Combining Datum-free Normal Equation Systems and Solutions with Full Covariance Information for Upcoming CRF Realizations

Andreas Iddink, Thomas Artz, Axel Nothnagel

Abstract Combination approaches within the International VLBI Service for Geodesy and Astrometry (IVS) are solely performed on the level of datum-free normal equations (NEQs). The procedure is used to compute the two major products of the IVS, i.e., time series of Earth orientation parameters (EOPs) and station coordinates. One shortcoming of the datum-free NEQ-based combination is the limitation to contributions based on the classical least squares adjustment and to analysis software packages supporting the output of datum-free NEQs. Hence, in order to increase the potential number of contributions, it would be a big gain to be able to include contributions based on solutions of a constrained NEQ.

In this paper, we present a method to mix the combination on the level of datum-free NEQ and on the solution level with full covariance information. We show the implementation of this approach in our existing software environment BonnSolutionCombination (BoSC) and discuss the prerequisites and the limitations. Furthermore, we show the benefits for upcoming Celestial Reference Frame (CRF) realizations.

Keywords VLBI, ICRF, intra-technique combination, datum-free normal equations, full variance-covariance information

1 Introduction

At present, six analysis centers regularly contribute their independent solutions to the International VLBI Service for Geodesy and Astrometry (IVS) [3, 6] for the computation of intra-technique combined products. This includes rapid solutions of Earth orientation parameters (EOPs) and quarterly solutions [2] of VLBI Terrestrial Reference Frames (VTRFs) [4] containing station positions and velocities. In multiple studies it has already been highlighted that the use of an intra-technique combination improves the stability and robustness of the combined results compared to the individual results [5]. In order to avoid that contributions are distorted by any constraints before combining them, the intra-technique combination is performed at the level of datum-free normal equation systems (NEQs). Thus, the same datum reference frames and an identical datum can be initially applied during the combination process. By using datum-free NEQs, it is guaranteed that the full variance-covariance information of all parameters and all individual contributions is rigorously transferred. The delivery and exchange of the NEQs is performed with the Solution Independent Exchange Format (SINEX).

Despite the advantages of the combination on the level of datum-free NEQs, some shortcomings can be identified. In the first place, the combination is limited to contributions based on the classical least squares adjustment. However, the similar parameterization of auxiliary parameters by linear splines in all solutions may lead to systematic effects. These might be mitigated by using solutions based on filter techniques as well since these are based on a different, i.e., stochastic, representation of the auxiliary parameters.

Secondly, analysis software packages which does not support the export of datum-free NEQs are excluded and cannot contribute to the combination. Hence, in order to overcome these shortcomings and to improve the robustness and stability of the combined

Institute of Geodesy and Geoinformation, University of Bonn

products even more, it would be a substantial progress to be able to include contributions based on solutions of constrained NEQs. Furthermore, by including source positions in the rigorous VLBI intra-technique combination, an improvement will be exploited for the Celestial Reference Frame (CRF) combination as well.

2 Reconstruction of Datum-free NEQs

In order to be able to compute TRFs, CRFs, and corresponding EOPs, it is essential that all contributing analysis centers (ACs) and software packages support the output of the entire set of parameters, including EOPs, all station coordinates, and all source positions. The common intra-technique combination is based on VLBI input data provided in the SINEX format in form of datum-free NEQs. In order to be able to include solutions with full covariance information in this combination procedure, the datum-free NEQs need to be reconstructed using information about the applied constraints. Due to the fact that the SINEX format allows for the storage of NEQs as well as of solutions of constrained NEQs, all necessary information for the reconstruction can be stored in the SINEX files.

In this case the file must contain the solution vector $\hat{x}$, the full covariance information of the estimated parameters $C_{\hat{x}\hat{x}}$, the a posteriori variance factor $\hat{\sigma}^2$, and the weighted square sum of residuals $\hat{v}^T P \hat{v}$ [1]. Furthermore, the most critical and important fact is that the full a priori variance-covariance matrix C_{xx} needs to be booked in the SINEX file as well. In order to be able to entirely reconstruct the datum-free NEQ matrix N, this matrix has to contain all applied constraints that were used for the parameter estimation. Under these prerequisites N can be reconstructed using [8]

$$N = \hat{\sigma}^2 C_{\hat{x}\hat{x}}^{-1} - \hat{\sigma}^2 C_{xx}^{-1} \tag{1}$$

$$y = \hat{\sigma}^2 C_{\hat{x}\hat{x}}^{-1} \hat{x} \tag{2}$$

$$l^T P l = (\hat{v}^T P \hat{v}) + y^T \hat{x} \tag{3}$$

Constraints which were used to stabilize the solution and refer to already reduced parameters are particularly critical. These hidden constraints are basically not booked in the SINEX file and consequently cannot be subtracted from the NEQs. In this case, the datum-free NEQs are not fully reconstructable. Existing correlations between remaining and reduced parameters can lead to deformations and systematic effects due to the fact that hidden constraints and new applied constraints might influence each other [7]. To avoid such incidents and in order to get an insight into the constraint characteristics, the number and type of rank defects need to be determined in a preceding step [9].

