On the Definition of Aberration

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Abstract There was a groundbreaking step in the history of astronomy in 1728 when the effect of aberration was discovered by James Bradley (1693–1762). Recently, the solar acceleration, due to the variations in the aberrational effect of extragalactic sources caused by it, has been determined from VLBI observations with an uncertainty of about 0.5 mm$\cdot$s$^{-1}\cdot$yr^{-1} level. As a basic concept in astrometry with a nearly 300-year history, the definition of aberration, however, is still equivocal and discordant in the literature. It has been under continuing debate whether it depends on the relative motion between the observer and the observed source or only on the motion of the observer with respect to the frame of reference. In this paper, we will review the debate and the inconsistency in the definition of the aberration since the last century, and then discuss its definition in detail, which involves the discussions on the planetary aberration, the stellar aberration, the proper motion of an object during the travel time of light from the object to the observer, and the way of selecting the reference frame to express and distinguish the motions of the source and the observer. The aberration is essentially caused by the transformation between coordinate systems, and consequently quantified by the velocity of the observer with respect to the selected reference frame, independent of the motion of the source. Obviously, this nature is totally different from that of the definition given by the IAU WG NFA (Capitaine, 2007) in 2006, which is stated as, "the apparent angular displacement of the observed position of a celestial object from its geometric position, caused by the finite velocity of light in combination with the motions of the observer and of the observed object."

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1 Introduction

In 1725, English astronomer James Bradley conceived of the diameter of the earth orbit around the sun as a base line to detect the apparent displacement of Gamma Draconis during one year, namely the effect of parallax. But what he observed at that moment is the effect of aberration rather than that of parallax, because the parallax effect of Gamma Draconis is three magnitudes smaller than that of the aberration effect. The stellar aberration was discovered by him with the observations of more 200 stars (Gualandi and Bnoli, 2009).

The effect of aberration, based on the apparent variations in positions of the objects detected from observations, can be simply modeled as:

$$\delta\mathbf{k} = \frac{1}{c}\mathbf{k} \times (\mathbf{k} \times \mathbf{V}), \tag{1}$$

where c is the velocity of light in vacuum and $\mathbf{k}$ is the geometric direction to the object. The immediate question is, from both the theoretical and practical point of view, what the velocity $\mathbf{V}$ in Equation 1 is. Concerned with this question, there have been a lot of discussions and debates since the beginning of the last century. In the specialized note taken down by Turner (1909), that very question was put forward directly as the following: "Does the correction for aberration in the position of an observed heavenly body P depend on the velocity of the observer relative to P, or upon his absolute velocity in space?" Even though to speak of the absolute velocity of a body may be somewhat improper and can be misleading, it nevertheless explicitly proposes the question if the motion of the observed object is involved in the effect of aberration. After that the definition of aberration remains equivocal.

In light of the inconsistency associated with the aberration, this paper is dedicated to clarify the definition of aberration, to discuss various kinds of its notion involved, and eventually to get a consistent and basic definition for both theoretical consideration and practical applications. To abstract the inconsistency here, there are three reasons to discuss the definition of aberration. First, as one of the basic concepts with great importance in astronomy, the definition has undergone for a long time a debate, which lasts until present. Secondly, the inconsistency of two kinds of notions leads to the ambiguity and practical impacts on the calculation of this effect; and some recent studies of aberration even obtain results that cannot withstand a careful inspection. Thirdly, as will be discussed later in this paper, the nominal explanation of aberration given officially by the IAU WG can be misleading, which should be clarified and revised.

Regarding the basic concept of aberration, we think that the effect of aberration depends on the motion of the observer with respect to a selected reference frame, but not at all on the motion of the object; it describes the apparent displacement of position of the observed object caused by the variation in the motion of the observer. Moreover, the planetary aberration is pervasively called aberration as well, but it is more than that; the definition of planetary aberration, which depends on the relative motion of the observer and the object, contains two effects: the stellar aberration and the proper motion of the object during the travel time of light. However, the planetary aberration, both its definition and application, should be abandoned in order to avoid the potential misunderstanding. One crucial reason is that it is only an approximation of modeling these two effects based on some assumptions, which are not satisfied in most circumstances.

