The Most Remote Point Method for the Site Selection of the Future VGOS Network

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Abstract The VLBI Global Observing System (VGOS) will be part of the Global Geodetic Observing System (GGOS) and will consist of globally well distributed geodetic observatories. The most remote point (MRP) method is used to identify gaps in the network geometry. In each iteration step the identified most remote points are assumed to become new observatory sites improving the homogeneity of the global network. New locations for VGOS observatories have been found in La Plata, Tahiti, O'Higgins, Galapagos, Colombo, and Syowa.[‡]

Keywords VGOS, MRP method, Voronoi, network densification

1 Introduction

The International Association of Geodesy (IAG) has launched the GGOS aiming for the unification of traditionally separated geometric and geophysical reference systems and for the realization of a 3D-position error of 1 mm on the global scale and a reference point velocity error of no more than 0.1 mm/yr. To reach these goals, the operation of 30–40 globally well-distributed geodetic observatories is needed (Beutler and Rummel, 2012).

The International VLBI Service for Geodesy and Astrometry (IVS) will meet the GGOS requirements with its VGOS initiative (Hase et al., 2012) requiring a new type of very fast radio telescopes with broadband capabilities as described in Petrachenko et al. (2012). In this study only this upcoming VGOS network is considered as it will outperform the currently used IVS network infrastructure. We assume that all VGOS stations deliver identical data quality.

We introduce the most remote point (MRP) method, a tool to find the best new geometrical locations with respect to existing reference observing stations. It will provide the optimal network configuration for further simulations.

2 Evaluation and Improvement of Global Geodetic Networks

2.1 The Most Remote Point (MRP) Method

Given a point distribution on a limited surface area and a requirement to add points in order to make the distribution more homogeneous, Hase (2000) suggested an iterative approach where new points are always added at the most remote location with respect to the others.

Figure 1 shows the properties of the Delaunay triangulation and its counterpart, the Voronoi diagram. The most remote point coincides with the Voronoi vertex, even if a three dimensional spherical surface replaces the two dimensional plane surface (Hase, 1999).

For an ideal homogeneous point distribution, the area of each Voronoi facet will be equivalent to the mean spherical surface area of *n* reference points. This goal can be reached either by filling up the largest gaps

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in the network by adding new points or by removing points which have the shortest distance to each other.

For the purpose of modeling the point distribution of a global geodetic network, a spherical Earth model is sufficient. We apply the MRP method in a non-rigorous way. This means that the most remote point suggests a vicinity in which the definite location has to be chosen according to other selection criteria. A rigorous application of the MRP method is not practical. For instance, if the most remote point is calculated to be in the ocean, a nearby island or continental point has to be chosen instead.

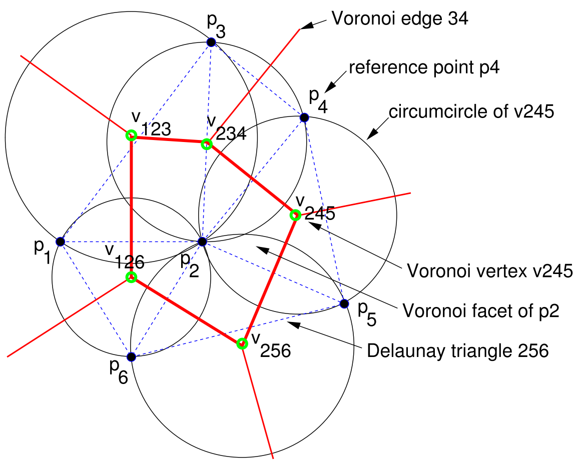

Fig. 1 Part of a network with reference points P_1, P_2, P_3, P_4, P_5, and P_6 as a given distribution showing the Voronoi facet of P_2. Within this facet no other reference point P_i is closer than P_2 itself. Therefore the Voronoi facet is the representative area of P_2. In this illustration the most remote point v_{123} is indicated by the largest circumcircle. If it will become a new reference point, i.e., P_7, the areas of the adjacent Voronoi facets of P_1, P_2, and P_3 will be decreased by a part which will surround P_7 (only the facet of P_2 is shown completely).

