SLR/LLR will provide added coverage of SLR data in an area very sparsely covered. This SLR/LLR system will be a GGOS component and should be co-located with a VLBI2010 compatible antenna.

4. General Relativity Tests using VLBI

VLBI currently attains accuracies better than 0.1 mas. This allows the use of VLBI as an excellent tool for GRT tests. Therefore geodetic VLBI has often been used to evaluate the space curvature parameter γ. Early tests of the general relativity theory involved using optical telescopes. These tests involved gravitational redshift measurements and determination of star light deflection at the Sun's limb during eclipses. A static and spherically symmetric metric (Schwarzschild) [7] can be used to describe the space-time geometry around the Sun. Eddington [3] created an isotropic formulation of Schwarzschild's original anisotropic version of the metric, which can be written as

$$ds^2 = -\left(1 - 2\frac{GM}{c^2r} + 2\left(\frac{GM}{c^2r}\right)^2\right)(cdt)^2 + \left(1 + 2\frac{GM}{c^2r}\right)\left[dx^2 + dy^2 + dz^2\right]. \qquad (1)$$

In Equation (1) the gravitational constant is given by G, the speed of light by c, and the mass of the star (Sun) by M. Considering the ten parameters in the PPN formalism, parameters γ and β are the most physically significant. Equation 1 can be written to include PPN parameters as

$$ds^2 = -\left(1 - 2\frac{GM}{c^2r} + 2\beta\left(\frac{GM}{c^2r}\right)^2\right)(cdt)^2 + \left(1 + 2\gamma\frac{GM}{c^2r}\right)\left[dx^2 + dy^2 + dz^2\right]. \qquad (2)$$

Considering the first term of Equation (2), β describes the degree of non-linearity in the superposition law for gravity (many gravity theories make provision for the assumption that gravity produces gravity). How much curvature is produced by unit rest mass is described in the second term (spatial part) by PPN parameter γ. This PPN parameter can be tested by deflection of light, bending of radio waves, and Shapiro delay experiments. In GRT both parameters γ and β are equal to unity, whereas the other eight PPN parameters are zero as given by Will and Nordtvedt [8].

5. Evaluation of PPN Parameter γ

Eddington's classical test of the deflection of light by the Sun [2] essentially measures the propagation of photons in curved space near the Sun. The amount of space curvature per unit mass is related to γ through the proportional relationship

$$\delta\theta = \frac{1}{2}(1+\gamma)\left(4Gm_{\odot}\big/c^2d\right)\left[(1+\cos\Phi)/2\right]. \qquad (3)$$

A radio signal (from a VLBI source) passing close to the Sun at distance d will be deflected by the angle $\delta\theta$.

In Equation (3) the mass of the Sun is $m_\odot$ and Φ is the angle between the direction of the radio signal from the radio source and the line between Earth and the Sun. The relative angular separation will change when the line-of-sight of the source moves closer to the Sun. Currently geodetic VLBI scheduling avoids sources closer than 15° to the Sun, which reduces sensitivity of using VLBI for GRT tests. This angular separation is given by

$$\delta\theta = \frac{1}{2}\left(1+\gamma\right)\left[-\frac{4m_\odot}{d}\cos\chi + \frac{4m_\odot}{d_r}\left(\frac{1+\cos\Phi_r}{2}\right)\right]. \tag{4}$$

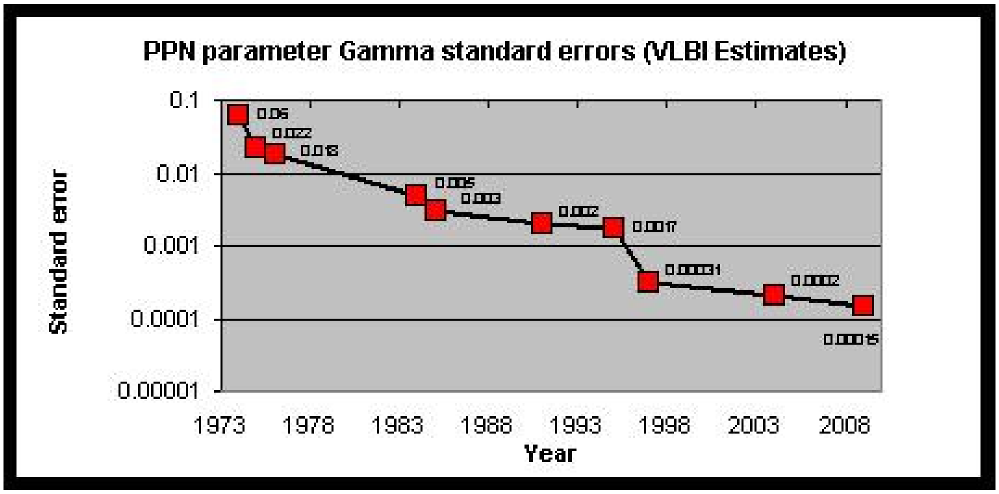

Figure 2. Plot of standard errors resulting from geodetic VLBI evaluations of PPN parameter γ. It is expected that improvement will continue as instrumentation, software, and models develop, particularly in the framework of GGOS.

The continuous improvement in valuation of PPN parameter γ is illustrated in Figure 2, which is based on the table of γ estimates spanning the period 1972 to 2009 as provided by Heinkelmann and Schuh [4]. Figure 2 contains the standard error of the various γ estimates, and an exponential fit constrained to the first and last estimate provides a value of $\pm 2.5 \times 10^{-5}$, when using the fitted function to extrapolate towards 2020. If this predicted accuracy level is achieved by VLBI, supported by the developments around VLBI2010 in the GGOS framework, it would be comparable to the accuracy (currently the best) of the estimate of γ achieved during the microwave tracking of the Cassini spacecraft on its approach to Saturn [1]. As VLBI error budgets are very robust due to the large volumes of data involved, it is expected that the VLBI solutions will be very reliable. Combined with a change in VLBI observation strategy (scheduling routine observations closer to the Sun), the increase in data useful for GRT tests should place VLBI into the ‘hot zone’ with errors at the level of $\pm\, 1.0 \times 10^{-6}$; therefore, it will soon be possibly the best technique to test PPN parameter γ reliably.

6. Conclusion

Combined with a change in VLBI observation strategy (scheduling routine observations closer to the Sun), advances due to VLBI2010, and other GGOS implementations, the increase in data useful for GRT tests should place VLBI into the 'hot zone' with errors at the level of $\pm\, 1.0 \times 10^{-6}$. Therefore it will be the best technique to test γ reliably. It is expected that GGOS will support VLBI, SLR, and LLR to improve their validations of GRT, but only to its fullest extent if all aspects of GGOS are addressed: networks, equipment, models, observing strategy, and processing strategies. Requirements for better results include scheduling of VLBI observations closer to the Sun, and new missions such as the construction of VLBI beacon/laser transponder units for placement in orbit, on the Moon, and on suitable planets.

GGOS currently only has three themes. It would be important not to exclude its fourth theme, fundamental physics. Millimeter accuracy in ALL the space geodetic techniques and reference frames is finally constrained by our understanding of the geometry of space. In fact, a *fourth* pillar of space geodesy (typically three pillars are given as (1) shape and deformation of Earth, (2) gravity and geoid, and (3) orientation and rotation) should be added, so that we have (4) *geometry of spacetime*. The geometry of spacetime affects all space geodetic techniques. It is fundamental to all observables and, as the fourth pillar of space geodesy, needs to be measured and incorporated at the highest level of accuracy into our modeling.

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