In Section 2 the exact expression for the aberration effect is derived in the context of Special Relativity, which is not an original work but is very helpful in explaining the pertinent notions of the next sections. In Section 3 we present the derivation of the model of the planetary aberration, and then we discuss the differences between planetary aberration and stellar aberration. Section 4 summarizes the arguments.

2 The Expression of the Aberration Effect

In the context of Special Relativity the derivation of the model of the aberration effect can be found in some excellent books, such as Green (1985). In order to have a clear understanding of its essence, a simple description of this procedure is given here.

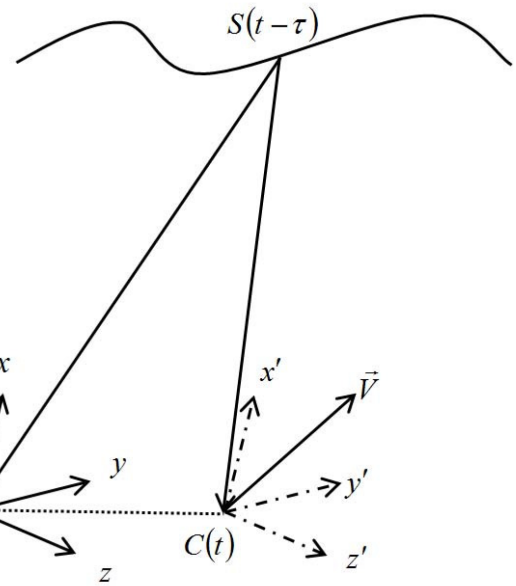

Fig. 1 Geometry of the observation.

As shown in Figure 1, imagine that an observer receives the photon at the time t_0 emitted from an object at an earlier time $(t_0 - \tau)$, τ being the travel time of the photon from the object to the observer. Suppose that two inertial reference frames, $O - xyz$ and $C - x'y'z'$, in relative motion with the velocity $\mathbf{V}$, exactly coincide at the observing time t_0. Let $C - x'y'z'$ be fixed to the observer and move with him. Therefore, the Lorentz transformation between these two frames, expressing C's coordinates in terms of O's, is given by:

$$\begin{aligned}
x' &= \left[1+\frac{\gamma^2 V_x}{(\gamma+1)c^2}\right]x + \frac{\gamma^2 V_x V_y}{(\gamma+1)c^2}y + \frac{\gamma^2 V_x V_z}{(\gamma+1)c^2}z \\
&\quad - \gamma V_x(t-t_0), \\
y' &= \frac{\gamma^2 V_y V_x}{(\gamma+1)c^2}x + \left[1+\frac{\gamma^2 V_y}{(\gamma+1)c^2}\right]y + \frac{\gamma^2 V_y V_z}{(\gamma+1)c^2}z \\
&\quad - \gamma V_y(t-t_0), \\
z' &= \frac{\gamma^2 V_z V_x}{(\gamma+1)c^2}x + \frac{\gamma^2 V_z V_y}{(\gamma+1)c^2}y + \left[1+\frac{\gamma^2 V_z}{(\gamma+1)c^2}\right]z \\
&\quad - \gamma V_z(t-t_0), \qquad (2)\\
t' &= -\frac{\gamma V_x}{c^2}x - \frac{\gamma V_y}{c^2}y - \frac{\gamma V_z}{c^2}z - \gamma(t-t_0), \\
\gamma &= \sqrt{(1-V^2/c^2)}.
\end{aligned}$$

Let the velocity of light in $C-x'y'z'$ be $\mathbf{V}(V_x,V_y,V_z)$. Its norm in general is a constant c independent of the observer. The travel path of the observed photon as measured in the frame $C-x'y'z'$ is expressed by the following form:

$$\begin{aligned}
x &= v_x(t-t_0), \\
y &= v_y(t-t_0), \qquad (3)\\
z &= v_z(t-t_0).
\end{aligned}$$