In this manner, the MRP method can be applied by the following sequence:

1. Compute the most remote point of the point distribution of a global network.
2. Define the new site to be added to the network close to the most remote point of the computation.
3. Continue iterating by computing and adding new most remote points to the network until a certain condition (i.e., total number of stations) is satisfied.

2.2 Network Quality Parameter

One criterion to measure the homogeneity of a point distribution can be derived from the Voronoi facet, which was introduced in Section 2.1. Thus, the standard deviation σ_i of a distribution of n Voronoi facet areas A_j in each iteration i will be a suitable parameter to measure the homogeneity of a global network. We introduce a quality parameter Q_i of the point distribution as:

$$Q_i := \frac{1}{\sigma_i} \qquad (1)$$

The quality parameter increases in each iteration step as the homogeneity improves.

2.3 Improvement Parameter

The MRP method is iterative. The improvement in each iteration step can be quantified either a) by the achieved reduction of the largest radius of the circumcircle of the Delaunay triangles or b) by the achieved increase of the quality parameter (Equation 1). They can be expressed as percentages.

Based on the radius r of the circumcircle, the relative improvement parameter I^r_{rel} and, based on the radius r_0 of the initial network configuration, the absolute improvement parameter I^r_{abs} are expressed as:

$$I^r_{rel} := 1 - \frac{r_{i+1}}{r_i} \quad , \quad I^r_{abs} := 1 - \frac{r_i}{r_0} \qquad (2)$$

In this study, the radius r of the circumcircle of the MRP corresponds to an arc of the great circle (and not to the baseline). The improvement of well placed new stations shortens the distance between neighboring stations.

The relative improvement parameter I^Q_{rel} is based on the area A of the Voronoi facets. In analogy to the previous formulae, an absolute improvement parameter I^Q_{abs} of the i-th iteration refers to the quality of the homogeneity of the initial configuration and can be expressed as:

$$I^Q_{rel} := \frac{Q_{i+1}}{Q_i} - 1 \quad , \quad I^Q_{abs} := \frac{Q_i}{Q_0} - 1 \qquad (3)$$

The improvement parameter (Equation 3) is based on the spherical surface area of the Voronoi facet and indicates how efficiently a site selection contributes to a more homogeneous point distribution.

3 Applying the MRP Method to the VGOS Network

New VGOS radio telescope projects are in execution at the following sites: Wettzell, DE; Yebes, ES; Flores Island, PT; Santa Maria Island, PT; Tenerife Island, ES; Ny-Ålesund, NO; Onsala, SE; Metsahovi, FI; Kaliningrad, RU; Kazan, RU; Zelenchukskaya, RU; Badary, RU; Ussuriysk, RU; Ishioka, JP; Sheshan, CN; Changchun, CN; Nanshan, CN; Hartebeesthoek, ZA; Yarragadee, AU; Hobart, AU; Katherine, AU; Warkworth, NZ; Kokee Park, US; Greenbelt, US; and Haystack, US. This incomplete future network will form the VGOS which is an appropriate example to demonstrate the MRP method for network densification.

Following Subsection 2.1 we use this VGOS network for the initial computation of the Voronoi grid and hence the most remote points (iteration 0).

Figure 2 shows the network configuration and the results in an equal-area Hammer-Aitoff projection. The three most remote points are found in South America and the South-East Pacific area. The first choice to improve this network is to add a new VGOS site to the VGOS network in South America.

