Toward Reliable Estimates of the Free Core and Inner Core Parameters from a Bayesian Inversion of VLBI and Gravimetric Data

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Abstract During the last decades, many authors have provided their own estimates of the FCN parameters. Some of them have also tried to detect and characterize the FICN. The agreement between all of these studies is far from satisfying, however, whatever data set is used, either VLBI or gravimetric data. Before providing another set of estimates from a joint Bayesian inversion of data from both aforementioned techniques, we endeavor in the present work to quantify the accuracy with which the FCN and FICN could be determined, considering the available data and the likely errors that distort them. Our synthetic tests suggest that if the FCN period and quality factor could be estimated with uncertainties smaller than one day and 10% respectively, the FICN is still much more difficult to characterize, assuming it can be routinely detected at all.

Keywords FCN, FICN, Chandler Wobble, nutations, VLBI, gravimetry, Bayesian inversion

1 Introduction

The resonance due to the Earth's outer core and its nutation (Free Core Nutation, FCN) has long been characterized consistently from two independent data sets, namely the nutations amplitudes [3, 6, 4, 11, 15] estimated from Very Long Baseline Interferometry (VLBI) and the tidal gravimetric factors estimated from surface gravity variations recorded by superconducting gravimeters (SG) worldwide [7, 2, 10, 11]. The parameters of the Free Inner Core Nutation (FICN) have also been estimated but, so far, these values have been notably less reliable due to the weakness of the associated resonance amplitude, compared to the data uncertainty. This fact, along with the improvement of data analysis beforehand and still longer time series in both above mentioned techniques, has motivated our new attempt at estimating the Earth's interior parameters. Our main concern in this work is the assessment of the reliability of the estimated geophysical parameters. In particular, we focus on the FICN period and quality factor, which are especially difficult to reliably determine and for which previous studies have yielded results which are not in good agreement. In this regard, it is worth noticing that even the theoretical framework describing the FICN is still the topic of important ongoing works [9, 1].

2 Synthetic Data

To assess our capability to accurately determine the FCN and FICN parameters, we generate and invert synthetic data in which we can introduce perturbations as needed; the inversion of actual VLBI and gravimetric data will be discussed elsewhere. The synthetic data are generated using the formalism of [6], which we will name MHB in this work. In brief, we use the MHB resonance formula, whose parameters are also given in [6], to compute the synthetic nutation data set to be inverted. An example of such a data set with some noise added is depicted in Figure 1. The uncertainties associated with each data point is borrowed from the Paris Observatory VLBI analysis to be as realistic as possible. Then, we can add diverse perturbations to

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these synthetic data: different levels of noise, outliers, or oceanic and atmospheric perturbations for example, and we check that we are able to properly recover the parameters used to generate the data.

In the following inversions, the annual prograde nutation is not included because of the abnormally small formal uncertainty associated with it, which puts an excessive weight on this specific data point. Indeed, because the amplitudes of the nutations have almost the same formal errors, the largest ones, such as the annual nutation, have especially small uncertainties, i.e. large weight, after normalization by the corresponding rigid Earth nutations, as required by the transfer function formalism of MHB. Apart from the annual prograde component, a few other nutations were also removed from the initial data set for similar reasons.

3 Synthetic Tests

We apply the Bayesian method to invert the synthetic nutations described in Section 2. Such an inversion technique is described in [4]; the key points of the Bayesian inversion are listed in [12], along with an interesting discussion of the philosophy behind it. We will not discuss here all the subtleties of the Bayesian inversion but it is worth reminding some of its important components. The estimated parameters are all given by a probability distribution instead of a single number with an associated uncertainty; these distributions reflect our limited knowledge. Thus, both the prior parameters and their posterior estimates take the form of distributions, the latter being summarized by *credible intervals* which have to be interpreted in the following way: knowing the data, there is a probability p that the real model (or parameter) is in the credible region (or interval) computed for value p. The credible intervals (CI) given here are computed for $p = 0.68$, which is similar to a 1-sigma interval in the case of a Gaussian posterior distribution. In the synthetic tests, to validate our method, we always check that the credible intervals provided by the Bayesian inversion are fully compatible with the expected results. The PyMC Python library [8] was used throughout this work to define the prior distributions and run the Monte Carlo Markov Chains on which our Bayesian inversion relies.

