The Evaluation of the Earth's Dynamical Flattening Based on the IAU Precession-nutation and VLBI Observations

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Abstract The dynamical flattening H_d is a fundamental Earth's parameter and a crucial scale factor in constructing the precession-nutation models. Its value has generally been derived from astronomical observations of the luni-solar precession in longitude at epoch, or from geophysical determinations of the Earth's moment of inertia. It should be noted that the observed precession rates in longitude and obliquity result from several theoretical contributions, some of them, as well as the nutation amplitudes, being also dependent on H_d. This paper discusses the rigorous procedure to be used for deriving H_d from the best available astronomical observations. We use the IAU 2006/2000 precession-nutation and VLBI observations of the celestial pole offsets spanning about 32 years in order to calculate the observed position of the CIP (Celestial intermediate pole) in the GCRS (Geocentric celestial reference system). Then, the value of H_d is evaluated by a least squares method with a careful consideration of the various theoretical contributions to the precession rates and of the largest terms of nutation. We compare the results with an indirect fit of H_d to the estimated corrections to the linear term in precession and the 18.6-yr nutation. We discuss the limit of accuracy, given the characteristics of the available observations and the uncertainties in the models, as well as the parameters on which H_d is dependent.

Keywords Astrometry, precession-nutation, VLBI, reference systems

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1 Introduction

Precession and nutation of the Earth originate from the tidal forces exerted by the Moon, the Sun, and the planets on the equatorial bulge of the Earth, for which the oblateness is described by the dynamical flattening H_d:

$$H_d = \frac{C - (A + B)/2}{C} \qquad (1)$$

where A, B, and C are the principal moments of inertia of the whole Earth. H_d is an essential parameter for the precession-nutation theory since the effect of luni-solar and planetary torques is proportional to this parameter; it is directly linked ($H_d = e/(1 + e)$) to the parameter e, which is one of the Basic Earth Parameters of the IAU 2000 precession-nutation (Mathews et al., 2002). Values of H_d deduced from precise astro-geodetic techniques, such as VLBI and LLR, have been used in developing recent precession nutation theories; some of them, along with the associated precession models, are listed in Table 1. The relative differences between these values are ranging from tenths to tens of parts per million (ppm).

Table 1 Values at J2000.0 used for computing the precession of the equator for various precession models.

Authors	$H_d \times 10^3$	$f_{01|_{LS}}$ in $''$/cy	Model
Lieske et al. (1977)	3.2739935	5494.3861	IAU 1976
Williams (1994)	3.2737634	5494.01083	W94
Mathews et al. (2002)	3.27379492		IAU 2000
Capitaine et al. (2003)	3.27379448	5494.062986	IAU 2006

Traditionally, the Earth's dynamical flattening has been derived from astronomical observations of the luni-solar precession in longitude at epoch, consider-

ing the first order-lunisolar precession rate at J2000.0, which is the most significant contribution (at the order of 5500″/cy), denoted $f_{01|_{LS}}$ (cf. column 3 of Table 1). However, other minor contributions to the precession rates in both longitude and obliquity as well as nutation are affected by H_d. This should be properly considered for a rigorous computation of H_d.

2 Estimation of H_d from the Observed Motion of the Celestial Intermediate Pole

The purpose of our work is to use a rigorous procedure for deriving the Earth's dynamical flattening H_d from the best available astronomical observations.

The first method we have used is a direct fit of H_d to the time series of the observed CIP (Celestial intermediate pole) location, which can be expressed by the coordinates, X and Y, of the CIP in the GCRS (Geocentric celestial reference system).

We have first computed the observed quantities X_{obs} and Y_{obs} by using the theoretical expressions of the CIP coordinates X_{IAU} and Y_{IAU}, based on the IAU 2006/2000 precession-nutation (Mathews et al., 2002, Capitaine et al., 2003, 2005) and the time series of the "celestial pole offsets" observed by VLBI, denoted dX and dY, as follows:

$$X_{obs} = X_{IAU} + \mathrm{d}X$$
$$Y_{obs} = Y_{IAU} + \mathrm{d}Y. \qquad (2)$$

The series for X_{obs} and Y_{obs} are shown in Figure 1.
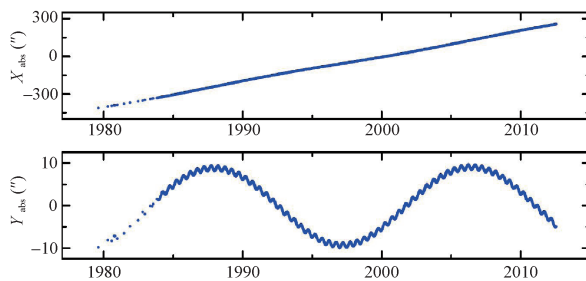

Fig. 1 Time series of the X and Y coordinates of the observed CIP in the GCRS over the period 1980–2012.