Recent contributions to the IVS from South America come from the German Transportable Integrated Geodetic Observatory (TIGO) and from Fortaleza in Brazil. In general it is easier to update an existing VLBI site rather than to set up a new one. Due to lacking financial support, TIGO will be moved from Concepción in Chile to La Plata in Argentina; both sites are closer to the computed remote point than Fortaleza. Therefore, a modernization project in La Plata to make the site VGOS compatible is meaningful. For this analysis, La Plata (LP) was chosen for the first iteration step (iteration 1). Figure 3 shows the consequence: a very significant improvement of 29.3% in the distance between the MRP and stations and of 11.2% of reduced inhomogeneity in the Voronoi facets.

Based on Figure 3, we chose Tahiti (Th) as the new site in the second iteration (Figure 4). This follows the idea of co-locating different geodetic space techniques at one site as we find already an SLR station on that island. A new VGOS radio telescope will improve the VLBI network further by 12.0% with respect to the previous iteration and create a very important fundamental reference point in the South Pacific.

The Antarctic region is very difficult for continuous VLBI operations. Only the German Antarctic Receiving Station O'Higgins and the Japanese base in Syowa are IVS network stations. The foundation of a radio telescope is restricted to areas with bedrock to obtain a long-term stable reference point. Access to the station is easier in the coastal regions. Both places are suitable

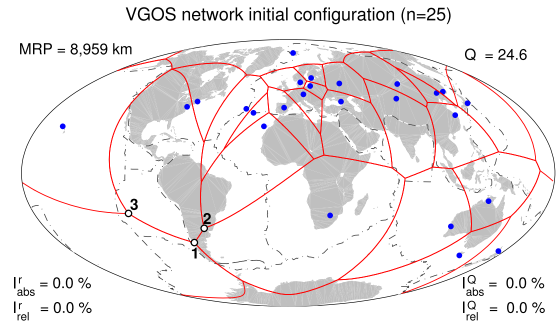

Fig. 2 The initial network configuration of the future VGOS network. New VGOS radio telescopes are marked with filled circles. The map shows the plate tectonic border lines (dotted lines) and the Voronoi edges (solid red lines) in an equal area projection. The Voronoi facets demonstrate the representative area of each individual station due to the given network configuration. The open black circles labeled with 1, 2, and 3 indicate the three most remote points with respect to the given network configuration or station distribution. The largest gaps in the network exist in the South America and South-East Pacific areas. The MRP is 8,959 km away from its nearest VGOS reference stations.

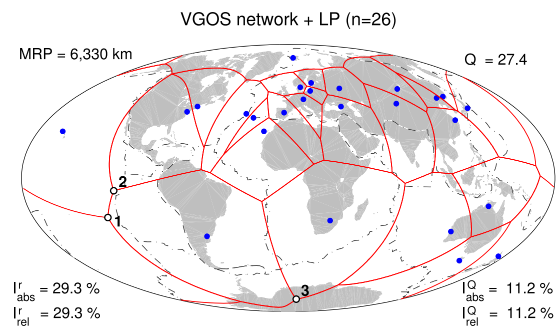

Fig. 3 The first iteration adds the site La Plata (LP) in Argentina to the initial network configuration of the future VGOS network. This improves the quality of the network by 11.2%. The largest gap in the network is shifted now to the South-East Pacific area.

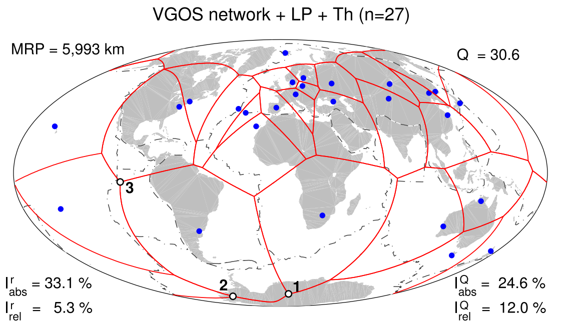

Fig. 4 The second iteration adds the site Tahiti (Th) in French Polynesia to the previous network configuration of the future VGOS network. This improves the network by 12.0% relative to the previous step and 24.6% relative to the initial configuration. The largest gap in the network is shifted to the Antarctic continent.

candidates for VGOS instrumentation. We selected in this step as the third added station to the VGOS network O'Higgins (Oh) site (Figure 5, iteration 3).