The first synthetic tests aim at estimating the influence of a broad but restrained random perturbation on all of the nutations. The origin of such perturbations can be diverse but in the synthetic tests, we simply generate them as a Gaussian noise whose standard deviation agrees with the claimed uncertainty of each data point. Of course, this agreement is not guaranteed to be valid in reality because the formal errors can be under- or overestimated for several nutations. For that reason, we will hereafter also discuss the influence of an outlier, a single strongly biased data point.

The inversion of noiseless synthetic data yields a period of -430.2 solar days for the FCN, as seen in the CRF, with a CI spanning 0.1 day. This value has to be compared with the target value of -430.21 days given by MHB and used to generate the synthetic data. The FCN quality factor is also properly determined, with the CI ranging from 19,800 to 20,200 for a target value of 20,046. The FICN is not so well characterized, even without any perturbation in the data: both its period and quality factor are determined with an uncertainty of about 10% to 20% and, for the period, the median estimate of $+995$ days is notably smaller than the target value of $+1,028$ days, keeping in mind that the data are noiseless. This fact suggests that the information contained in the data is not sufficient to strongly constrain the FICN parameters.

Adding noise in the data as described above, the results are similar for the FCN, despite, of course, somewhat larger CI. The resonance associated with the FCN is thus relatively insensitive to a realistic level of noise. On the contrary, several tests with the same level of noise but different noise values suggest that the estimation of the FICN parameters can be much more influenced by the noise. The difference between the median estimate and the target value can be as large as 20% for both the period and quality factor. Considering the weakness of the resonance associated with the FICN, it is surprising, however, that its parameters can still be generally well retrieved in spite of the substantial perturbations. Figure 2 depicts 1,000 models sampled by the Bayesian inversion for a synthetic data set with noise. The FCN is obviously well characterized whereas the spreading of the samples is much more visible for the FICN, even if, as stated before, the associated resonance is still reasonably well adjusted. When the noise level is increased by a factor of 2, the FICN cannot be retrieved anymore. The reason is twofold: firstly, the resonance becomes indistinguishable from the noise, and secondly, the uncertainty associated with the data does not reflect the actual er-