The quantities X_{IAU} and Y_{IAU} are calculated using the IAU SOFA software, while the quantities (dX, dY) used in this analysis are those of the IVS solution

opa2012a (of Paris Observatory): the time span is about 32 years (1980–2012) and the free core nutation (FCN) is removed using the empirical model provided in the IERS Conventions (2010). The weighted mean root square for dX and dY are 0.137 and 0.173 mas, respectively.

We have then expressed the X_{obs} and Y_{obs} quantities as a function of H_d:

$$X_{obs} = H_d \left(\dot{X}_{th} t + X_{nut} \right) + \left(\dot{X}_g + \ddot{X}_{nl} \right) t + \delta X \quad (3)$$
$$Y_{obs} = H_d \left(\dot{Y}_{th} t + Y_{nut} \right) + \dot{Y}_{nl} t + \delta Y, \qquad (4)$$

where the subscripts "th", "nut", "g", and "nl" represent the theoretical contributions to precession rates, contributions from nutation, geodesic precession, and non-linear effect, respectively. The time t is measured in Julian centuries since J2000.0.

The Earth's dynamical flattening H_d has been estimated from the above equations by using a weighted least squares fit among the time series of X_{obs} and Y_{obs}. The expressions for $\dot{X}_g + \ddot{X}_{nl}$ and $\dot{Y}_g$ are taken from Capitaine et al., (2003) and Capitaine et al., (2005), respectively. A rigorous expression of $\dot{X}_{th}$ as a function of H_d can be obtained from Bourda and Capitaine, (2004). Different fits have been performed by using (i) equation (3) only, (ii) equation (4) only, and (iii) both equations (3) and (4) simultaneously. As the most significant effect of H_d is in the X component, especially for the contribution of the first order lunisolar term, the relative errors (for both theoretical and observational) in precession and nutation in the X component are much smaller than those in the Y component.

The value for H_d obtained in this way, from (i), is such that its relative difference with respect to the reference value H_{P03} of the IAU 2006 precession is about -14 ppm, which is much larger than expected.

The second method we have used in this work is an indirect estimation of H_d from the celestial pole offsets. We have used the time series, opa2012, of VLBI celestial pole offsets corrected for the FCN, but this time for determining H_d from the estimated corrections to the two most sensitive terms of the IAU precession-nutation to H_d, namely the t^1 term in precession and the 18.6-yr term of nutation. The celestial pole offsets dX and dY can then be written as follows (Capitaine et al., 2009):

$$\mathrm{d}X = A_0 + A_1 t + A_s \sin\Omega + A_c \cos\Omega$$
$$\mathrm{d}Y = B_0 + B_1 t + B_s \sin\Omega + B_c \cos\Omega, \qquad (5)$$

where Ω is the mean longitude of the ascending node of the Moon.

The coefficients of the above equations have then been derived by a weighted least squares fit as shown in Table 2. Then, we have fitted to A_1 the correction dH_d

Table 2 Weighted fits of a linear term plus a 18.6 yr periodic term to the opa2012 time series (1980–2012) of VLBI celestial pole offsets.

	coefficient	value (mas or mas/cy)	uncertainty
	A_0	0.02745	0.00221
X	A_1	0.14252	0.03139
	A_s	−0.01397	0.00267
	A_c	0.05762	0.00263
	B_0	−0.08655	0.00254
Y	B_1	0.09964	0.03614
	B_s	0.01886	0.00307
	B_c	−0.07189	0.00298

to the reference value H_{P03}. This gives a correction of 0.071 ppm in the Earth dynamical flattening, which is quite negligible.

3 Discussion

Possible ways to derive an accurate value for H_d using the best available precession-nutation models and VLBI observations are investigated; for that purpose, various theoretical contributions are considered for the first time in studying this Earth's global parameter.

The differences between the results obtained by two different methods of least squares fit need to be further investigated with considering all error sources for each term in the CIP X and Y coordinates. A possible reason for the discrepancy is due to the out-of-phase components of the main nutation terms, because most of the effects in those terms are not proportional to H_d. This paper is a first step of the work, which will be continued by the authors. Publication of this work is under preparation and is planed to be submitted to A&A in the near future.

Acknowledgements

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