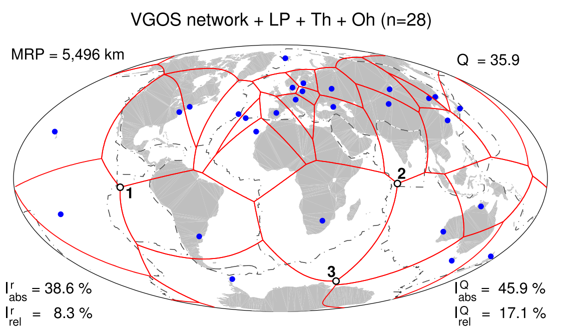

Fig. 5 The third iteration adds the site O'Higgins (Oh) in Antarctica to the previous network configuration of the future VGOS network. This improves the network by 17.1% relative to the previous step and 45.9% relative to the initial configuration. The largest gap in the network is shifted back to the East Pacific.

Due to the lack of continental surface the East Pacific does not offer many possibilities to set up a new observatory. In this fourth iteration, the Galapagos Islands (GI) have been chosen (Figure 6, iteration 4).

The Indian Ocean is populated with small and remote islands where it would be difficult to operate a VGOS station for logistical reasons. Therefore the town of Colombo (Co), Sri Lanka was chosen to become the fifth new site in the VGOS network (Figure 7, iteration 5).

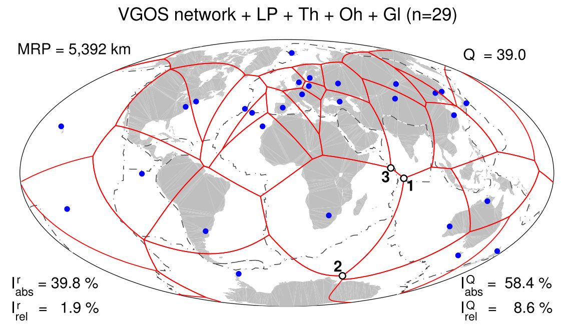

Fig. 6 The fourth iteration adds the site Galapagos Islands (GI) to the previous network configuration of the future VGOS network. This improves the network by 8.6% relative to the previous step and 58.4% relative to the initial configuration. The largest gap in the network is now shifted back to the Indian Ocean.

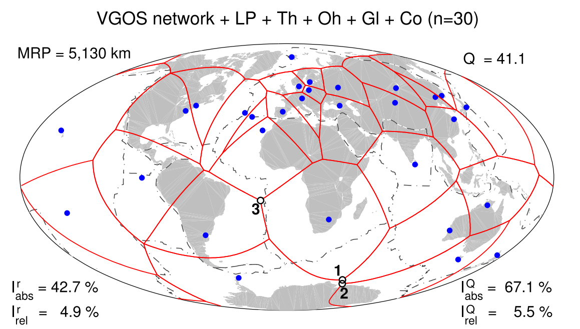

Fig. 7 The fifth iteration adds the site Colombo (Co) in Sri Lanka to the previous network configuration of the future VGOS network. This improves the network by 5.5% relative to the previous step and 67.1% relative to the initial configuration. The largest gap in the network is now shifted again to Antarctica.

Antarctica is a large and extended region. The sixth iteration adds Syowa base (Sy) as a new VGOS network station (Figure 8, iteration 6).

After six iterations, the improvements in the VGOS network have closed the largest gaps—all in the Southern hemisphere. The distance to the most remote point has almost halved (46.8%) and the network quality improved by nearly 86%. As one can easily see in the comparison of Figure 2 with Figure 8, the new Voronoi facets in the Southern hemisphere look much more equal in size compared to the Northern hemisphere. This confirms that the most remote point method is suitable for global network planning. It also underlines the argument that a network can be created in a much more efficient way with a global perspective.