When the direction of the light path is the normalized velocity vector itself, then we have the apparent direction of the object in the rest frame of the observer, $C(t)S(t-\tau)$ denoted by a unit vector $\mathbf{k}'$:

$$\mathbf{k} = (-\frac{dx}{cdt}, -\frac{dy}{cdt}, -\frac{dz}{cdt}). \qquad (4)$$

Then, it is a straightforward matter to consider this process in the frame $O-xyz$. In this frame, denoting the velocity of light by $\mathbf{v}(v_x,v_y,v_z)$, its path is given by:

$$\begin{aligned}
x' &= v'_x t', \\
y' &= v'_y t', \qquad (5)\\
z' &= v'_z t'.
\end{aligned}$$

Consequently, the direction of the object as measured in the frame $O-xyz$, denoted by $\mathbf{k}$, is analogically expressed as:

$$\mathbf{k}' = (-\frac{dx'}{cdt'}, -\frac{dy'}{cdt'}, -\frac{dz'}{cdt'}). \qquad (6)$$

By using Lorentz transformation to relate Equations 4 and 6, we have the expression of $\mathbf{k}'$ in terms of $\mathbf{k}$,

$$\mathbf{k}' = [\frac{1}{\gamma}\mathbf{k} + \frac{\mathbf{V}}{c} + \frac{\gamma(\mathbf{k}\cdot\mathbf{V})\mathbf{V}}{(\gamma+1)}]/(1+\frac{\mathbf{k}\cdot\mathbf{V}}{c}). \qquad (7)$$

which is, in the context of Special Relativity, the complete formula for the aberration effect. It becomes more apparent when expanding it using the binomial theorem,

$$\begin{aligned}
d\mathbf{k} = \mathbf{k}' - \mathbf{k} &= \frac{1}{c}\mathbf{k}\times(\mathbf{V}\times\mathbf{k}) + \\
&+ \frac{1}{2c^2}\left[2(\mathbf{k}\cdot\mathbf{V})^2\mathbf{k} - (\mathbf{k}\cdot\mathbf{V})\mathbf{V} - \mathbf{V}^2\mathbf{k}\right] \qquad (8)\\
&+ O\left(\mathbf{V}^3/c^3\right).
\end{aligned}$$

The first term on the right-hand side of this equation is the classical model of the aberration effect, Equation 1, which merely contains the first order of the actual effect.

This process demonstrates how the aberration effect is introduced when one makes the Lorentz transformation. That is why, in the group delay model of VLBI observations, there is no need to consider the correction of the aberration effect for the source position in a direct way as the atmospheric effect does; it is implicitly hidden in that model as a result of the coordinate transformation (Petit & Luzum 2010). The effect of aberration indeed describes the apparent displacement in the positions of an object obtained by different observers in relative motion. So, when analyzing the measurements of direction, we need to portrait the motions of observers in a given frame and correspondingly make the correction for aberration according to Equations 7 or 8.

3 Planetary Aberration

The term planetary aberration is conventionally used to describe the displacement between the geometric direction and the apparent direction of the object at the time of observation t; while the term stellar aberration describes the displacement between the geometric direction of the object at the time of emission $t-\tau$ and its apparent direction at the time of reception t. Therefore, the planetary aberration actually includes the stellar aberration and the light-time correction, arising from the object's movement during the time interval τ.

Suppose that an observation is made at the instant of time t. Figure 2 shows the geometry of the object and the observer for this observation. Let the points C and S represent the observer and the object, respectively.