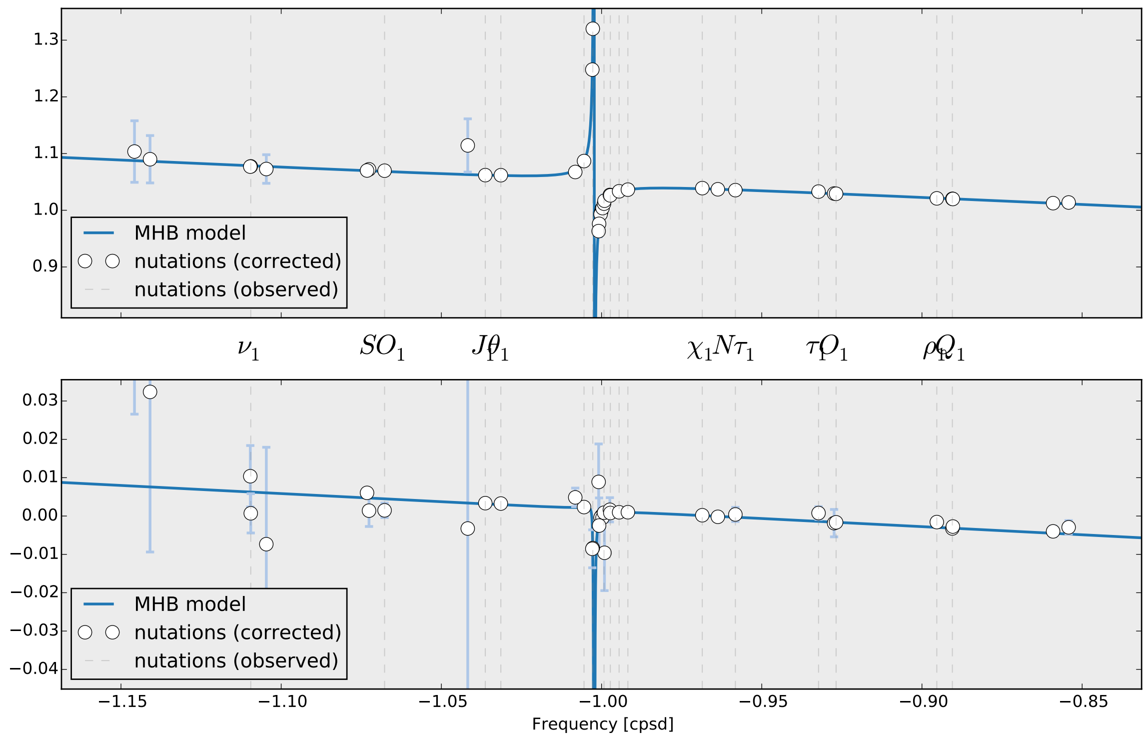

Fig. 1 Real (top) and imaginary (bottom) part of the MHB transfer function (blue line) and the generated synthetic data points with some noise added (white dot). The error bars are borrowed from the actual data of the Paris Observatory VLBI analysis. The frequency is in solar days in the terrestrial reference frame. Some nutations have been annotated with the corresponding tidal wave.

ror anymore, which is larger than expected, and the inversion fails to circumvent that problem. It is thus primordial not to underestimate the error on the nutations amplitudes before doing such an inversion.

Focusing on the FICN, we have also made synthetic tests with an outlier just next to the FICN resonance and synthetic tests with the FICN shifted toward higher or lower frequencies. These tests assess our ability to properly detect the FICN even if its period is somewhat larger or smaller than the actual standard estimate of about 1,000 days. This point is not critical for the FCN whose associated resonance is much bigger and surrounded by many nutations, contrary to the FICN. The results show that, even for the weak FICN resonance, neither an isolated outlier nor a departure of the resonance frequency from the expected value are enough to prevent a proper estimation of the eigenmode parameters. In the worst case, the CI become quite large (several tens of percent of uncertainty), especially regarding the quality factor, but the resonance can still be characterized with the target value of its parameters lying within the CI.

The last synthetic test we will discuss here is a complement to the noise tests, addressing the sensitive question of the atmospheric perturbation. Such a perturbation can reach up to 100 μas, with the largest perturbations being in the annual and semi-annual nutations. The oceans are a bigger perturbation for the nutations than the atmosphere, but they can be better corrected or even included in the model parameters, which is why we focus on the atmospheric effect here. The perturbations of the nutations amplitudes used in these tests were computed using the ECMWF atmospheric model. The results of adding the oceanic effect slightly differ from the results obtained when adding a realistic noise in the data. The target values are not within the CI anymore, except for the FICN period, which is probably just a lucky coincidence. The FCN period and quality factor do not dramatically depart from the target value, however, with an error on the period smaller than half a day and a quality factor close to 16,000 instead of 20,000. On the contrary, the quality factor of the FICN is notably overestimated, with a median value exhibiting both a large bias (1,500 instead of 640) and a large