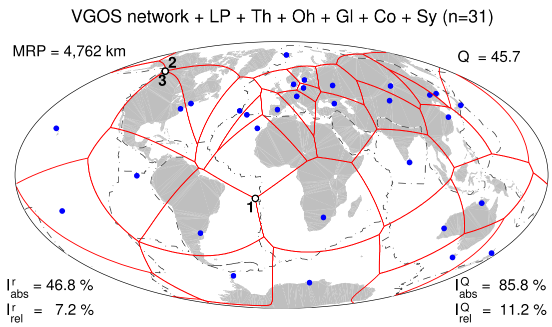

Fig. 8 The sixth iteration adds the site Syowa (Sy) in Antarctica to the previous network configuration of the future VGOS-network. This improves the network by 11.2% relative to the previous step and 85.8% relative to the initial configuration. With this iteration step the largest gaps in the network fall now into the Southern Atlantic and first time to the upper northern latitudes near Alaska.

Table 1 summarizes the convergence of radius of the MRP, of the improvement parameters (Equations 2 and 3) and of the quality parameter (Equation 1), indicating a much more homogeneous site distribution after adding six new sites with the MRP method.

Table 1 VGOS network: Improvement and quality parameters with six iterations. With only six additional VGOS stations the radius of the MRP shrinks from 8,959 km to 4,762 km, and a much more homogeneous global site distribution can be reached.

iter.	r_i of MRP	n	I^r_{rel} %	I^r_{abs} %	I^Q_{rel} %	I^Q_{abs} %	Q
0	8,959 km	25	0.0	0.0	0.0	0.0	24.6
1	6,330 km	26	29.3	29.3	11.2	11.2	27.4
2	5,993 km	27	5.3	33.1	12.0	24.6	30.6
3	5,496 km	28	8.3	38.6	17.1	45.9	35.9
4	5,392 km	29	1.9	39.8	8.6	58.4	39.0
5	5,130 km	30	4.9	42.7	5.5	67.1	41.1
6	4,762 km	31	7.2	46.8	11.2	85.8	45.7

The achievements by applying the MRP method to the densification of the global network can be illustrated by Figure 9, showing the circumcircles of the Voronoi-points in a concentric way comparing the initial and the sixth iteration. The circles are color-coded and distinguish Voronoi points of the Northern hemisphere from those of the Southern hemisphere.

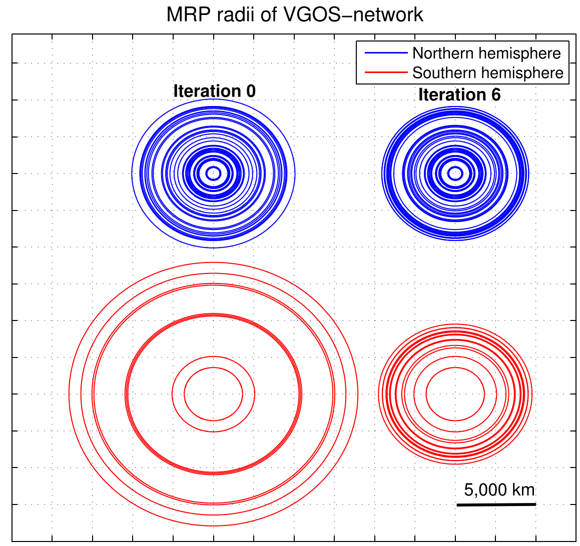

Fig. 9 Radii of the MRP of the initial VGOS network configuration (iteration 0) vs. those of the last iteration 6. This figure shows the MRP radii in the Northern hemisphere in blue above those in the Southern hemisphere in red. Initially the largest gaps were found in the Southern hemisphere indicated by large circumcircles. After the introduction of six new points with the MRP method, the situation has improved. The concentration of circles shows homogeneity as it can also be seen by comparing Figure 2 with Figure 8.

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