Selecting Sources that Define a Stable Celestial Reference Frame with the Allan Variance

Karine Le Bail[1], David Gordon[1], Chopo Ma[2]

Abstract The ICRF2 was adopted by the IAU in 2009 and was based on the positions of 3,414 radio sources determined by VLBI. Discussions on the next realization of the ICRF (ICRF3) have been underway within the IAU and IVS since 2012. VLBI has made significant advances since ICRF2. From the latest GSFC solution, we extract a set of sources that defines a stable celestial reference frame, as shown by Feissel-Vernier 2003 [1] using tools such as the Allan variance and the drift of the position time series. This method also allows us to highlight a set of the least stable sources that may need special handling.

Keywords ICRF, radio sources, source selection, Allan variance, defining sources, special handling sources

1 Introduction

In this paper, we investigate two questions of interest for the realization of the ICRF3. The first objective is to select a set of sources that define a stable celestial reference frame. The method used is inspired by the study of Feissel-Vernier 2003 [1] using the Allan variance and the drift of the position time series. The second objective is to identify sources that degrade the stability of the celestial reference frame to be able to handle them in the same way as the set of sources that were called the 'special handling' sources in ICRF2. The method proposed here uses the results of the previous method, but also takes into account the type of noise determined by the Allan variance.

In the second section of this paper, we discuss the solution of source position time series we studied. In the third section, we detail our analysis as well as the tools used. The fourth section shows the results obtained and proposes tools and criteria organized in a detailed method to determine different sets of sources: "stable" sources and sources that would need to be handled differently in the analysis because they degrade the celestial reference frame (called special handling sources in ICRF2).

2 Data

The set of VLBI position time series we analyzed in this paper was produced with the Calc/Solve software by D. Gordon at GSFC. It used VLBI sessions from August 1979 through October 2015. It contains 4,081 sources, including the VCS sources.

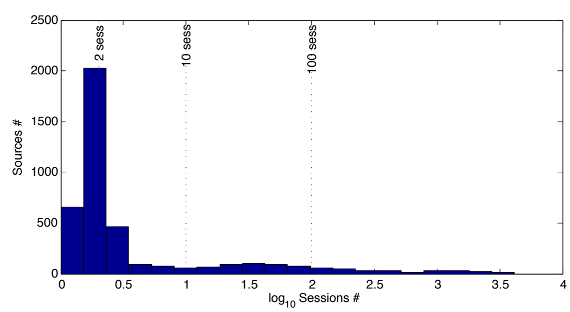

Fig. 1 Distribution of the number of sessions each source was observed in for the 4,081 sources studied in this paper.

1. NVI, Inc., NASA GSFC, Greenbelt Rd, Greenbelt, MD 20701
2. NASA Goddard Space Flight Center

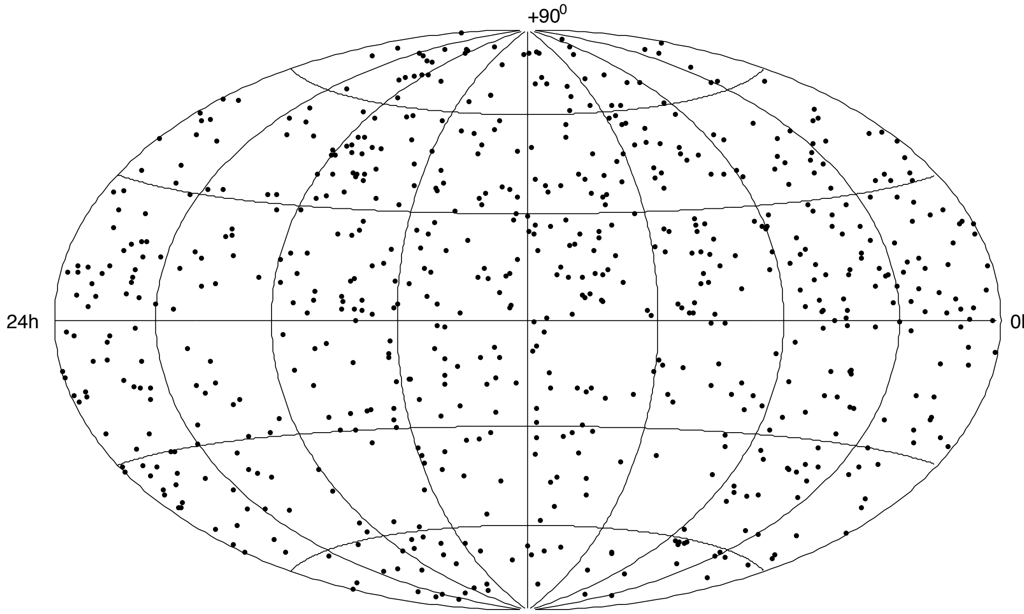

Fig. 2 Map of the 724 sources studied in this paper.