Let R be the displacement vector of the object due to its movement during τ, and U be its velocity relative to the barycenter of the solar system. Denote the vectors $C(t)S(t)$ and $C(t)S(t-\tau)$ by $\rho\mathbf{k}$ and $\rho'\mathbf{k}'$, where $\mathbf{k}$ and $\mathbf{k}'$ are unit vectors and then represent the geometric direction of the object at the instant $t-\tau$ and t, respectively. Clearly the geometry shows that

$$\rho\mathbf{k} - \rho'\mathbf{k}' = \mathbf{R}, \tag{9}$$

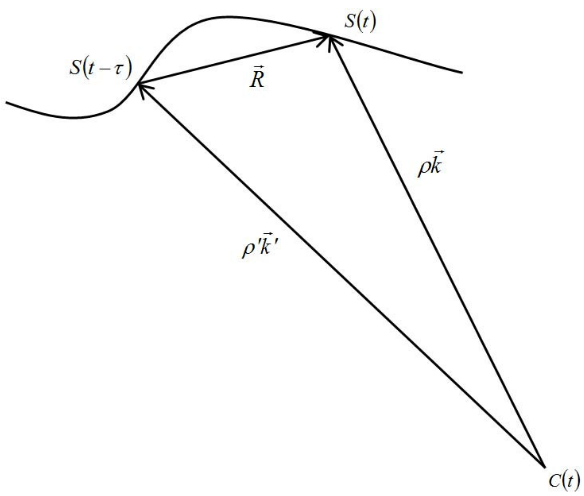

Fig. 2 Displacement of the source due to its motion during the light transmitting time.

Actually, this is the exact formula for the light-time correction. But, for its implicit and useful formula, two assumptions are required. If $R \ll \rho$, which satisfies that $\rho' \approx \rho$ and $\mathbf{k}' \cdot \mathbf{k} \approx 1$, after a little reduction, this equation can be written as,

$$\mathbf{k}' - \mathbf{k} = -\frac{1}{\rho}\mathbf{k} \times (\mathbf{R} \times \mathbf{k}). \tag{10}$$

The time interval τ by definition is $\tau = \frac{\rho}{c}$, and then the displacement vector of the object during this time can be found through integration,

$$\mathbf{R} = \int_{t-\tau}^{t} \mathbf{u}dt. \tag{11}$$

However, if we make the second assumption that within this interval τ the object is moving with a uniform velocity, it is more convenient to simply write Equation 11 as

$$\mathbf{R} = \mathbf{u}\tau = \frac{\mathbf{u}}{c}\rho. \tag{12}$$

The approximate expression for the light-time correction, by substituting Equation 12 into 10, can be found to be

$$\mathbf{k}' - \mathbf{k} = -\frac{1}{c}\mathbf{k} \times (\mathbf{u} \times \mathbf{k}). \tag{13}$$

This expression for the light-time correction closely parallels that for the aberration effect, as Equation 7. But as we can see from the derivation of it, provided that the motion of the object within the traveling time of the light can be regarded as constant, the light-time correction only depends on the motion of the object, which has the same form as that of stellar aberration to the first order. In addition, the light-time correction cannot have the terms of higher orders in Equation 8, and thus, the relative velocity of the observer and the object cannot be applied for the second order of Equation 8 or the exact expression (7). Their similarity ends to the first order and is mathematically accidental.

The correction for planetary aberration, according to its definition and Equations 1 and 13, then is given by

$$d\mathbf{k} = \frac{1}{c}\mathbf{k} \times [(\mathbf{V} - \mathbf{u}) \times \mathbf{k}]. \tag{14}$$

As one might expect, the displacement of planetary aberration depends on the relative motion of the observer and the object.

From both theoretical and practical considerations, the definition of planetary aberration, that aberration depends on the relative motion of the observer and the object, should be abandoned, because it can never be used for the stars and even for planets it is insufficient.

4 Discussion and Conclusions

In astrometry, the directly observed positions of a celestial object are affected by some systematical variations, such as parallax, aberration, atmospheric refraction, gravitational deflection, and so on. The basic principle for the data analysis, in order to make the measurements obtained by different observers comparable, is to take into account all kinds of effects caused by the personalities of the observers, referring to such a common fictitious observer as the barycenter of the solar system. After that, the proper motion caused by the motion of the object often remains, if detectable, as a signal embedded in the resulting time series of positions that can be analyzed and studied later. The aberration effect depends on the variation of the motion of the observer and is independent of the motion of the object.

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