Updated Estimation of the Free Core and Inner-Core Nutation Resonance Parameters Using 2023 VLBI Solutions

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Abstract The elliptical fluid core and solid inner core undergo rotational wobbles within the rotating Earth. Under tidal forcing, the presence of these rotational modes induces resonances in the Earth's nutations as observed by VLBI. The strengths and frequencies of these Free Core Nutation (FCN) and Free Inner Core Nutation (FICN) resonances depend on a few fundamental geophysical parameters of the Earth's deep interior. Since the first estimates of the FCN and FICN resonance parameters in 2000, nutation measurements by VLBI have accumulated with a better accuracy due to a number of improvements in the modeling of the delay, including a more reliable celestial reference frame with a highest number of reference points of high precision. We propose an updated determination of the FCN and FICN resonance parameters using a Bayesian inversion of the most recent VLBI solutions. Various nutation series are analyzed to retrieve a combined solution with consistent uncertainties. We obtain estimates of the FCN resonant period and quality factor that are in agreement with previous estimates. The FICN resonant parameters are also deduced from the coupling constant at the inner core boundary. The latter is, however, much less constrained, resulting in a large uncertainty in the value of the FICN frequency.

Keywords free core nutation, free inner core nutation, VLBI

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1 Introduction

Knowledge of the Earth's deep interior is limited by the weakness of the signals used to probe deep inside. The resonances associated with the Earth's normal modes are, however, helpful for evaluating geophysical parameters of the deep interior. The elliptical fluid core and solid inner core in the rotating elliptical Earth undergo rotational wobbles that induce resonances under the tidal forcing. These Free Core Nutation (FCN) and Free Inner Core Nutation (FICN) are rotational modes whose strengths and frequencies depend on a few fundamental geophysical parameters related to the coupling existing at the core boundaries. While the resonance associated with the FCN has been clearly observed and precisely analyzed using VLBI (Very Long Baseline Interferometry), measurements of nutation (e.g., [1]), or both space nutation and surface gravity data [8], no FICN resonance has ever been detected at a comfortable level [6]. The inversion of the geophysical parameters from the observed VLBI nutations was performed by [4]. Their model, referred to as MHB, was adopted by the International Astronomy Union (IAU) to represent the Earth's response to tidal forcing. Later, [3] re-estimated the values of some of these parameters using a Bayesian inversion of the VLBI nutation in the time domain. Since the MHB work, one has accumulated 20 years of VLBI observations. In this work, we present a re-analysis of VLBI nutation observations to evaluate the FCN and FICN resonance parameters. We particularly check the sensitivity of the nutation terms to the FICN.

2 Data

We used time series of nutation offsets to the MHB model from four Analysis Centers of the International VLBI Service for Geodesy and Astrometry (IVS, [5]): usn2023a, bkg2023a, gsf2023a, and iaa2021a. These series were obtained using state-of-the-art analysis of the complete (or almost complete) VLBI observational data base since 1979. Though the USNO, BKG, and GSFC series were computed using the traditional Calc/Solve VLBI analysis software package of the Goddard Space Flight Center, the IAA series was obtained through a different network and independent package (QUASAR and the OCCAM/GROSS software packages [7]). The analysis configuration, which is the choice of the analyst, may also differ from one series to another (e.g., frequency of troposphere parameter estimation, clock offset modeling, exclusion of some data), producing relatively small differences between the final series. The dispersion of these differences constitutes an empirical error often more representative than the standard error derived from the least-squares inversion. Although one cannot say that the four series are truly independent, the multiplicity of the nutation series will therefore be of interest hereafter to assess the robustness of the results. A hybrid series is also obtained by averaging these four series.

Nutation amplitudes were then estimated for each of the five series as in [1]. In particular, we fitted the amplitudes of a set of corrections to the 21 prograde and retrograde nutations used by the MHB authors to fit their geophysical model. These 42 values will be used in the next section to adjust the resonance parameters.

3 Analysis

We formulate the reduced resonance function [4] as

$$\delta = \frac{e-\sigma}{1+e}\left[1+(1+\sigma)\left(\frac{\tilde{N}_1}{\sigma-\tilde{\sigma}_1}+\frac{\tilde{N}_2}{\sigma-\tilde{\sigma}_2}+\frac{\tilde{N}_3}{\sigma-\tilde{\sigma}_3}\right)\right]$$

where we have ignored the Inner Core Wobble. The σ frequencies are normalized by the mean rotation rate of the Earth [4] and expressed in cpsd (cycles per sidereal days). The eigenfrequencies of the Chandler Wobble (CW), FCN, and FICN rotational modes are:

$$\tilde{\sigma}_1 = \frac{A}{A_m}(e-\tilde{\kappa}),$$
$$\tilde{\sigma}_2 = -\frac{A}{A_m}\left(e_f-\tilde{\beta}+K^{CMB}+\frac{A_s}{A_f}K^{ICB}\right)-1,$$
$$\tilde{\sigma}_3 = \left(\alpha_2 e_s+\nu-K^{ICB}\right)-1,$$

where A, A_m, A_f, and A_s are the mean equatorial moments of inertia of the Earth, the mantle, the fluid outer core, and the solid inner core, respectively. The flattenings e_f and e_s are relevant to the fluid and solid inner cores. The compliance $\tilde{\kappa}$ expresses the deformability at the surface under degree-2 tidal forcing, and the compliance $\tilde{\beta}$ characterizes the deformability of the core-mantle boundary (CMB) under the centrifugal force. K^{CMB} and K^{ICB} are the complex coupling strength parameters representing the influence of the visco-magnetic torques at the CMB and inner-core boundary (ICB), respectively. α_2 is related to the strength of the gravitational torque acting on the inner core, and ν is very small because it is proportional to the fluid core viscosity ([4]). In the following, we use $\Re$ and $\Im$ to respectively designate real and imaginary parts of a complex quantity. In this work, $e_f+\Re(K^{CMB})$, $\Im(K^{CMB})$, $\Re(K^{ICB})$, and $\Im(K^{ICB})$ are the four parameters we invert, other parameters being fixed to the MHB values [4].

We first compute the sensitivity of the 42 nutation terms as used in [4] to these parameters by varying 2,000 times each model parameter value noted m and computing the relative difference in nutation amplitudes between the MHB value d_0 and the new value d (real and imaginary parts) as $\frac{|d-d_0|}{\sigma(m)}$, where σ is the standard deviation of the parameter values. We represent then the logarithm of this relative difference in Figure 1. We finally use the Metropolis-Hastings algorithm to sample the distributions in the Markov-chain Monte-Carlo Bayesian inversion [6, 8] of the 42 nutation terms determined from the four individual series and for the hybrid series. Because the hybrid series provides similar results to the inversion of each individual series, we only give the results for the mean series. We do not assume that the coupling at the CMB and ICB are only due to the visco-magnetic torques so that, in a first step, we do not impose the positivity of $\Re(K^{ICB})$ in the inversion as was done in previous works [3, 4]. In a second step, we impose $\Re(K^{ICB}) > 0$. Results of the inversion are compared with values from previous studies in Table 1, and the obtained model (in the case

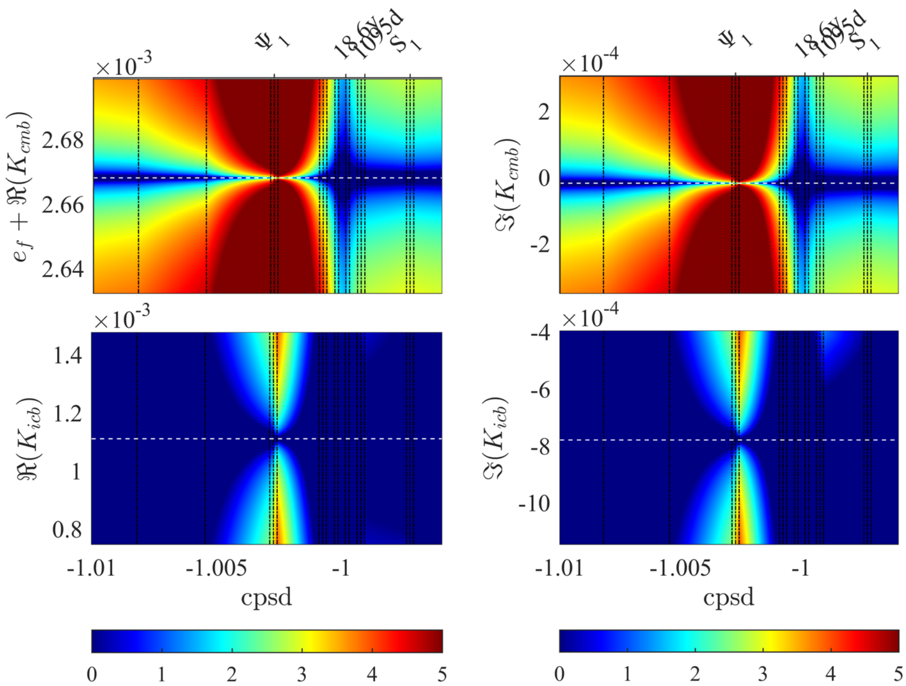

Fig. 1 Sensitivity of nutation terms (vertical black lines, labels indicate the 18.6-yr and 1,095-day nutation terms as well as the Ψ_1 and S_1 tides respectively corresponding to the retrograde annual and prograde annual nutation terms) to the Earth's coupling parameters at the core boundaries. Relative amplitude of the sensitivity is in logarithm and with respect to MHB nutation amplitudes. Frequencies are in cycles per sidereal days (cpsd). Coupling parameter values for MHB are indicated by the horizontal white lines.

Table 1 Inversion results for the Earth's boundary coupling parameters, for the FCN and FICN parameters (period in sidereal days and Q) from the mean series (without and with imposing $\Re(K^{ICB}) > 0$) compared with the MHB values and with the values from previous studies ([3, 8]).

Authors	$e_f + \Re(K^{CMB})$ $\times 10^{-3}$	$\Im(K^{CMB})$ $\times 10^{-5}$	$\Re(K^{ICB})$ $\times 10^{-3}$	$\Im(K^{ICB})$ $\times 10^{-3}$	T_{FCN} days	Q_{FCN}	T_{FICN} days	Q_{FICN}
This work	2.6834	-0.90	-2.96	-1.08	-429.4	27655	197	450
	$\pm 3 \times 10^{-4}$	± 0.04	± 0.01	± 0.03	± 0.1	± 1044	± 0.6	± 20
This work $\Re(K^{ICB}) > 0$	2.6522	-1.12	1.59	-0.70	-429.8	29000	6000	900
	$\pm 3 \times 10^{-3}$	± 0.1	± 0.4	± 0.3	± 1.0	± 5000	± 5000	± 400
MHB [4]	2.6680	-1.85	1.11	-0.78	-430.2	20000	1035	677
	$\pm 2 \times 10^{-3}$	± 0.15	± 0.1	± 0.14	± 0.3	± 600	± 105	± 56
Koot et al. (2010) [3]	2.6753	-1.78	1.01	-1.09	-429.05	19741	924	464
	$\pm 2 \times 10^{-4}$	± 0.02	± 0.02	± 0.03	± 0.1	± 328	± 33	± 24
Ziegler et al. (2020) [8]	–	–	–	–	-430.0	16200	900	–
	–	–	–	–	± 0.2	± 500	± 400	–

where we do not impose $\Re(K^{ICB}) > 0$) is plotted together with nutation observations in Figure 2.

4 Results and Conclusion

Sensitivity to the coupling parameters at the ICB is elusive outside the FCN resonance (Figure 1). The semi-annual prograde nutation (P_1 tide) constrains the FICN resonance to be around 200 days (Figure 2). Using older datasets (1984–2015), [6], had shown that the annual prograde nutation (S_1 tide) forced the FICN resonance to be between 362 and 414 days. Indeed, the semi-annual prograde nutation (P_1 tide) is among the nutation terms with the smallest uncertainty (Figure 2), hence having a strong weight in the inversion. The estimation of this term is quite consistent between the

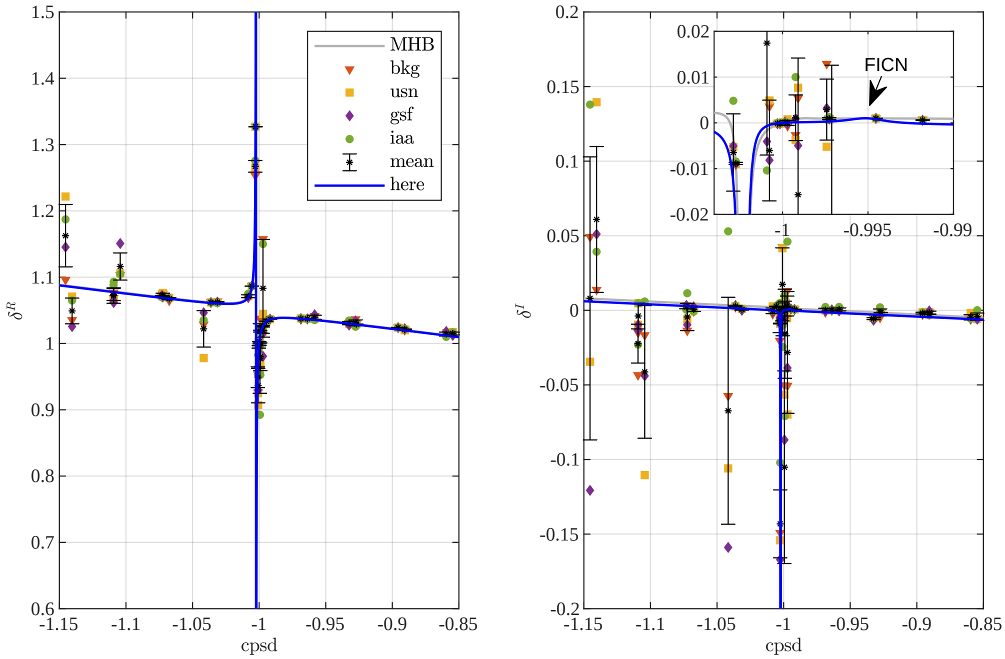

Fig. 2 Observed and modeled (gray: MHB, blue: this study) resonance functions (real and imaginary parts) for 42 nutation terms from the four individual VLBI series considered here and from the mean series. Frequencies are in cycles per sidereal days (cpsd).

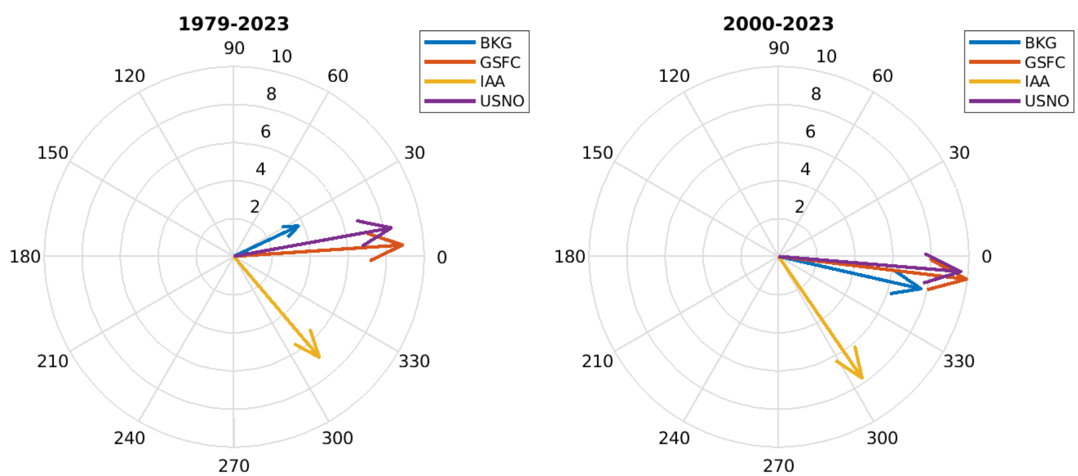

Fig. 3 Amplitude and phase of P_1 (prograde semi-annual nutation) estimated by the four Analysis Centers between (left) 1979–2023 and (right) 2000–2023.

various Analysis Centers, with little more differences between the Analysis Centers when considering only the time period after 2000 (1979–2023 vs. 2000–2023, see Figure 3).

Previous studies assumed $\Re(K^{ICB}) > 0$ [3, 4]. Our best solution is obtained when $\Re(K^{ICB}) < 0$, which disagrees with the definition of the visco-magnetic coupling strength that should be positive. The influence of the inner core obliquity could also play a role. But it enters Equation 1 as a negative quantity [2], so it contributes to the FICN frequency in the same way as $\Re(K^{ICB})$ and then could not make an apparent $\Re(K^{ICB})$ negative. One possible interpretation for a negative $\Re(K^{ICB})$ is that other torques compensate visco-magnetic coupling at the ICB. The preferred conclusion is that we cannot constrain the FICN resonance given the current precision on prograde nutation components, as already shown by the sensitivity analysis (Figure 1) and given the large uncertainties on prograde nutation terms.

Simulations are needed to evaluate what should be improved in the VLBI observation analyses to reach the best precision in the determination of the prograde nutation terms and having a chance to constrain the FICN resonance from nutation observations by VLBI.

Acknowledgements

We would like to thank all the VLBI communities for acquiring, preparing, and sharing the data processed and inverted in this study. This project was financially supported by CNES (Centre National d'Etudes Spatiales).

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