To be significant, the Allan variance analysis needs to be done on a reasonable number of points. For this reason, we used only 724 sources that were observed in more than ten sessions, shown in Figure 1. These sources are still reasonably distributed as seen in Figure 2.

In previous studies (Feissel-Vernier 2003 [1], Le Bail and Gordon 2010 [2], Le Bail et al. 2014 [3]), the analysis was done independently on Right Ascension and Declination, and then a stability index was built by combining the values obtained on each coordinate.

To take into account both coordinates at the same time, we decided to convert the series (Right Ascension, Declination) into Arc Lengths (see Bolotin and Lytvyn 2009 [4]). We compute the Arc Length as the angular distance between the position $P_i(\alpha_i, \delta_i)$ of a given source at time t_i and its average position over time $P_m(\alpha_m, \delta_m)$. As this distance is rather small, we use the haversine formula which is better conditioned:

$$l_{im} = 2\arcsin\left\{\sin^2\left(\frac{\delta_m - \delta_i}{2}\right) + \cos(\delta_i)\cos(\delta_m)\sin^2\left(\frac{\alpha_m - \alpha_i}{2}\right)\right\}^{0.5}$$

Figure 3 illustrates an example for the source 0642+449.

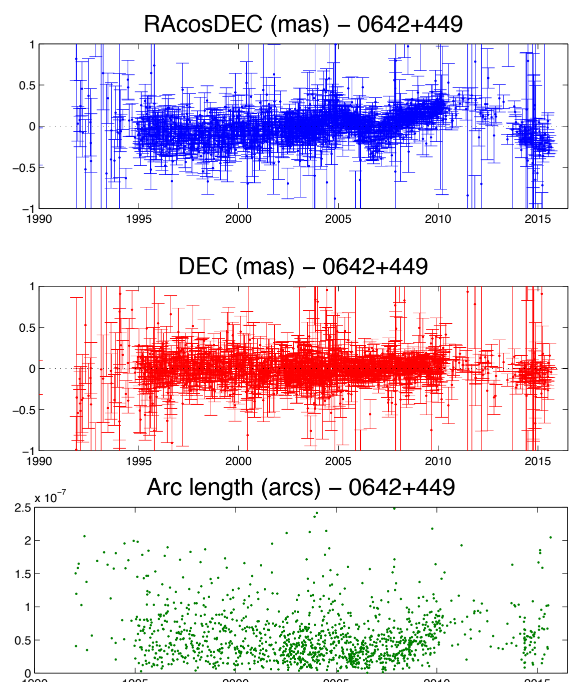

Fig. 3 Right Ascension, Declination and Arc Length of the source 0642+449.

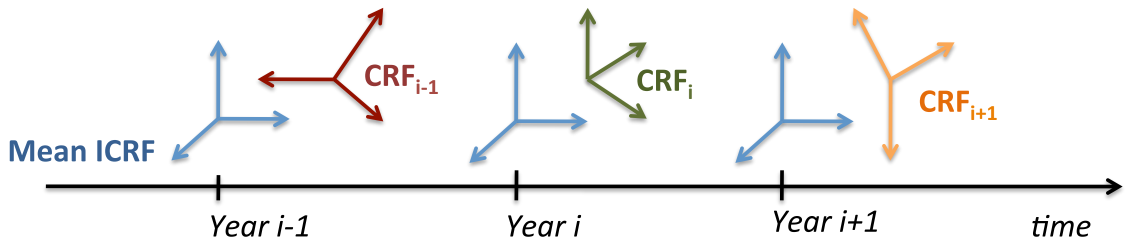

Fig. 4 Stability test scheme (see [2]).

3 Analysis

To obtain the type and the level of noise, we use the Allan variance. If $(x_i)_i$ are the measurements and τ the sampling time, the Allan variance at τ is defined by: $\sigma^2(\tau) = \frac{1}{2} < (\widehat{x_{i+1}} - \widehat{x_i}) >^2$. The type of noise is obtained by computing the slope of the Allan variance curve in a plot $(log_{10}(\sigma^2(\tau)), log_{10}(\tau))$. A slope of -1 indicates white noise, 0 indicates flicker noise, and $+1$ indicates random walk.

The time series are analyzed over the period January 1990 to October 2015. The 724 remaining sets of time series are not all suitable for our analysis with the Allan variance (e.g., data span not long enough, large gaps in between consecutive data, poor number of points in a year). So we eliminated the time series of 207 sources. Our method is applied on a reduced set of 517 sources.

We construct a source stability index *SIndex* inspired by Feissel-Vernier 2003 [1] as the combination of the normalized value of the drift and the Allan standard deviation at one-year sampling time ($\sigma^2(\tau = 1\ year)$). We derived two *SIndex* per source: one studying the series (Right Ascension, Declination) and a second one obtained from the Arc Length time series.

The sources are then sorted depending on the *SIndex* from the less stable to the most stable. To test the stability of a given subset of sources, we calculate the rotation parameters between two celestial reference frames realized by this subset: one is the yearly mean realization and the other one is the mean computed over the entire period (see Equation below and Figure 4). We obtain $(A_1(i), A_2(i), A_3(i), dz(i))_i$ for each year i and we look at the standard deviation and the mean of the quantity $(A_1(i), A_2(i), A_3(i), dz(i))_i$ for each subset.

$$\begin{cases} (\alpha_m - \alpha_i)\cos\delta_m = & A_1(i)\ \tan\delta_i \cos\alpha_i \\ & +A_2(i)\ \tan\delta_i \sin\alpha_i\ - A_3(i) \\ \delta_m - \delta_i = & -A_1(i)\ \sin\alpha_i \\ & +A_2(i)\ \cos\alpha_i \qquad + dz(i) \end{cases}$$

where α_i and α_m are the Right Ascension of the source at year i and mean, respectively; δ_i and δ_m are the Declination of the source at year i and mean, respectively.

This scheme is applied to (Right Ascension, Declination) and then to the series (Arc Length).

4 Results

The standard deviations and means for each subset are presented in Figure 5. The study made when analyzing Right Ascension and Declination independently is shown in blue circles, and when analyzing Arc Length in red diamonds.

The two approaches give similar results, especially for the smallest subsets of sources (right side of the plots).

If we choose the 270 most stable sources, 201 sources are common for both selection methods.

The less stable sources seem to impact significantly the frame. As the source stability indices are computed as a combination of the drift and the level of noise (Allan variance for a sampling time of one year), this may be due to the instability of the source (e.g., structure) and/or poor accuracy due to the small number of points in the time series. It tends to show that this stability index may not be sufficient to detect sources that need special handling. However, we said previously that the Allan variance gives also an indication on the type of noise: sources exhibiting flicker noise or random walk can be considered as unstable and should be studied in more details.

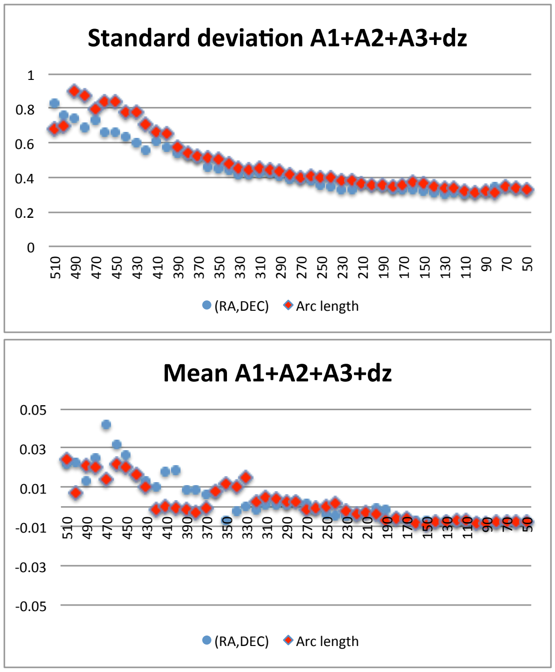

Fig. 5 Standard deviation and mean of the quantity A1+A2+A3+dz.

We have then two criteria on which to judge the stability of a source: the stability index and the type of noise.

Let's take the set of 39 special handling sources that were listed in ICRF2. Thirty of them are studied in this paper. All 30 sources are considered unstable: eight are considered unstable by the stability index *SIndex*, fifteen by the type of noise, and seven by both criteria.

But this study permitted the detection of other non-special handling sources in ICRF2, that are unstable. An example is the source 0642+449 which coordinates are represented in Figure 3. When computing the Allan variance on each coordinate (Right Ascension, Declination, and Arc Length), the corresponding slopes are respectively 0.23 ± 0.24, 0.08 ± 0.35, and 0.41 ± 0.03, which points on flicker noise in the time series. This is a very important conclusion, especially because 0642+449 is an ICRF2 defining source.

5 Conclusions

The methods used in this study allows us to select sources that would define a stable celestial reference frame. When combining the source stability index and the study of the type of noise with the Allan variance, the selection can be also extended to sources that need special handling.

The limitation of this method is that it depends greatly on the Allan variance which is statistically significant when the sources are regularly observed. When averaging yearly, 336 of the 724 sources had less than ten points per year each year.

To remedy the problem and to be able to investigate all sources of a set, more criteria should be considered:

1. criteria that could be computed even with a low number of observations: level of noise using the regular standard deviation, drift of the time series;
2. criteria that are significant when the source is sufficiently observed: level of noise using the Allan variance at different sampling time, type of noise using the Allan variance on regularized series averaged in different periods from 7 days to 1 year;
3. criteria that indicate the physical nature of structure: Structure Index SI from Fey & Charlot 1997 [5], time series of flux values.

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