3 Software Implementation

The reconstruction of datum-free NEQs has been implemented in our existing software environment called BonnSolutionCombination (BoSC) (Figure 1).

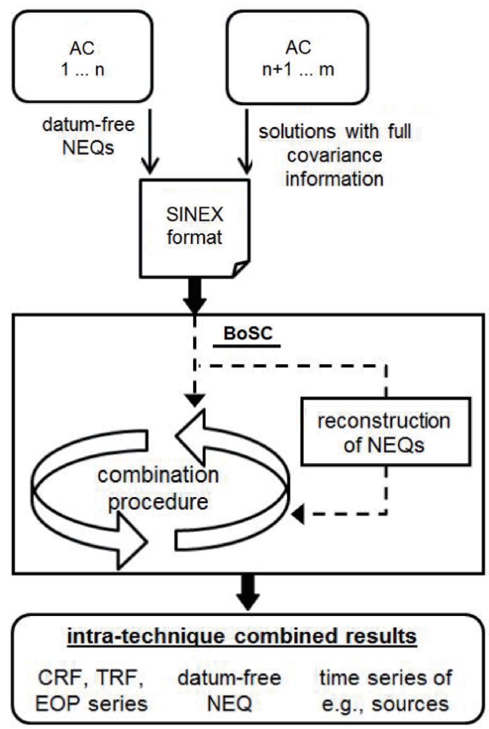

Fig. 1 Reconstruction of datum-free NEQs implemented in our software environment BonnSolutionCombination (BoSC).

Through this addition, the software obtains the capability to include contributions based on solutions with full variance-covariance information in the common combination strategy on the level of datum-free NEQs. Based on this, BoSC offers several capabilities and final products. It is possible to compute consistent VLBI intra-technique CRF and TRF with corresponding EOP series. Furthermore, a monolithic datum-free NEQ can be determined or time series of sources, stations, and EOPs can be illustrated. The full variance-covariance matrix of all parameters is carried forward from each AC and each observing session to the final products. All upcoming CRF investigations can profit from these capabilities, because the complemented software is not only suitable to generate a combined CRF but also to support any studies and investigations required previously.

4 Conclusion and Future Work

In this paper, an approach for combining datum-free NEQs and solutions with full variance-covariance information has been presented. Up to now the implemented reconstruction has been successfully tested with well-chosen and self-generated input data. In the next step we plan to use this new implemented feature for an inclusion of stand-alone catalogs with full variance-covariance information like K-band and Ka-band reference frames. Investigations concerning the properties and features of such a combined CRF have to be made in upcoming studies. This includes also the expectable difficulty with hidden constraints in real input data.

References

1. D. Angermann, H. Drewes, M. Krügel, B. Meisel, M. Gerstl, R. Kelm, H. Müller, W. Seemüller, V. Tesmer. ITRS Combination Centre at DGFI - A terrestrial reference frame realization 2003. *Deutsche Geodätische Kommission*, Reihe B, Nr. 313, Munich, 2004
2. S. Bachmann, M. Lösler. Activities and Products at IVS combination center at BKG. In: N. Zubko and M.Poutanen (Eds.): *Proceedings of the 21st Meeting of the European VLBI Group for Geodesy and Astrometry*, pp. 169–173, ISBN: 978-951-711-296-3, 2013
3. D. Behrend. Data Handling within the International VLBI Service. *Data Science Journal*, Vol. 12, pp. WDS81–WDS84, ISSN 1683-1470, 17 February 2013. DOI 10.2481/dsj.WDS-011
4. S. Böckmann, T. Artz, A. Nothnagel. VLBI terrestrial reference frame contributions to ITRF2008. *Journal of Geodesy*, 84, pp. 201–219, doi: 10.1007/s00190-009-0357-7, 2010
5. S. Böckmann, T. Artz, A. Nothnagel, V. Tesmer. International VLBI Service for Geodesy and Astrometry: Earth orientation parameter combination methodology and quality of the combined products. *Journal of Geophysical Research*, Vol. 115, doi:10.1029/2009JB006465, 2010
6. H. Schuh, D. Behrend. VLBI: A fascinating technique for geodesy and astrometry. *Journal of Geodynamics*, 61, doi: 10.1016/j.jog.2012.07.007, pp. 68–80, 2012
7. M. Seitz. Kombination geodätischer Raumbeobachtungsverfahren zur Realisierung eines terrestrischen Referenzsystems. *Reihe C 630 Deutsche Geodätische Kommision*, Munich, 2009
8. M. Seitz, D. Angermann, M. Bloßfeld, H. Drewes, M. Gerstl. The 2008 DGFI Realization of the ITRS: DTRF2008. *Journal of Geodesy*, Volume 86, Issue 12, pp. 1097–1123 , DOI:10.1007/s00190-012-0567-2, 2012
9. R. Kelm. Rank Defect Analysis and Variance Component Estimation for Inter-Technique Combination. In: B. Richter, W. Schwegmann and W. Dick (Eds.): *Proceedings of the IERS Workshop on Combination Research and Global Geophysical Fluids*, Bavarian Academy of Sciences, Munich, Germany, 18–21 November 2002. (IERS Technical Note 30) Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie