Adjusting Intensive Formal Errors

Christopher Dieck[1], Nathan Secrest[1], Phil Cigan[1]

Abstract Multiple analyses have demonstrated that the reported formal errors of UT1–UTC values determined from different Intensive series understate the true uncertainty of the measurements. If the formal errors of UT1–UTC measurements captured their true uncertainty, then the median formal error would be very close to the dispersion of UT1–UTC residuals with respect to a reference series. For users of VLBI EOP products to properly make use of them, the formal errors must be corrected. We investigate multiple statistical approaches to adjust the UT1–UTC formal errors so that the resulting values reflect the true uncertainty of the measurements of the series.

Keywords UT1, Intensives, Formal Errors

1 Introduction

The most precise and accurate estimates of the Earth's rotation phase are made using the Very Long Baseline Interferometry (VLBI) space-geodetic technique. This phase is expressed as a time, UT1, and is conventionally reported as UT1–UTC, the difference between UT1 and Coordinated Universal Time (UTC), a time scale determined by atomic clocks. The International VLBI Service for Geodesy and Astrometry (IVS) and other organizations, such as the United States Naval Observatory (USNO), provide estimates of UT1–UTC with latencies under 24 hours by running observing sessions referred to as "Intensives." These sessions involve a small number of VLBI stations (typically two) that observe many bright extragalactic radio sources in a short period of time, typically 60–90 minutes. This is in contrast to "24-hour" sessions, such as the `IVS-R1` and `IVS-R4` sessions organized by the IVS, which are 24 hours in duration and include several VLBI stations. The two types of sessions also contrast in that the UT1–UTC estimate latency from a 24-hour session is typically around two weeks and, due to their relatively limited geometry and short durations, Intensives are less precise than 24-hour sessions.

UT1–UTC is highly variable and challenging to predict, and because of this, Intensives hold value due to their low latency. Many applications use UT1–UTC and, in order for them to properly assign uncertainties on their products, they require proper representation of the UT1–UTC measurement uncertainty, historically estimated by the formal error.

Multiple analyses have shown that the reported formal errors of UT1–UTC measurements from VLBI Intensives understate the true uncertainty of those measurements (Böhm et al., 2010; Dieck et al., 2023). This is illustrated by looking at the residuals between UT1–UTC values from Intensive series with respect to a reference series. A statistic describing the dispersion of the residuals, such as the standard deviation (σ_{res}), represents the true precision of the series as a whole. We contend that the median formal error ($Med(FE)$) of the UT1–UTC estimates should also represent the precision of the series as a whole. Thus, if the formal errors were properly stated these two values would be expected to be close.

In this work, we use estimates of UT1–UTC from the `IVS-INT-1` series between the KOKEE (Kk) and WETTZELL (Wz) stations, the `USNO-INT-N` series between the MK-VLBA (Mk) and PIETOWN (Pt) stations of the Very Long Baseline Array (VLBA), and the

1. US Naval Observatory

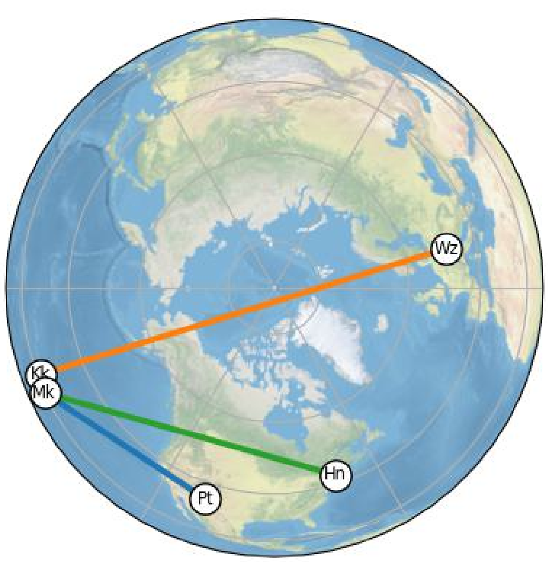

Fig. 1 Illustration of the geometries for the Kk-Wz, Hn-Mk, and Mk-Pt baselines.

`USNO-INT-P` series between the HN-VLBA (Hn) and MK-VLBA stations of the VLBA as determined with the `usn2023c` solution[1] made with the `Calc/Solve` analysis software (Ryan et al., 1980; Ma et al., 1986). Those baselines are shown in Figure 1. The IERS 20 C04 series, provided by the Paris Observatory, is used as the reference series[2]. Comparing the two statistics that describe the precision of each series, shown in Table 1, we see again that $Med(FE)$ is consistently considerably below σ_{res}.

Table 1 The name of the series and its participating baseline, as well as two different statistics describing the series' precision.

Series	Baseline	SD wrt IERS C04 (µs)	Median FE (µs)
`IVS-INT-1`	Kk-Wz	26.4	11.1
`USNO-INT-P`	Hn-Mk	31.4	8.2
`USNO-INT-N`	Mk-Pt	47.9	16.4

There are numerous theories regarding the contributors to this discrepancy, such as the neglect of correlations between parameters during estimation and unaccounted for variation in the tropospheric delay. It is likely that there are many contributors to the variance that are not being accounted for. So far, there is no comprehensive approach that accounts for all of the missing variance, and very few tools have been developed that can be readily and widely incorporated into geodetic VLBI analysis software packages. While work continues on that effort, we wanted to explore statistical methods of adjusting the formal errors of UT1–UTC estimates from Intensives. To aid in adopting any statistical method, it is important that it is relatively simple to implement and can be applied outside of the analysis software. Such approaches have the potential to temporarily overcome the noted discrepancy while methods of improving the determination of formal errors in the estimation process are researched. With these considerations in mind, we explore two methods of adjusting Intensive formal errors.

2 Adjustment Method 1: Add the Missing Variance

The variance observed in the UT1–UTC residuals is not represented in the formal error; some is "missing" from the formal errors. Perhaps the simplest approach is to add the missing variance determined over the whole series to the formal errors of each session. The total variance of a distribution is the sum of the variances due to different factors. Thus,

$$Var_{total} = Var_{reported} + Var_{missing}. \tag{1}$$

Applied to the situation at hand, Var_{total} is simply the variance of the residuals, the median of individual sessions' formal errors—which largely capture only measurement errors—represents $Var_{reported}$, and $Var_{missing}$ represents all of the unaccounted for physical causes of additional noise in a session. In this way

$$Var_{missing} = \sigma_{res}^2 - Med(FE)^2. \tag{2}$$

Interpreting the session formal error (FE) as an uncertainty of one standard deviation, the adjusted formal error of any given Intensive session is then

$$FE_{adjusted} = \sqrt{FE^2 + Var_{missing}}. \tag{3}$$

By construction, adding the missing variance of a series calculated in this way to the formal errors results in a $Med(FE)$ that matches σ_{res}. As an exam-

[1] `https://crf.usno.navy.mil/data_products/RORFD/EOP/EOPI/current/int_last.eopi`

[2] `https://hpiers.obspm.fr/iers/eop/eopc04/eopc04.1962-now`

ple, Figure 2 shows the residuals and formal errors of the USNO-INT-P series, both before and after adjustment. As a consequence of adding in quadrature a single value to all of the formal errors in a series ($Var_{missing} = 918.7\ \mu s^2$ for Hn-Mk), there is now a floor to the distribution of formal errors. Most formal errors now clump around the value of the square root of the missing variance (30.3 μs), with only those that had relatively high formal errors before still rising above the grouping, though this effect is accentuated for this series relative to the others because the difference between σ_{res} and $Med(FE)$ is so large.

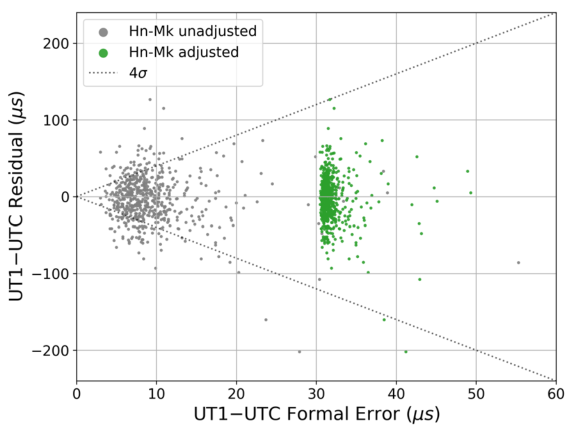

Fig. 2 Hn-Mk UT1–UTC residuals compared to their original formal errors in gray and their adjusted formal errors in green. The dashed lines denote where the normalized residual is ±4.

Before the adjustment, many sessions had normalized residuals (calculated by dividing the residual of a UT1–UTC estimate by its formal error) greater than 4. Following adjustment, with all the sessions having lower normalized residuals, that number is greatly reduced. This is illustrated in Figure 3 which depicts histograms of the normalized residuals of the USNO-INT-P series both before and after adjustment alongside the standard normal distribution.

After adjustment, the histogram resembles the $\mathcal{N}(0,1)$ distribution. This makes intuitive sense in that the UT1–UTC estimate residual of any one session is drawn from $\mathcal{N}(0,\sigma)$, given the formal error, σ, of the session and assuming that the formal error is describing the standard deviation of a normal distribution. Normalizing the residual thus makes it drawn from $\mathcal{N}(0,1)$, and the distribution of a series of UT1–UTC estimate residuals would approach $\mathcal{N}(0,1)$ with additional sessions. Though the distribution of normalized residuals after adjustment with this method of adding variance resembles $\mathcal{N}(0,1)$, it has an overabundance of normalized residual values close to zero and a general deficiency of normalized residual values in the sides and toward the tails. A method that gives a distribution of normalized residuals that is closer to $\mathcal{N}(0,1)$ may give more realistic formal errors.

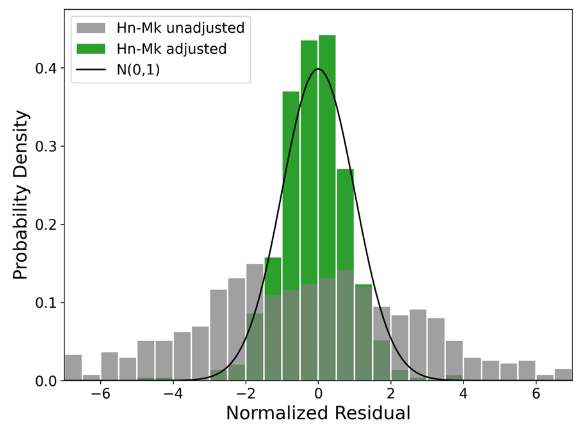

Fig. 3 Histogram of the normalized residuals of the Hn-Mk Intensive sessions prior to adjustment shown in grey and after adjustment shown in green, overlaid with the standard normal distribution.

Relative to an unadjusted series, this approach provides a more realistic view of the precision of a series, but not necessarily for individual sessions, so it may not fully satisfy the goal of providing downstream users a proper representation of the uncertainty of an Intensive's UT1–UTC estimate. One of the drawbacks to this approach is that the missing variance is calculated from all the sessions of the series. The missing variance could be updated and re-applied at a regular interval, but it would still be calculated from all sessions. Thus, whenever this adjustment factor is calculated and applied, it has no time dependence which could take into account seasonal variations due to available target sources, weather patterns, etc. Also, as the series gets longer and more sessions are used in the calculation of the missing variance, the marginal impact of another session on that value will be small. A method that can both result in a standard normal distribution of the normalized residuals and vary the adjustment factor with time is desirable.

3 Adjustment Method 2: Bayesian Update

Updating the probability that a UT1–UTC estimate is correct (i.e., modifying its formal error) through Bayesian inference is a method that has the potential to both provide a time-varying adjustment factor and result in a standard normal distribution of the normalized residuals. The method works by incorporating new information about previous estimates as it becomes available. With many sessions already observed in a series, there is a lot of previous information that can be used to update the uncertainties of all sessions and, in particular, the next session observed.

Bayes' Theorem restated for the purposes of this application is

$$P(FE|D) \propto \mathcal{L}(FE|D) \cdot P(FE), \tag{4}$$

where $P(FE|D)$ is the posterior probability distribution of the formal error value given the data, D, $\mathcal{L}(FE|D)$ is the likelihood of the formal error value given the data, and $P(FE)$ is the prior probability distribution of the formal error value. The new formal error is the value of the formal error at the maximum of the posterior distribution. The likelihood is set by the observed dispersion of the UT1–UTC residuals. For any given session, the prior distribution is the posterior distribution of the previous session convolved with a Gaussian to control the correlation between the formal errors of nearby sessions. The form of the Gaussian is tuned for each series during the first several sessions of the series, the burn-in period, and then kept the same for the rest of the sessions. It is tuned so that the predicted dispersion of the residuals matches the observed dispersion of the residuals.

After applying this to the Intensive series, this approach does indeed provide what we had desired. The formal error adjustment factor varies with time as shown in Figure 4, and the normalized residuals form a distribution very close to $\mathcal{N}(0,1)$. Unlike with the method of adding missing variance, the resulting adjusted formal errors, shown in Figure 5, span a large range of values. However, the underlying physical reasons for the added uncertainty have yet to be determined.

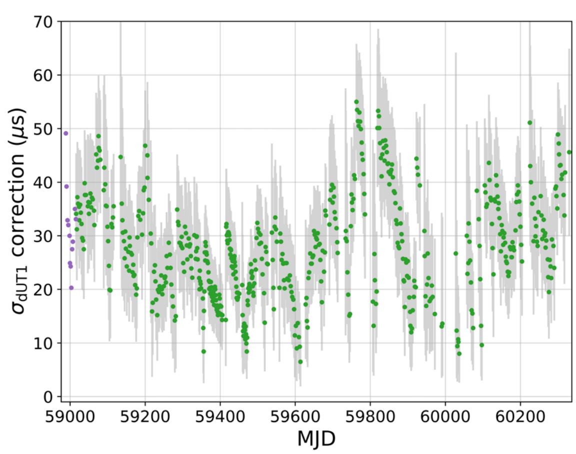

Fig. 4 For the `USNO-INT-P` series, the adjustment factors, in green, to be added in quadrature to a session's original formal error. The purple points represent the burn-in period where the tuning is established for the series. The gray bands denote the 32–68% credible interval.

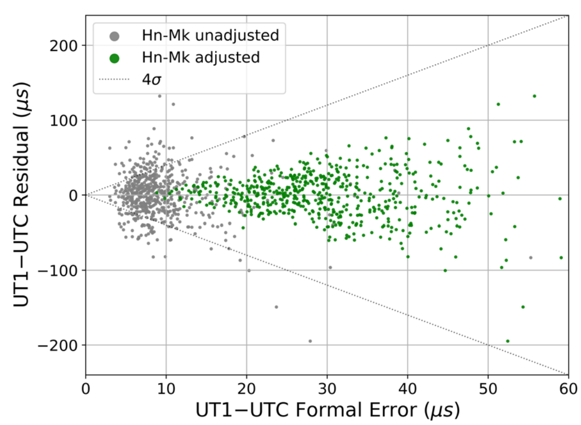

Fig. 5 Hn-Mk UT1–UTC residuals compared to their original formal errors in gray and their formal errors adjusted through the Bayesian update method in green. The dashed lines denote where the normalized residual is ±4.

4 Conclusions and Future Work

The formal errors of the UT1–UTC estimates in the `IVS-INT-1`, `USNO-INT-P`, and `USNO-INT-N` series calculated by the `Calc/Solve` analysis software in the `usn2023c` solution currently overestimate the precision of the whole series by understating the true uncertainty of individual estimates. This issue is not limited to `Calc/Solve` or the USNO VLBI AC.

A likely solution to this problem is to incorporate more physical information into the estimation process and take into account off-diagonal elements of the variance-covariance matrix. It will take considerable time and effort to research the physical processes affecting the uncertainty and ensure that it is properly accounted for in the uncertainties. In an effort to address the problem more quickly, we explored the possibility of using statistical methods to adjust the formal errors of UT1–UTC estimates from Intensive sessions.

Based on this preliminary work, we conclude that it is possible to use statistical techniques to adjust individual Intensive formal errors so that, in aggregate, the formal errors align with the dispersion of the UT1–UTC measurement residuals. While the method of adding the missing variance brings the median formal error to the standard deviation of the residuals, it is inflexible to seasonal changes in the uncertainty, and the resulting distributions of formal errors and normalized residuals are too narrow. The Bayesian update method is certainly an improvement over adding the missing variance in that the adjustment value is time-dependent. It also does better with regards to the distributions of the formal errors and normalized residuals. This is not a surprise though, as it was designed to have the resulting formal error values create a standard normal distribution of the normalized residuals.

The techniques involve modifying the formal errors of a series after UT1–UTC estimates have been made, and the techniques are thus readily applied by any VLBI Analysis Center to any Intensive series. The principal downside to these approaches is that there is no longer a clear physical interpretation of why a particular session has the formal error that it does. Another shortcoming in their current forms is that the residuals are calculated with respect to the IERS 20 C04 series, the most recent value of which lags 30 days behind the present. To make use of the Bayesian update technique in particular, some method of determining the adjustment factor for an Intensive made in the present needs to be developed. This could be done with a reference series that continues through the present, but that has the potential to be problematic if the USNO VLBI AC series are included in the sources of data from which that reference series is developed. Finally, the validation of both of the methods to adjust the formal errors is based on the assumption that the distribution of normalized residuals should approach $\mathcal{N}(0,1)$, and the Bayesian Update method is tuned toward this same distribution. This assumption requires mathematical validation.

As the physical sources of the missing variance are identified and included, thus bringing the formal errors closer to a reasonable value of the uncertainty, these approaches would still work. In such a case, the adjustment would simply be decreased in magnitude. In this way, any statistical method used to adjust the formal error values would be phased out as the formal errors determined as part of the UT1–UTC estimation process account for an increasing amount of the variance in the estimates, fulfilling the goal of using a statistical method to overcome the current discrepancy between the dispersion of UT1–UTC residuals and their formal errors.

References

1. J. Böhm, T. Hobiger, R. Ichikawa, T. Kondo, Y. Koyama, A. Pany, H. Schuh, and K. Teke. "Asymmetric Tropospheric Delays from Numerical Weather Models for UT1 Determination from VLBI Intensive Sessions on the Baseline Wettzell-Tsukuba." Journal of Geodesy 84, no. 5 (May 2010): 319–25. https://doi.org/10.1007/s00190-010-0370-x.
2. C. Dieck, M. C. Johnson, and D. S. MacMillan. "The Importance of Co-Located VLBI Intensive Stations and GNSS Receivers." Journal of Geodesy 97, no. 3 (March 3, 2023): 21. https://doi.org/10.1007/s00190-022-01690-1.
3. C. Ma, T. A. Clark, J. W. Ryan, T. A. Herring, I. I. Shapiro, B. E. Corey, H. F. Hinteregger, et al. "Radio-Source Positions from VLBI." The Astronomical Journal 92 (November 1, 1986): 1020–29. https://doi.org/10.1086/114232.
4. G. Petit and Brian Luzum. "IERS Conventions (2010)." IERS Technical Note 36 (January 1, 2010): 1.
5. J. W. Ryan, C. Ma, and B. P. Schupler. "Mark 3 Interactive Data Analysis System." In Radio Interferometry: Techniques for Geodesy. Cambridge, MA, 1980. https://ntrs.nasa.gov/citations/19800020331.