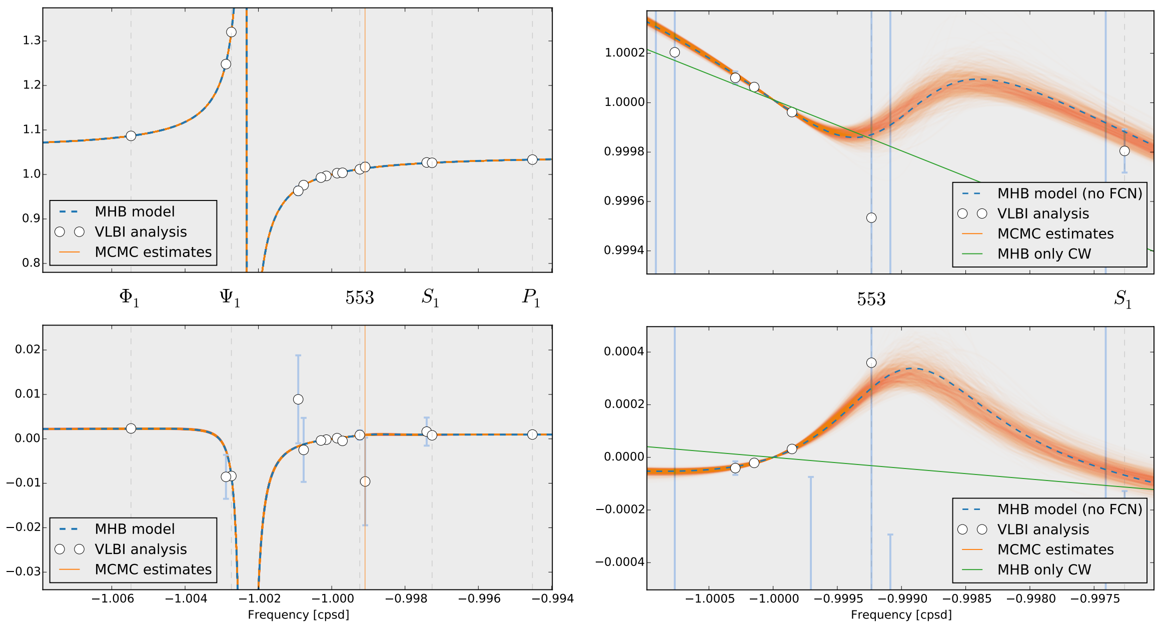

Fig. 2 One thousand models sampled by the Monte Carlo Markov Chain algorithm (orange) plotted against the MHB transfer function (dashed blue line). The white dots with blue error bars are the synthetic data points visible in Figure 1. The large resonance in the left plots is the FCN; at this scale the FICN is not visible but is indicated by the vertical orange line. On the right plots, the FCN was removed, and we have zoomed on the FICN frequency band. The green line indicate the transfer function with only the CW to highlight the effect of the FICN resonance.

uncertainty ($\pm 50\%$) which does not even compensate for the bias. The amplitude of the FICN resonance is significantly overestimated as well. These results prove that an uncorrected perturbation as small as the atmospheric one can have an especially strong influence on the estimation of the parameters of the FCN and FICN. Combining the atmosphere with other sources of errors, it is thus difficult to estimate the net effect of these perturbations when inverting real data.

The results of these synthetic tests confirm the high necessity to be particularly cautious when interpreting the parameters of the FCN and, especially, FICN.

4 Joint Inversion

After this systematic work on synthetic data, we can then return to the inversion of actual nutations amplitudes derived from VLBI nutation time series and tidal gravimetric factors estimated from SGs' time series. We will discuss elsewhere the preprocessing of such data and the actual numerical results obtained from their joint inversion. As yet, we rather discuss the potential advantages and pitfalls of such a joint inversion.

The combined inversion of VLBI and gravimetric data is much more complex than a mere numerical combination of each individual resonance parameter. Even if the FCN and, especially, the FICN are usually harder to characterize using gravimetric data, it does not prove a priori that the end result of the inversion will not be improved using both data set simultaneously. The gravimetric factors computed from gravity time series have larger uncertainties than the nutations obtained from VLBI and normalized by the rigid Earth nutations, but they also provide additional data points at some frequencies where the nutations are poorly determined. In addition, the larger uncertainties associated with the gravimetric factors better reflect the actual error made on their estimated value and are thus better suited to the Bayesian inversion, which is very sensitive to the data uncertainty, as stated earlier. Last but not least, because the gravimetric factors are derived from a completely different and independent technique, their joint use with VLBI data may also mitigate systematic errors in the latter.

5 Conclusion

Our synthetic tests suggest that we should be able to determine the FCN period with an uncertainty of at most one day, assuming the actual errors in the data are not much larger than expected. It might be interesting now to investigate whether the small variations of the FCN period from one study to the other, with an order of magnitude of one day or more, are of artificial or physical origin, as some authors have suggested a variability of the FCN frequency [13, 14]. Similarly, we have also found that the FCN quality factor could be probably determined now with an accuracy not larger than 10% (and not better than one percent), although not all recent studies have this level of agreement yet, for some unknown reason.

The FICN is still much harder to properly characterize. As illustrated and quantified by our tests, the resonance associated with this eigenmode is sensitive to most uncorrected perturbations in the inverted data set, and we are far from being able to determine its parameters accurately. Even in the idealistic case where there are no errors in the data, it is difficult to determine the FICN period with an error smaller than 30 days and an uncertainty smaller than 100 days. As soon as there are some uncorrected perturbations in the inverted data, the error can rise up to several hundreds of days. Thus, we might be able to detect and roughly characterize the FICN, but the interpretation of the estimated values for its period and, especially, quality factor must be made with great care. Such a conclusion also holds for the amplitude of the associated resonance and, importantly, for quantities related to the FICN parameters such as the electromagnetic coupling constant introduced by MHB and discussed more recently in [5].

Acknowledgements

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References

1. Crossley, D. J. and Rochester, M. G, 2014, “A new description of Earth’s wobble modes using Clairaut coordinates 2: results and inferences on the core mode spectrum”, Geophys. J. Int., 198, pp. 1890–1905, doi: 10.1093/gji/ggu232.
2. Defraigne, P., Dehant, V., Hinderer, J., 1994, “Stacking gravity tide measurements and nutation observations in order to determine the complex eigenfrequency of the nearly diurnal free wobble”, J. Geophys. Res., 99, pp. 9203–9213, and Correction 100, pp. 2041–2042, doi: 10.1029/94JB00133.
3. Herring, T. A., Gwinn, C. R., Shapiro, I. I., 1986, “Geodesy by Radio Interferometry: Studies of the Forced Nutations of the Earth - 1. Data Analysis”, J. Geophys. Res., 91, pp. 4745–4754, doi:10.1029/jb091ib05p04745.
4. Koot, L., Rivoldini, A., de Viron, O. and Dehant, V., 2008, “Estimation of Earth interior parameters from a Bayesian inversion of very long baseline interferometry nutation time series”, J. Geophys. Res. (Solid Earth), 113, B08414, doi: 10.1029/2007JB005409.
5. Koot, L., Dumberry, M., Rivoldini, A., de Viron, O., Dehant, V., 2010, “Constraints on the coupling at the core-mantle and inner core boundaries inferred from nutation observations”, Geophys. J. Int., 182, pp. 1279–1294, doi:10.1111/j.1365-246x.2010.04711.x.
6. Mathews, P. M., Herring, T. A. and Buffett, B. A., 2002, “Modeling of nutation and precession: New nutation series for nonrigid Earth and insights into the Earth’s interior”, J. Geophys. Res. (Solid Earth), 107, pp. 3–26, doi: 10.1029/2001jb000390.
7. Neuberg, J., Hinderer, J. and Zürn, W., 1987, “Stacking gravity tide observations in central Europe for the retrieval of the complex eigenfrequency of the nearly diurnal free-wobble”, Geophys. J. Int., 91, pp. 853–868, doi: 10.1111/j.1365-246x.1987.tb01671.x.
8. Patil, A., Huard, D. and Fonnesbeck, C., 2010, “PyMC: Bayesian Stochastic Modelling in Python”, J. Stat. Software, 35(4), pp. 1–81, doi: 10.18637/jss.v035.i04.
9. Rogister, Y. and Valette, B. 2009, “Influence of liquid core dynamics on rotational modes”, Geophys. J. Int., 176, pp. 368–388, doi: 10.1111/j.1365-246X.2008.03996.x.
10. Rosat, S., Florsch, N., Hinderer, J. and Llubes, M., 2009, “Estimation of the Free Core Nutation parameters from SG data: Sensitivity study and comparative analysis using linearized least-squares and Bayesian methods”. J. Geodyn., 48, pp. 331–339, doi: 10.1051/0004-6361/200811489.
11. Rosat, S., Lambert, S. B., Gattano, C. and Calvo, M., 2017, “Earth’s core and inner core resonances from analysis of VLBI nutation and superconducting gravimeter data”, Geophys. J. Int., 208, pp. 211–220, doi: 10.1093/gji/ggw378.
12. VanderPlas, J., 2014, “Frequentism and Bayesianism: A Python-driven Primer”, ArXiv e-prints, arXiv: 1411.5018 [astro-ph.IM].
13. Vondrák, J., Ron, C., 2009, “Stability of Period and Quality Factor of Free Core Nutation”, Acta Geodynamica et Geomaterialia.
14. Xu, J., Sun, H., 2009, “Temporal variations in free core nutation period”, Earthquake Science, 22, 4, pp. 331–336, doi:10.1007/s11589-009-0331-8.
15. Zhu, P., Rivoldini, A., Koot, L. and Dehant, V., 2017, “Basic Earth’s Parameters as estimated from VLBI observations”. Geodesy Geodyn., 8, pp. 427–432, doi: 10.1016/j.geog.2017.04.007.